Omnidimensional Hypercomplex Spectral System (OHSS-17)

Pre-Spacetime Foundational and the Minimal Relational Architecture Before Spacetime

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1 Primitive Structural Fluidity and the Minimal Relational Architecture Before Spacetime

1.1 Scope of the Primitive Layer

The primitive layer is prior to spacetime, prior to geometry, prior to physical units, and prior to any effective algebraic realization. It does not contain space, time, distance, metric, curvature, mass, energy, momentum, field, probability amplitude, observer, measurement apparatus, physical phase, charge, spin, or force.

The primitive layer is a state of structural fluidity. By structural fluidity we mean that the structure has not yet crystallized into fixed physical constants, metric scale, spacetime coordinates, material fields, quantum states, or observed bodies. What exists at this level is only primitive distinguishability, relational generability, closure admissibility, and central readability conditions.

Thus the primitive layer is not described by a physical manifold, not by a metric space, not by a Hilbert space, not by a field bundle, and not by a pre-given algebra. It is described only by a finite relational architecture:

$$\mathfrak{U}_{\mathrm{pre}} = \Big(\mathcal{A}, \mathcal{R}_{\mathrm{pre}}, \mathfrak{i}, \mathfrak{k}, \mathcal{L}, \mathrm{Reach}, \mathrm{Cl}, \mathrm{Couple}\Big). \tag{1}$$

Here $\mathcal{A}$ is the primitive carrier set, $\mathcal{R}_{\mathrm{pre}}$ is the primitive relational state-space generated from the carriers, $\mathfrak{i}$ is the direct-readability condition, $\mathfrak{k}$ is the mediated-closure condition, $\mathcal{L}$ is the primitive relational state-shift, Reach is the primitive relational reach function, Cl is a primitive binary closure operation on admissible objects, and Couple records whether two primitive classes are allowed to couple operationally after generation.

None of the symbols in (1) denotes a physical quantity.

1.2 Primitive Carriers and Their Non-Geometric Meaning

The primitive layer begins with a finite set of distinguishable carriers:

$$\mathcal{A} = \{a_1, a_2, a_3\}. \tag{2}$$

The elements a_1, a_2, a_3 are not spatial axes, not coordinates, not dimensions, not physical particles, not fields, and not basis vectors of an already-given algebra. They are primitive distinguishable carriers.

The first requirement is pairwise distinguishability:

$$a_i \not\equiv a_j \qquad \text{for all } i \neq j. \tag{3}$$

The second requirement is primitive non-derivability:

$$a_k \notin \mathrm{Gen}(a_i, a_j) \qquad \text{for every distinct } i, j, k \in \{1, 2, 3\}. \tag{4}$$

Equation (4) states that no primitive carrier is generated from the other two at the primitive level. This prevents the carrier set from collapsing into a smaller generating family.

The third requirement is non-geometric independence:

$$\mathrm{Ind}_{\mathrm{pre}}(a_1, a_2, a_3) = 1. \tag{5}$$

The predicate $\mathrm{Ind}_{\mathrm{pre}}$ is not a determinant, not a rank, not an angle condition, and not a metric orthogonality condition. It means only that the primitive carriers are not reducible to each other or to a smaller carrier family.

1.3 Why the Primitive Carrier Number Is Exactly Three

The number of primitive carriers is not chosen to match the observed number of spatial dimensions. The primitive layer contains no space. The number three is forced by minimal relational self-generation.

A one-carrier layer has the form

$$\mathcal{A}_1 = \{a_1\}. \tag{6}$$

This layer cannot internally generate distinction:

$$|\mathcal{A}_1| = 1 \Longrightarrow \text{no internal primitive contrast.} \tag{7}$$

A two-carrier layer has the form

$$\mathcal{A}_2 = \{a_1, a_2\}. \tag{8}$$

It can generate contrast:

$$a_1 \not\equiv a_2. \tag{9}$$

However, it cannot generate a closed relational architecture without importing an additional closure rule from outside the layer:

$$\mathcal{A}_2 \Longrightarrow \text{contrast only} \quad \text{but not self-contained relational closure.} \tag{10}$$

Thus a two-carrier layer has the form

$$\mathcal{A}_2 + \text{external closure rule.} \tag{11}$$

A primitive layer that requires an external closure rule is not self-generating. Therefore $|\mathcal{A}| = 2$ is structurally insufficient.

A four-carrier primitive layer would have the form

$$\mathcal{A}_4 = \{a_1, a_2, a_3, a_4\}. \tag{12}$$

This is not minimal, because the first three carriers already generate non-collapsing relational plurality without requiring an external closure rule. The fourth carrier either satisfies

$$a_4 \in \mathrm{Gen}(a_1, a_2, a_3), \tag{13}$$

in which case it is not primitive, or satisfies

$$a_4 \notin \mathrm{Gen}(a_1, a_2, a_3), \tag{14}$$

in which case it introduces an additional primitive independence not required by the minimal closure problem.

Therefore a fourth primitive carrier breaks minimality:

$$|\mathcal{A}| = 4 \Longrightarrow \text{non-minimal primitive carrier layer.} \tag{15}$$

The minimal carrier layer satisfying primitive distinguishability, non-collapsing relational plurality, and closure potential without an external rule is therefore

$$|\mathcal{A}| = 3. \tag{16}$$

This establishes $\mathcal{A} = \{a_1, a_2, a_3\}$ as the minimal primitive carrier set. A fourth primitive carrier would not refine the primitive architecture; it would add an unrequired primitive degree and therefore break the minimal relational logic.

1.4 Relational Roots Generated by the Three Carriers

Relations are not added as primitive carriers. They are generated as relational offspring of the carrier set.

The elementary unordered carrier-pair roots are

$$\mathcal{P}_2(\mathcal{A}) = \Big\{\{a_1, a_2\}, \{a_2, a_3\}, \{a_3, a_1\}\Big\}. \tag{17}$$

Since $|\mathcal{A}| = 3$, the number of elementary pair-roots is

$$|\mathcal{P}_2(\mathcal{A})| = \binom{3}{2} = 3. \tag{18}$$

We denote these three primitive relational roots by

$$\rho_1 = \{a_1, a_2\}, \qquad \rho_2 = \{a_2, a_3\}, \qquad \rho_3 = \{a_3, a_1\}. \tag{19}$$

The ρ_i are not products of carriers. They are not algebraic multiplications. They are not physical interactions. They are relational roots generated by the coexistence of the three primitive carriers.

Hence the primitive distinction is

$$\text{primitive carrier} \neq \text{relational offspring}. \tag{20}$$

If relational offspring were treated as additional primitive carriers, the primitive layer would cease to be minimal. If they were treated as algebraic products, the primitive layer would already presuppose an algebra. Both options are excluded.

1.5 The Direct-Readability and Mediated-Closure Distinction

A primitive relation cannot be merely binary if the architecture is required to distinguish direct readability from mediated closure.

We impose the structural distinction

$$\text{direct readability} \neq \text{mediated closure}. \tag{21}$$

This is not a physical distinction. It does not refer to an observer, measurement apparatus, time, force, or field. It states only that a primitive relational state may close in a directly readable way or may enter a mediated closure condition that must return to readability under a second closure.

A two-state cycle

$$\mathbb{Z}_2 = \{0, 1\} \tag{22}$$

cannot support two distinct closure positions. It gives one state and one complement, but not direct readability and mediated closure as distinct closure statuses.

A three-state cycle

$$\mathbb{Z}_3 = \{0, 1, 2\} \tag{23}$$

is non-binary, but it does not provide two closure positions separated by transition positions. It therefore cannot support a balanced primitive distinction between direct readability, transition, mediated closure, and return-correction.

The smallest cycle that contains two distinct closure positions and two transition positions is

$$\mathbb{Z}_4 = \{0, 1, 2, 3\}. \tag{24}$$

Thus

$$4 = \min \{n : \mathbb{Z}_n \text{ contains two distinct closure positions and two transition positions}\}. \tag{25}$$

The four-state cycle is therefore not an arbitrary numerical choice. It is the minimal closure cycle compatible with the distinction in (21).

1.6 The Four-State Relational Cycle

For each primitive relational root ρ_i, define four relational states:

$$\rho_i^{(m)} \qquad i \in \{1, 2, 3\}, \quad m \in \mathbb{Z}_4. \tag{26}$$

Let $\mathcal{L}$ be the primitive relational state-shift:

$$\mathcal{L}\rho_i^{(m)} = \rho_i^{(m+1 \bmod 4)}. \tag{27}$$

The internal four-state closure is

$$\mathcal{L}^4 \rho_i^{(m)} = \rho_i^{(m)}. \tag{28}$$

The four states have structural roles only:

$$m = 0 \iff \text{direct readability state,} \tag{29}$$

$$m = 1 \iff \text{transition state,} \tag{30}$$

$$m = 2 \iff \text{mediated closure state,} \tag{31}$$

$$m = 3 \iff \text{return-correction state.} \tag{32}$$

These states are not physical phases, not temporal steps, not angles, and not motions. They are primitive relational closure states.

1.7 Emergence of the Twelve Relational States

The primitive relational state-space is defined by

$$\mathcal{R}_{\text{pre}} = \left\{ \rho_i^{(m)} : i \in \{1, 2, 3\},\ m \in \mathbb{Z}_4 \right\}. \tag{33}$$

Therefore

$$|\mathcal{R}_{\text{pre}}| = 3 \times 4 = 12. \tag{34}$$

The twelve states are not externally added. They are generated by the three primitive carriers through their three relational roots and the minimal four-state closure cycle.

The generation chain is

$$\mathcal{A} \Longrightarrow \mathcal{P}_2(\mathcal{A}) \Longrightarrow \mathcal{R}_{\text{pre}}. \tag{35}$$

Equivalently,

$$\text{3 primitive carriers} \Longrightarrow \text{3 primitive relational roots} \Longrightarrow \text{12 primitive relational states.} \tag{36}$$

At this level, the twelvefold set $\mathcal{R}_{\text{pre}}$ is not a physical rim, not a quantum sector, not a phase space, not a field sector, and not an internal symmetry space. It is only a primitive relational state-space.

1.8 Non-Central Primitive Class and Self-Return

Before any effective algebraic realization is introduced, the primitive structure contains two kinds of non-central objects.

The first kind is the primitive carrier:

$$a_i \in \mathcal{A}. \tag{37}$$

The second kind is the generated relational state:

$$\rho_i^{(m)} \in \mathcal{R}_{\text{pre}}. \tag{38}$$

Although these two kinds are not identical, neither of them is a central condition. Therefore both belong to the same primitive non-central class:

$$\mathcal{N}_{\text{pre}} = \mathcal{A} \cup \mathcal{R}_{\text{pre}}. \tag{39}$$

The central conditions are excluded from this class:

$$\mathfrak{i} \notin \mathcal{N}_{\text{pre}}, \qquad \mathfrak{k} \notin \mathcal{N}_{\text{pre}}. \tag{40}$$

Every element of $\mathcal{N}_{\text{pre}}$ possesses a primitive self-return status:

$$x \in \mathcal{N}_{\text{pre}} \Longrightarrow \text{Self}(x) \text{ is defined.} \tag{41}$$

This self-return status is not a metric square, not a norm, not a physical length, not a probability, and not an algebraic product. It states only that a non-central primitive object, when reflected into itself, does not generate a new primitive carrier, does not generate a new relational root, and does not become a central condition by itself.

Thus, for all $x \in \mathcal{N}_{\text{pre}}$,

$$\text{Self}(x) \notin \mathcal{A}, \qquad \text{Self}(x) \notin \mathcal{R}_{\text{pre}}, \qquad \text{Self}(x) \notin \{\mathfrak{i}, \mathfrak{k}\}. \tag{42}$$

The self-return class of non-central objects is structurally distinct from the mediated closure-return class:

$$\text{Self}(x) \neq \mathfrak{k} \circ \mathfrak{k} \qquad \text{for all } x \in \mathcal{N}_{\text{pre}}. \tag{43}$$

No sign is assigned at the primitive level:

$$\text{Self}(x) \text{ has no positive or negative sign at the primitive level.} \tag{44}$$

Equation (43) is the primitive source of a later signed separation between non-central self-return and mediated central closure return. The sign itself can appear only after an effective algebraic realization is introduced.

1.9 Central Conditions Are Not Primitive Carriers

The primitive structure requires two central conditions, but these are not primitive carriers and are not relational states.

The first is the direct-readability condition:

$$\mathfrak{i}. \tag{45}$$

The second is the mediated-closure condition:

$$\mathfrak{k}. \tag{46}$$

They are not members of the primitive carrier set:

$$\mathfrak{i} \notin \mathcal{A}, \qquad \mathfrak{k} \notin \mathcal{A}. \tag{47}$$

They are not relational offspring:

$$\mathfrak{i} \notin \mathcal{R}_{\mathrm{pre}}, \qquad \mathfrak{k} \notin \mathcal{R}_{\mathrm{pre}}. \tag{48}$$

The direct-readability class is represented by the $m = 0$ state:

$$m = 0 \Longrightarrow \mathfrak{i}. \tag{49}$$

The mediated-closure class is represented by the $m = 2$ state:

$$m = 2 \Longrightarrow \mathfrak{k}. \tag{50}$$

The mediated-closure condition returns to direct readability under second closure:

$$\mathfrak{k} \circ \mathfrak{k} = \mathfrak{i}. \tag{51}$$

Equation (51) is not a physical equation and not an algebraic multiplication law. It is a primitive closure-return condition.

It prevents two failures. First, mediated closure must not generate an infinite sequence of new central conditions:

$$\mathfrak{k} \longmapsto \mathfrak{k}_1 \longmapsto \mathfrak{k}_2 \longmapsto \mathfrak{k}_3 \longmapsto \cdots . \tag{52}$$

Second, mediated closure must not collapse into nullity:

$$\mathfrak{k} \circ \mathfrak{k} \neq \varnothing. \tag{53}$$

The minimal non-degenerate return condition is therefore (51).

This is the primitive source of a later hyperbolic closure image. At the primitive level it is only a closure-return condition, not yet an algebraic square.

1.10 No Carrier-Offspring Coupling

The relation between a primitive carrier and its relational offspring is generative, not interactive.

The generative relation is allowed:

$$\mathcal{A} \Longrightarrow \mathcal{R}_{\mathrm{pre}}. \tag{54}$$

Direct carrier-offspring coupling is forbidden:

$$\mathrm{Couple}\Big(a_j, \rho_i^{(m)}\Big) = \varnothing \qquad \text{for all } j, i, m. \tag{55}$$

This is the No Carrier-Offspring Coupling Principle.

Its meaning is not that the offspring has no origin in the carrier. Its meaning is that a carrier does not directly interact with the relational state once that state has been generated.

The origin relation and the operational coupling relation are distinct:

$$\mathcal{A} \Longrightarrow \mathcal{R}_{\mathrm{pre}} \neq \mathrm{Couple}(\mathcal{A}, \mathcal{R}_{\mathrm{pre}}). \tag{56}$$

The carrier generates the relational state, but the generated relational state is not allowed to directly modify, mix with, or recursively act back on its generator.

The following loop is forbidden:

$$a_j \Longrightarrow \rho_i^{(m)} \Longrightarrow a_j \Longrightarrow \rho_{i'}^{(m')} \Longrightarrow \cdots . \tag{57}$$

If such a loop were allowed, the primitive layer would require an additional stabilization law to prevent uncontrolled self-modification. Since no external law is allowed at the primitive level, direct carrier-offspring coupling is forbidden.

Therefore the primitive structure permits common origin without direct coupling:

$$\text{common primitive origin} \quad \not\Rightarrow \quad \text{direct primitive coupling}. \tag{58}$$

1.11 Primitive Relational Reach and the Origin of the Window

The primitive structure must specify which relational states may be compared or closed without leaving the minimal closure cycle. This is done by a primitive reach function.

Arrange the twelve relational states in the cyclic order induced by $\mathcal{L}$. Let

$$\mathrm{pos}\Big(\rho_i^{(m)}\Big) \in \mathbb{Z}_{12} \tag{59}$$

denote the induced twelve-state position.

Define the primitive cyclic reach between two relational states by

$$\mathrm{Reach}\Big(\rho_i^{(m)}, \rho_j^{(n)}\Big) = \min\left(\Big|\mathrm{pos}\Big(\rho_i^{(m)}\Big) - \mathrm{pos}\Big(\rho_j^{(n)}\Big)\Big|,\, 12 - \Big|\mathrm{pos}\Big(\rho_i^{(m)}\Big) - \mathrm{pos}\Big(\rho_j^{(n)}\Big)\Big|\right). \tag{60}$$

This is not a spatial distance. It is not a metric. It is only a relational reach count inside the twelve-state relational cycle.

Because each relational root has exactly four closure states, the largest admissible reach that remains within one minimal closure cycle is

$$\mathrm{Reach} \leq 4. \tag{61}$$

States with

$$\mathrm{Reach} = 5 \quad \text{or} \quad \mathrm{Reach} = 6 \tag{62}$$

exceed the internal four-state closure range or approach the half-cycle opposition of the twelve-state structure. They are therefore inadmissible at the primitive level:

$$\mathrm{Reach} > 4 \Longrightarrow \text{inadmissible primitive relational closure.} \tag{63}$$

This is the primitive origin of the later admissibility window. The number 4 is not inserted as an external cutoff; it is inherited from the minimal four-state closure cycle of each relational root.

1.12 Primitive Exchange Orientation

The relational roots in (19) are unordered at the level of generation. However, when a relational exchange is operationally compared, an orientation of exchange may be required.

Define a primitive exchange-orientation predicate

$$\mathrm{Ori}(a_i, a_j) \tag{64}$$

with the antisymmetry condition

$$\mathrm{Ori}(a_i, a_j) = -\,\mathrm{Ori}(a_j, a_i) \qquad (i \neq j). \tag{65}$$

The symbol $-$ in (65) is not a physical sign and not a metric sign. It denotes reversal of exchange orientation.

Orientation is not required for direct readability or mediated closure positions. It is required only for transition-like relational comparisons where exchange direction matters.

Therefore exchange orientation is not a new primitive carrier:

$$\mathrm{Ori}(a_i, a_j) \notin \mathcal{A}. \tag{66}$$

It is also not a new relational root:

$$\mathrm{Ori}(a_i, a_j) \notin \mathcal{P}_2(\mathcal{A}). \tag{67}$$

It is a primitive ordering predicate attached to exchange comparison. Its later algebraic image may become an antisymmetric sign rule on those effective channels that require orientation, while direct readability and mediated closure channels need no exchange-orientation sign.

1.13 Primitive Left-Nested Closure Path

The primitive structure does not assume general associativity. Associativity is an algebraic property and cannot be imposed before an algebraic realization without adding an external structural law.

Therefore any multi-step closure must be defined by a single admissible closure path.

Let Cl denote a primitive binary closure operation on admissible primitive objects. For an ordered finite string

$$(x_1, x_2, \dots, x_n), \tag{68}$$

define the primitive left-nested closure path recursively by

$$\mathrm{Cl}_L(x_1, x_2) = \mathrm{Cl}(x_1, x_2), \tag{69}$$

and

$$\mathrm{Cl}_L(x_1, x_2, \ldots, x_n) = \mathrm{Cl}\Big(\mathrm{Cl}_L(x_1, x_2, \ldots, x_{n-1}), x_n\Big) \qquad (n \geq 3). \tag{70}$$

Equivalently,

$$\mathrm{Cl}_L(x_1, x_2, \ldots, x_n) = \mathrm{Cl}\Big(\cdots \mathrm{Cl}(\mathrm{Cl}(x_1, x_2), x_3) \cdots, x_n\Big). \tag{71}$$

This is not a physical time-ordering. It is not a causal order and not a spacetime order. It is a primitive generation order.

The reason for selecting the left-nested path is that the generated closure status at each step becomes the input status for the next step:

$$\text{previous closure result} \Longrightarrow \text{next closure input}. \tag{72}$$

Thus primitive closure is forward-generative and non-recursive:

$$\text{carrier} \Longrightarrow \text{relational offspring} \Longrightarrow \text{next closure state}. \tag{73}$$

Direct return of the offspring to couple with the carrier is already forbidden by the No Carrier-Offspring Coupling Principle.

If all possible bracketings were allowed, then the primitive structure would require an additional rule identifying or comparing the different bracketed outcomes. Such a rule would be external to the primitive layer. Therefore only one closure path is admitted at the primitive level.

The later algebraic image of this principle is the left-nesting rule:

$$x_1 x_2 \cdots x_n = (\cdots((x_1 x_2) x_3) \cdots x_n). \tag{74}$$

Equation (74) is not assumed at the primitive level as algebraic associativity. It is the effective algebraic image of the primitive left-nested closure path.

1.14 No Primitive Geometry and No Primitive Physics

The primitive structure contains no physical geometry. Therefore the following are not primitive objects:

$$\text{distance}, \quad \text{angle}, \quad \text{time}, \quad \text{space}, \quad \text{metric}. \tag{75}$$

The primitive structure contains no physical quantities:

$$\text{mass}, \quad \text{energy}, \quad \text{momentum}, \quad \text{field}, \quad \text{curvature}, \quad \text{force}, \quad \text{quantum state}. \tag{76}$$

The primitive structure also contains no physical constants:

$$c, \quad \hbar, \quad G \quad \text{are not primitive objects}. \tag{77}$$

All such objects may appear only after representation, projection, and calibration.

Direct readability is not physical observation:

$$\mathfrak{i} \neq \text{physical observation}. \tag{78}$$

Mediated closure is not a force, not a field, and not an energy:

$$\mathfrak{k} \neq \text{physical interaction}. \tag{79}$$

1.15 Conditional Uniqueness of the Primitive Relational Architecture

The architecture in (1) is unique up to relabeling under the following requirements:

$$\text{primitive distinguishability is required;} \tag{80}$$

$$\text{one carrier gives no internal distinction;} \tag{81}$$

$$\text{two carriers require an external closure rule;} \tag{82}$$

$$\text{three carriers are the minimal non-collapsing carrier set;} \tag{83}$$

$$\text{a fourth primitive carrier breaks minimality;} \tag{84}$$

$$\text{relations are generated as offspring, not added as carriers;} \tag{85}$$

$$\text{direct readability and mediated closure must be distinguished;} \tag{86}$$

$$\text{the minimal closure cycle satisfying this distinction has four states;} \tag{87}$$

$$\text{carrier-offspring coupling is forbidden after generation;} \tag{88}$$

$$\text{central conditions are not primitive carriers and not relational offspring;} \tag{89}$$

$$\text{multi-step closure must use one non-associative closure path;} \tag{90}$$

$$\text{the admissible reach is bounded by the four-state closure cycle.} \tag{91}$$

Under these requirements, the minimal primitive architecture is

$$\mathfrak{U}_{\text{pre}} = \Big(\mathcal{A}, \mathcal{R}_{\text{pre}}, \mathfrak{i}, \mathfrak{k}, \mathcal{L}, \mathrm{Reach}, \mathrm{Cl}, \mathrm{Couple}\Big), \tag{92}$$

where

$$\mathcal{A} = \{a_1, a_2, a_3\}, \tag{93}$$

and

$$\mathcal{R}_{\text{pre}} = \Big\{\rho_i^{(m)} : i \in \{1,2,3\}, m \in \mathbb{Z}_4\Big\}, \tag{94}$$

with

$$|\mathcal{R}_{\text{pre}}| = 12. \tag{95}$$

The total primitive pattern is therefore

$$3 \text{ primitive carriers} + 12 \text{ relational states} + 2 \text{ central conditions.} \tag{96}$$

The two central conditions are not counted as primitive carriers. They are boundary conditions of direct readability and mediated closure.

1.16 Core Statement of the First Block

The primitive carriers generate the relational state-space, but they do not directly couple to it after generation:

$$\mathcal{A} \Longrightarrow \mathcal{R}_{\mathrm{pre}}, \qquad \mathrm{Couple}(\mathcal{A}, \mathcal{R}_{\mathrm{pre}}) = \varnothing. \tag{97}$$

The relational state-space has twelve states because three primitive relational roots each require the minimal four-state closure cycle:

$$|\mathcal{R}_{\mathrm{pre}}| = |\mathcal{P}_2(\mathcal{A})| \cdot |\mathbb{Z}_4| = 3 \cdot 4 = 12. \tag{98}$$

The central conditions are not primitive carriers:

$$\mathfrak{i}, \mathfrak{k} \notin \mathcal{A} \cup \mathcal{R}_{\mathrm{pre}}. \tag{99}$$

The mediated closure condition returns to direct readability:

$$\mathfrak{k} \circ \mathfrak{k} = \mathfrak{i}. \tag{100}$$

The non-central self-return class is distinct from mediated closure return:

$$\mathrm{Self}(x) \neq \mathfrak{k} \circ \mathfrak{k} \qquad (x \in \mathcal{N}_{\mathrm{pre}}). \tag{101}$$

The primitive relational reach is bounded by the four-state cycle:

$$\mathrm{Reach} \leq 4. \tag{102}$$

Multi-step closure is not associative; it is defined only through the primitive left-nested closure path:

$$\mathrm{Cl}_L(x_1, x_2, \ldots, x_n) = \mathrm{Cl}\Big(\mathrm{Cl}_L(x_1, x_2, \ldots, x_{n-1}), x_n\Big). \tag{103}$$

This completes the primitive relational layer before spacetime. No spacetime, no physical units, no geometry, no material field, and no effective algebra have been assumed.

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2 Induced Axioms of the First Effective Algebraic Realization

2.1 Purpose of the Effective Realization

The primitive architecture $\mathfrak{U}_{\mathrm{pre}}$ constructed in Section 1 is relational and non-algebraic. It contains primitive carriers, generated relational states, central readability conditions, a primitive reach bound, a primitive closure path, and a no carrier-offspring coupling principle.

It does not yet contain an algebraic product, a norm, a physical metric, a physical observer, spacetime coordinates, physical units, or dynamical fields.

The purpose of the first effective algebraic realization is to represent the primitive relational architecture by a finite algebraic image that preserves all primitive distinctions:

$$\mathfrak{U}_{\mathrm{pre}} \Longrightarrow \mathfrak{H}_{\mathrm{eff}}. \tag{104}$$

The realization $\mathfrak{H}_{\mathrm{eff}}$ is not a physical spacetime theory. It is only the first effective algebraic image of the primitive layer.

The realization must preserve the following primitive requirements:

$$\text{three primitive carriers,} \tag{105}$$

$$\text{twelve primitive relational states,} \tag{106}$$

$$\text{one direct-readability condition,} \tag{107}$$

$$\text{one mediated-closure condition,} \tag{108}$$

$$\text{no direct carrier-offspring coupling,} \tag{109}$$

$$\text{bounded relational reach,} \tag{110}$$

$$\text{primitive left-nested closure.} \tag{111}$$

Thus the effective algebraic axioms are not introduced independently. They are induced as the minimal algebraic representation of the primitive relational constraints.

2.2 Effective Images of the Primitive Classes

Let the first effective realization map be denoted by

$$\Phi : \mathfrak{U}_{\mathrm{pre}} \longrightarrow \mathfrak{H}_{\mathrm{eff}}. \tag{112}$$

The three primitive carriers are represented by three effective carrier images:

$$\Phi(a_1) = e_1, \qquad \Phi(a_2) = e_2, \qquad \Phi(a_3) = e_3. \tag{113}$$

Define the effective carrier sector by

$$F = \mathrm{span}\{e_1, e_2, e_3\}. \tag{114}$$

The twelve primitive relational states are represented by twelve effective relational images:

$$\Phi\Big(\mathcal{R}_{\mathrm{pre}}\Big) = \{e_4, e_5, \ldots, e_{15}\}. \tag{115}$$

Define the effective relational sector by

$$R = \mathrm{span}\{e_4, e_5, \ldots, e_{15}\}. \tag{116}$$

The direct-readability condition is represented by one effective central image:

$$\Phi(\mathfrak{i}) = e_0. \tag{117}$$

The mediated-closure condition is represented by one effective central image:

$$\Phi(\mathfrak{k}) = e_{16}. \tag{118}$$

Define the effective central sector by

$$C = \mathrm{span}\{e_0, e_{16}\}. \tag{119}$$

Therefore the first effective algebraic carrier set is

$$S_{17} = \{e_0, e_1, e_2, \ldots, e_{15}, e_{16}\}. \tag{120}$$

Its sector decomposition is

$$\mathfrak{H}_{\text{eff}} = C \oplus F \oplus R. \tag{121}$$

The number 17 is therefore not introduced as an arbitrary count. It is forced by the primitive pattern:

$$17 = 2 + 3 + 12, \tag{122}$$

where 2 counts central conditions, 3 counts primitive carrier images, and 12 counts primitive relational-state images.

Equivalently,

$$17 = 2 + 3 + (3 \times 4). \tag{123}$$

2.3 Central Readability Induces the Effective Identity

In the primitive architecture, $\mathfrak{i}$ is the direct-readability condition. It is not a primitive carrier and not a relational state. Its role is to provide the completed readable status of a relational closure.

Under the effective realization,

$$\Phi(\mathfrak{i}) = e_0. \tag{124}$$

Because $\mathfrak{i}$ is the direct-readability condition, its effective image must act as the neutral readability image. Hence, for every effective basis element $e_a \in S_{17}$,

$$e_0 e_a = e_a e_0 = e_a. \tag{125}$$

In particular,

$$e_0^2 = e_0. \tag{126}$$

Thus the effective identity property of e_0 is induced by the primitive direct-readability condition. It is not an independent algebraic decoration.

2.4 Mediated Closure Induces the Hyperbolic Closure Image

In the primitive architecture, mediated closure satisfies

$$\mathfrak{k} \circ \mathfrak{k} = \mathfrak{i}. \tag{127}$$

Under the effective realization,

$$\Phi(\mathfrak{k}) = e_{16}, \qquad \Phi(\mathfrak{i}) = e_0. \tag{128}$$

Therefore the effective image of mediated closure return is

$$e_{16}^2 = e_0. \tag{129}$$

Equation (129) is the effective algebraic image of primitive mediated closure return. It is the first hyperbolic closure signature of the effective algebra, because the closure image returns to the readability image under second closure.

The element e_{16} is therefore not an additional primitive carrier. It is the effective image of $\mathfrak{k}$, the primitive mediated-closure condition.

2.5 Non-Central Self-Return Induces the Effective Negative Return Class

The primitive non-central class is

$$\mathcal{N}_{\text{pre}} = \mathcal{A} \cup \mathcal{R}_{\text{pre}}. \tag{130}$$

Its effective image is

$$\Phi(\mathcal{N}_{\text{pre}}) = \{e_1, e_2, \ldots, e_{15}\}. \tag{131}$$

In the primitive layer, every $x \in \mathcal{N}_{\text{pre}}$ possesses a self-return class that is structurally distinct from mediated closure return:

$$\text{Self}(x) \neq \mathfrak{k} \circ \mathfrak{k} \qquad (x \in \mathcal{N}_{\text{pre}}). \tag{132}$$

The effective algebraic realization must preserve this distinction. Since mediated closure return is represented by

$$e_{16}^2 = +e_0, \tag{133}$$

the non-central self-return class must be represented by the opposite effective return class:

$$e_a^2 = -e_0 \qquad \text{for } a = 1, 2, \ldots, 15. \tag{134}$$

Equation (134) does not insert a primitive sign into the pre-spacetime layer. The sign appears only after the effective algebraic realization is introduced. It records the structural distinction between non-central self-return and mediated closure return.

Thus the later signed separation is induced by the primitive distinction:

$$\text{Self}(\mathcal{N}_{\text{pre}}) \neq \mathfrak{k} \circ \mathfrak{k}. \tag{135}$$

2.6 No Carrier-Offspring Coupling Induces $F \times R = 0$

The primitive layer allows generation:

$$\mathcal{A} \Longrightarrow \mathcal{R}_{\text{pre}}. \tag{136}$$

But it forbids direct carrier-offspring coupling:

$$\text{Couple}(\mathcal{A}, \mathcal{R}_{\text{pre}}) = \varnothing. \tag{137}$$

Under the effective realization,

$$\Phi(\mathcal{A}) = F, \qquad \Phi(\mathcal{R}_{\text{pre}}) = R. \tag{138}$$

Therefore the effective algebraic image of no carrier-offspring coupling is

$$F \times R = 0. \tag{139}$$

Equation (139) is not a separation imposed after the fact. It is the algebraic image of the primitive principle that a carrier may generate its relational offspring but may not directly couple back to that offspring after generation.

This principle prevents the recursive instability

$$\text{carrier} \Longrightarrow \text{offspring} \Longrightarrow \text{carrier} \Longrightarrow \cdots. \tag{140}$$

Hence the effective sector F and the effective sector R have common primitive origin but no direct algebraic product.

2.7 Carrier Independence Induces $F \times F \subseteq C$

Primitive carrier independence requires that no primitive carrier be generated from the other two:

$$a_k \notin \mathrm{Gen}(a_i, a_j) \qquad \text{for distinct } i, j, k. \tag{141}$$

Under effective realization, this prohibits products of effective carrier images from producing another effective carrier image. Therefore the product of two carrier-sector elements must not close inside F.

The smallest allowed effective closure target is the central sector:

$$F \times F \subseteq C. \tag{142}$$

Equation (142) is therefore induced by primitive carrier independence. If $F \times F$ produced elements of F, then at the effective level one carrier image could be generated from products of the others, contradicting the primitive independence condition.

The self-return part of (142) is already fixed by (134):

$$e_a^2 = -e_0 \qquad (a = 1, 2, 3). \tag{143}$$

For distinct carrier images, closure must be central and must not generate a carrier:

$$e_a e_b \in C \qquad (a, b \in \{1, 2, 3\},\ a \neq b). \tag{144}$$

The mediated closure image is the only non-readability central closure image. Therefore the distinct-carrier product is represented by the mediated central channel:

$$e_a e_b = \sigma_F(a, b) e_{16} \qquad (a, b \in \{1, 2, 3\},\ a \neq b), \tag{145}$$

where $\sigma_F(a, b)$ is an exchange-orientation sign satisfying

$$\sigma_F(a, b) = -\sigma_F(b, a). \tag{146}$$

A fixed orientation convention may be chosen by

$$e_1 e_2 = e_{16}, \qquad e_2 e_3 = e_{16}, \qquad e_3 e_1 = e_{16}, \tag{147}$$

with reversed products

$$e_2 e_1 = -e_{16}, \qquad e_3 e_2 = -e_{16}, \qquad e_1 e_3 = -e_{16}. \tag{148}$$

Equations (147)–(148) are the effective oriented image of primitive carrier-pair exchange. They preserve carrier independence because no product of two carrier images produces another carrier image.

2.8 Relational-State Closure Induces $R \times R \subseteq C$

The sector R is the effective image of the primitive relational state-space:

$$R = \Phi(\mathcal{R}_{\mathrm{pre}}). \tag{149}$$

Relational states are offspring, not primitive carriers. Therefore the closure of relational states must not generate primitive carrier images. Hence

$$R \times R \not\subseteq F. \tag{150}$$

The closure of two relational-state images must return to a central readability or mediated-closure channel:

$$R \times R \subseteq C. \tag{151}$$

Equation (151) is the effective algebraic image of the fact that relational-state interactions do not generate new primitive carriers. They return to central conditions.

Thus both carrier-pair closure and relational-state closure return to the same effective central sector:

$$F \times F \subseteq C, \qquad R \times R \subseteq C. \tag{152}$$

The difference is that F represents primitive carrier images, while R represents relational offspring images.

2.9 The Twelve Relational States Induce the Effective Rim Order

The primitive relational state-space has the form

$$\mathcal{R}_{\text{pre}} = \{\rho_i^{(m)} : i \in \{1, 2, 3\}, m \in \mathbb{Z}_4\}. \tag{153}$$

Its effective image is ordered as

$$\Phi(\rho_1^{(0)}) = e_4, \qquad \Phi(\rho_2^{(0)}) = e_5, \qquad \Phi(\rho_3^{(0)}) = e_6, \tag{154}$$

$$\Phi(\rho_1^{(1)}) = e_7, \qquad \Phi(\rho_2^{(1)}) = e_8, \qquad \Phi(\rho_3^{(1)}) = e_9, \tag{155}$$

$$\Phi(\rho_1^{(2)}) = e_{10}, \qquad \Phi(\rho_2^{(2)}) = e_{11}, \qquad \Phi(\rho_3^{(2)}) = e_{12}, \tag{156}$$

$$\Phi(\rho_1^{(3)}) = e_{13}, \qquad \Phi(\rho_2^{(3)}) = e_{14}, \qquad \Phi(\rho_3^{(3)}) = e_{15}. \tag{157}$$

Define the four effective relational layers:

$$T_0 = \{e_4, e_5, e_6\}, \tag{158}$$

$$T_1 = \{e_7, e_8, e_9\}, \tag{159}$$

$$T_2 = \{e_{10}, e_{11}, e_{12}\}, \tag{160}$$

$$T_3 = \{e_{13}, e_{14}, e_{15}\}. \tag{161}$$

Then

$$R = T_0 \cup T_1 \cup T_2 \cup T_3. \tag{162}$$

The effective rim order is therefore the algebraic image of

$$3 \text{ relational roots} \times 4 \text{ closure states}. \tag{163}$$

2.10 Primitive State-Shift Induces the Effective Rim Generator

The primitive relational state-shift is

$$\mathcal{L}\rho_i^{(m)} = \rho_i^{(m+1 \bmod 4)}. \tag{164}$$

On the twelve-state relational ordering, this induces a twelve-step effective rim shift. Denote its effective image by

$$L_{16}. \tag{165}$$

The effective rim shift acts by

$$L_{16}(e_k) = e_{k\oplus 1} \qquad (k = 4, \dots, 15), \tag{166}$$

where $\oplus 1$ denotes the next position in the cyclic order on the twelve rim elements.

Since the primitive relational cycle closes after twelve effective relational states, the induced rim shift satisfies

$$L_{16}^{12}\Big|_R = \mathrm{Id}_R\,. \tag{167}$$

The central and carrier sectors do not participate in the twelvefold relational-state cycle. The mediated closure image acts on them as a two-return closure condition:

$$L_{16}^{2}\Big|_{C\cup F} = \mathrm{Id}_{C\cup F}\,. \tag{168}$$

Equations (167) and (168) are induced by the primitive distinction between the twelve-state relational offspring cycle and the central mediated-closure return.

2.11 Primitive Reach Induces the Effective Window

The primitive reach condition is

$$\mathrm{Reach} \le 4. \tag{169}$$

In the effective rim, define the rim position of $e_k \in R$ by

$$p(e_k) = k - 4 \qquad (k = 4, \dots, 15), \tag{170}$$

with values in $\mathbb{Z}_{12}$.

For $e_i, e_j \in R$, define

$$\Delta(i,j) = \min\left(|p(e_i) - p(e_j)|, 12 - |p(e_i) - p(e_j)|\right). \tag{171}$$

The effective admissibility window is the algebraic image of primitive reach:

$$\Delta(i,j) \le 4. \tag{172}$$

If

$$\Delta(i,j) \in \{5, 6\}, \tag{173}$$

then the pair exceeds the primitive four-state closure reach or reaches the half-cycle opposition of the twelvefold relational ordering. Therefore it is inadmissible:

$$\Delta(i,j) \in \{5, 6\} \Longrightarrow e_i e_j = 0. \tag{174}$$

This is the effective domain-as-zero image of primitive inadmissibility. It is not an arbitrary cutoff.

2.12 Degree Classes and Channel Separation

The four-state primitive closure cycle induces four effective degree classes on the rim. Define

$$\deg(e_k) = k - 4 \pmod 4 \qquad (k = 4, \ldots, 15). \tag{175}$$

For an admissible rim pair $e_i, e_j \in R$, define the channel degree

$$d(i,j) = \deg(e_i) + \deg(e_j) \pmod 4. \tag{176}$$

The direct-readability class corresponds to

$$d(i,j) = 0. \tag{177}$$

The mediated-closure class corresponds to

$$d(i,j) = 2. \tag{178}$$

The transition-like classes correspond to

$$d(i,j) \in \{1,3\}. \tag{179}$$

Therefore the even channels require no exchange-orientation sign:

$$d(i,j) = 0 \Longrightarrow e_i e_j = +e_0, \tag{180}$$

$$d(i,j) = 2 \Longrightarrow e_i e_j = +e_{16}. \tag{181}$$

The transition-like odd channels require exchange orientation:

$$d(i,j) \in \{1,3\} \Longrightarrow e_i e_j = \sigma(i,j)\chi_{d(i,j)}, \tag{182}$$

where $\chi_{d(i,j)} \in \{e_0, e_{16}\}$ is the central target prescribed by the channel convention, and

$$\sigma(i,j) = -\sigma(j,i). \tag{183}$$

The sign $\sigma(i,j)$ is the effective algebraic image of the primitive exchange-orientation predicate Ori. It is not used on the even channels because direct readability and mediated closure are central closure classes, not transition-like exchange classes.

2.13 Centrality of Effective Closure Products

The primitive structure has only two central conditions:

$$\mathfrak{i}, \qquad \mathfrak{k}. \tag{184}$$

Therefore every admissible effective closure product of two non-central images must return to the effective central sector:

$$e_a e_b \in C \qquad (a, b \in \{1, \ldots, 15\}) \tag{185}$$

whenever the product is admissible.

This gives

$$F \times F \subseteq C, \qquad R \times R \subseteq C, \qquad F \times R = 0. \tag{186}$$

The first inclusion follows from primitive carrier independence. The second follows from relational-state closure. The third follows from no carrier-offspring coupling.

Thus the centrality of closure is not an independent axiom; it is induced by the fact that the primitive layer has central conditions but no central primitive carriers.

2.14 Primitive Left-Nested Closure Induces GC-1

The primitive layer does not assume associativity. Multi-step closure is defined by the left-nested closure path:

$$\mathrm{Cl}_L(x_1, x_2, \ldots, x_n) = \mathrm{Cl}\Big(\mathrm{Cl}_L(x_1, x_2, \ldots, x_{n-1}), x_n\Big). \tag{187}$$

Under the effective algebraic realization, this becomes the left-nesting rule:

$$x_1 x_2 \cdots x_n = (\cdots((x_1 x_2)x_3)\cdots x_n). \tag{188}$$

Equation (188) is the effective algebraic image of primitive left-nested closure. It is not a consequence of associativity. It replaces general associativity by a single admissible closure path.

If different bracketings were allowed, an additional rule would be required to identify or compare their outcomes:

$$(x_1 x_2)x_3 \overset{?}{=} x_1(x_2 x_3). \tag{189}$$

Such a rule is not present in the primitive layer. Therefore the effective realization must preserve left-nested closure as its only multi-product evaluation path.

2.15 Conjugation and Central-First Readability

The primitive architecture distinguishes non-central self-return from central readability. The effective algebraic realization represents this distinction by an involutive conjugation.

Define conjugation on basis images by

$$\overline{e_0} = e_0, \qquad \overline{e_{16}} = e_{16}, \tag{190}$$

and

$$\overline{e_a} = -e_a \qquad (a = 1, 2, \ldots, 15). \tag{191}$$

This conjugation is the effective image of the primitive distinction between central conditions and non-central objects:

$$C \quad \text{is fixed,} \qquad F \oplus R \quad \text{is reversed.} \tag{192}$$

The effective central projection is

$$\Pi_C : \mathfrak{H}_{\mathrm{eff}} \longrightarrow C. \tag{193}$$

The effective direct-readability projection is

$$\Pi_{e_0} : C \longrightarrow \mathrm{span}\{e_0\}. \tag{194}$$

The central-first readability map is

$$\mathrm{Read}(X) = \Pi_{e_0}\left(N(\Pi_C(X))\right), \tag{195}$$

where N is the effective central return functional.

This map is not a physical observation map at the primitive stage. It is the effective algebraic image of the primitive rule that readability is central and must not be extracted directly from non-central relational states.

2.16 The Effective Algebraic Image of Structural Fluidity

Structural fluidity means that the primitive layer does not yet contain fixed physical interpretation. The effective algebraic realization is the first crystallization of the primitive relational constraints into a finite symbolic structure.

This crystallization has the form

$$\mathfrak{H}_{\mathrm{eff}} = C \oplus F \oplus R, \tag{196}$$

with

$$C = \operatorname{span}\{e_0, e_{16}\}, \tag{197}$$

$$F = \operatorname{span}\{e_1, e_2, e_3\}, \tag{198}$$

$$R = \operatorname{span}\{e_4, \ldots, e_{15}\}. \tag{199}$$

The central sector C represents central readability and mediated closure. The carrier sector F represents the three primitive carrier images. The relational sector R represents the twelve generated relational-state images.

The effective algebra is therefore not introduced as a free algebraic invention. It is the first finite algebraic crystallization of

$$3 \text{ primitive carriers} + 12 \text{ relational states} + 2 \text{ central conditions}. \tag{200}$$

2.17 Summary of Induced Effective Axioms

The induced effective axioms are summarized as follows.

Direct readability induces the effective identity:

$$e_0 e_a = e_a e_0 = e_a \qquad (a = 0, 1, \ldots, 16). \tag{201}$$

Mediated closure return induces the hyperbolic closure image:

$$e_{16}^2 = e_0. \tag{202}$$

Non-central self-return induces the effective negative return class:

$$e_a^2 = -e_0 \qquad (a = 1, \ldots, 15). \tag{203}$$

Carrier independence induces carrier-sector central closure:

$$F \times F \subseteq C. \tag{204}$$

No carrier-offspring coupling induces direct sector separation:

$$F \times R = 0. \tag{205}$$

Relational-state closure induces rim-sector central closure:

$$R \times R \subseteq C. \tag{206}$$

The primitive twelve-state relational cycle induces the rim shift:

$$L_{16}^{12}\Big|_{R} = \mathrm{Id}_R\,. \tag{207}$$

The mediated closure return induces the central-carrier two-return condition:

$$L_{16}^{2}\Big|_{C\cup F} = \mathrm{Id}_{C\cup F}\,. \tag{208}$$

Primitive reach induces the admissibility window:

$$\Delta(i,j) \le 4 \qquad (i,j \in R). \tag{209}$$

Primitive inadmissibility induces domain-as-zero:

$$\Delta(i,j) \in \{5,6\} \Longrightarrow e_i e_j = 0. \tag{210}$$

The four-state closure cycle induces the degree policy:

$$\deg(e_k) = k - 4 \pmod 4 \qquad (k = 4, \ldots, 15). \tag{211}$$

Direct readability and mediated closure induce the even channel policy:

$$d = 0 \Longrightarrow +e_0, \qquad d = 2 \Longrightarrow +e_{16}. \tag{212}$$

Primitive exchange orientation induces the odd-channel orientation sign:

$$d \in \{1,3\} \Longrightarrow \sigma(i,j) = -\sigma(j,i). \tag{213}$$

Primitive left-nested closure induces the effective left-nesting rule:

$$x_1 x_2 \cdots x_n = (\cdots((x_1 x_2)x_3)\cdots x_n). \tag{214}$$

The effective axioms are therefore induced by the primitive relational architecture. They are not independent assumptions detached from the pre-spacetime structure.

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3 Spectral Lorentzian Arm Form

3.1 Purpose of the Section

The purpose of this section is to identify the first Lorentzian-type structure that appears after the effective algebraic realization, while preserving the distinction between a spectral form and a physical spacetime form.

This section does not derive physical special relativity directly from the primitive layer. It derives a spectral Lorentzian arm form from the effective arm sector. The physical Lorentzian interval appears only after a later projection and calibration step.

The logical order is

$$\text{primitive relational architecture} \Longrightarrow \text{effective algebraic realization} \Longrightarrow \text{direct central spectral arm proj} \tag{215}$$

At the stage treated in this section, there is no physical spacetime, no physical time coordinate, no physical spatial coordinate, no physical metric field, and no physical speed of light. There is only a spectral signed structure carried by the effective arm sector.

3.2 Arm Sector as the Effective Image of Primitive Carriers

The primitive carrier set is

$$\mathcal{A} = \{a_1, a_2, a_3\}. \tag{216}$$

Under the first effective algebraic realization,

$$\Phi(a_1) = e_1, \qquad \Phi(a_2) = e_2, \qquad \Phi(a_3) = e_3. \tag{217}$$

Thus the effective arm sector is

$$F = \mathrm{span}\{e_1, e_2, e_3\}. \tag{218}$$

The elements e_1, e_2, e_3 are not physical spatial axes at this stage. They are effective algebraic images of the three primitive carriers. Their later interpretation as physical spatial directions requires projection and calibration.

3.3 Direct Central Spectral Projection of the Arm Sector

The spectral Lorentzian arm form appears directly from the arm sector because the arm sector closes into the direct-readability carrier e_0 through self-return. It does not need to pass through the relational offspring sector R, and it does not need the mediated closure image e_{16} in order to produce the first signed arm form.

For the effective arm images,

$$e_1^2 = -e_0, \qquad e_2^2 = -e_0, \qquad e_3^2 = -e_0. \tag{219}$$

The direct-readability image satisfies

$$e_0^2 = e_0. \tag{220}$$

Therefore the self-return contributions of the arm sector land directly on the e_0-line:

$$e_1^2, e_2^2, e_3^2 \in \mathrm{span}\{e_0\}. \tag{221}$$

This is a spectral central projection, not yet a physical projection to spacetime.

The mediated closure image remains governed by

$$e_{16}^2 = e_0. \tag{222}$$

However, equation (222) is not needed to form the first spectral Lorentzian arm quadratic form. The arm self-return already lands directly on e_0.

Thus the arm sector carries the spectral signed pattern

$$(+ - - -)_S. \tag{223}$$

The subscript S indicates that the signature is spectral. It is not yet a physical spacetime signature.

3.4 Why the Lorentzian-Type Pattern Appears Directly

The direct-readability contribution is represented by the e_0-line:

$$e_0^2 = e_0. \tag{224}$$

The three arm self-return contributions are represented by

$$e_1^2 = -e_0, \qquad e_2^2 = -e_0, \qquad e_3^2 = -e_0. \tag{225}$$

Hence the arm sector provides one direct-readability contribution and three non-central self-return contributions. This produces the spectral sign pattern

$$\text{one direct-readable contribution} \quad \text{and} \quad \text{three arm self-return contributions.} \tag{226}$$

In effective signed form, this becomes

$$(+ - --)_S. \tag{227}$$

This is why the Lorentzian-type structure appears directly from the arms. It is not imported from physical spacetime. It is induced by the effective return rules of e_0, e_1, e_2, e_3.

3.5 Spectral Arm Differential with an Uncalibrated Spectral Constant

Define the spectral arm differential by

$$dX_S = \chi_S \, d\tau_S \, e_0 + d\xi^1 e_1 + d\xi^2 e_2 + d\xi^3 e_3. \tag{228}$$

Here $d\tau_S$ is a spectral readability parameter, not physical time. The quantities $d\xi^1, d\xi^2, d\xi^3$ are spectral arm increments, not physical spatial coordinate differentials. The coefficient χ_S is an uncalibrated spectral conversion constant. It is not yet the physical speed of light.

Thus

$$d\tau_S \neq dt, \qquad d\xi^1 \neq dx^1, \qquad d\xi^2 \neq dx^2, \qquad d\xi^3 \neq dx^3, \qquad \chi_S \neq c \tag{229}$$

before physical projection and calibration.

The use of χ_S is essential. At this stage one must not write c, because c is a physical calibration constant. The spectral arm form contains only the uncalibrated spectral constant χ_S.

3.6 Spectral Lorentzian Arm Form

The signed spectral arm quadratic form is

$$q_S(dX_S) = \chi_S^2 \, d\tau_S^2 - (d\xi^1)^2 - (d\xi^2)^2 - (d\xi^3)^2. \tag{230}$$

Equation (230) is the spectral Lorentzian arm form.

It is Lorentzian in sign pattern but spectral in status. It is not yet the physical spacetime interval:

$$q_S(dX_S) \neq ds^2. \tag{231}$$

The source of (230) is the direct central spectral projection of the arm sector:

$$e_0^2 = e_0, \qquad e_1^2 = -e_0, \qquad e_2^2 = -e_0, \qquad e_3^2 = -e_0. \tag{232}$$

Thus the spectral form arises before physical spacetime. It is a signed arm return form, not yet a physical metric interval.

3.7 Direct Spectral Projection Versus Physical Projection

There are two different notions of projection, and they must not be confused.

The first is the direct central spectral projection of the arm sector:

$$F_{\text{self}} \longrightarrow \operatorname{span}\{e_0\}. \tag{233}$$

This projection is already present at the effective algebraic level because

$$e_1^2 = -e_0, \qquad e_2^2 = -e_0, \qquad e_3^2 = -e_0. \tag{234}$$

The second is the physical projection-calibration map:

$$\mathcal{P}_{\text{phys}}. \tag{235}$$

The first projection gives the spectral signed form. The second projection converts the spectral signed form into a physical interval with physical units.

Therefore,

$$\text{direct spectral projection to } e_0 \neq \text{physical projection to spacetime.} \tag{236}$$

The arm sector projects spectrally to e_0 directly, but it does not project physically to spacetime until the calibration map is imposed.

3.8 Distinction from the Central-First Readability Map

The spectral Lorentzian arm form must not be confused with the final central-first readability map.

The central-first readability map has the form

$$\operatorname{Read}(X) = \Pi_{e_0}\left(N(\Pi_C(X))\right). \tag{237}$$

This map is a final central readability map. It is not the tool used to extract the signed Lorentzian arm form.

Therefore

$$q_S(dX_S) \neq \operatorname{Read}(dX_S). \tag{238}$$

The signed spectral arm form must be constructed before final central readability is applied. If final readability is applied too early, the signed Lorentzian distinction may be lost or converted into a nonnegative central readout.

Thus the correct order is

$$\text{arm signed return structure} \Longrightarrow \text{spectral Lorentzian arm form} \Longrightarrow \text{physical projection and calibration} \tag{239}$$

3.9 Physical Projection and Calibration

The physical spacetime interval appears only after the physical projection-calibration map is introduced:

$$\mathcal{P}_{\text{phys}} : q_S(dX_S) \longrightarrow ds^2. \tag{240}$$

The calibration of the spectral readability component is

$$\mathcal{P}_{\text{phys}}(\chi_S \, d\tau_S) = c\, dt. \tag{241}$$

The calibration of the three spectral arm increments is

$$\mathcal{P}_{\mathrm{phys}}(d\xi^1) = dx^1, \qquad \mathcal{P}_{\mathrm{phys}}(d\xi^2) = dx^2, \qquad \mathcal{P}_{\mathrm{phys}}(d\xi^3) = dx^3. \tag{242}$$

Only after (241) and (242) does the spectral Lorentzian arm form become the physical Lorentzian interval:

$$\mathcal{P}_{\mathrm{phys}}\Big(q_S(dX_S)\Big) = ds^2 = c^2dt^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2. \tag{243}$$

Thus c is not assumed inside the spectral arm form. It enters only at the physical projection-calibration boundary.

The spectral constant χ_S is the pre-calibration placeholder whose physical image is fixed by

$$\chi_S\, d\tau_S \longmapsto c\, dt. \tag{244}$$

3.10 Status of the Spectral Lorentzian Arm Form

The result of this section is the following.

First, the primitive relational layer does not contain spacetime:

$$\mathfrak{U}_{\mathrm{pre}} \not\supset \text{physical spacetime}. \tag{245}$$

Second, the first effective algebraic realization contains the arm sector:

$$F = \mathrm{span}\{e_1, e_2, e_3\}. \tag{246}$$

Third, the arm sector has direct central spectral self-return:

$$e_1^2 = -e_0, \qquad e_2^2 = -e_0, \qquad e_3^2 = -e_0. \tag{247}$$

Fourth, together with

$$e_0^2 = e_0, \tag{248}$$

this induces the spectral signed pattern

$$(+---)_S. \tag{249}$$

Fifth, this yields the spectral Lorentzian arm form

$$q_S(dX_S) = \chi_S^2\, d\tau_S^2 - (d\xi^1)^2 - (d\xi^2)^2 - (d\xi^3)^2. \tag{250}$$

Sixth, the physical Lorentzian interval appears only after projection and calibration:

$$q_S(dX_S) \overset{\mathcal{P}_{\mathrm{phys}}}{\Longrightarrow} ds^2 = c^2dt^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2. \tag{251}$$

Therefore the Lorentzian structure that appears directly from the arms is spectral, not physical. The physical form of special relativity is obtained only after the spectral form is projected and calibrated. ===

A Conditional Minimality and Representation Theorem for the First Effective Algebraic Realization

A.1 Purpose and Scope of the Appendix

This appendix proves a conditional minimality theorem for the first effective algebraic realization of the primitive relational architecture introduced in Section 1.

The theorem is not an unrestricted uniqueness theorem over all possible mathematical systems. It does not claim that no alternative formal universe can be constructed from different primitive assumptions. Such a statement would require a complete classification over all possible primitive architectures, which is not the purpose of this manuscript.

The theorem proved here is the following conditional result:

> Given the primitive relational architecture $\mathfrak{U}_{\text{pre}}$, and requiring faithfulness, linear faithfulness, readability preservation, non-degenerate signed return, no carrier-offspring coupling, bounded relational reach, and primitive left-nested closure, the first minimal effective algebraic realization has exactly seventeen linearly independent basis images. These split canonically into two central-condition images, three carrier images, and twelve relational-state images. The induced effective algebraic structure has the sector form $C \oplus F \oplus R$.

Thus the appendix establishes that, under the stated primitive and representation requirements, the 17-basis structure is not a free numerical choice. It is the minimal faithful algebraic image of the primitive relational architecture.

The result should be read as a conditional representation theorem:

$$\mathfrak{U}_{\text{pre}} + \text{faithful minimal realization requirements} \Longrightarrow \mathfrak{H}_{\text{eff}} = C \oplus F \oplus R, \qquad \dim \mathfrak{H}_{\text{eff}} = 17. \tag{252}$$

The theorem does not yet introduce physical spacetime, physical units, fields, particles, curvature, mass, energy, or experimental observables. It only proves the minimal effective algebraic image of the pre-spacetime relational architecture.

A.2 Primitive Data Recalled

The primitive architecture is

$$\mathfrak{U}_{\text{pre}} = \Big(\mathcal{A}, \mathcal{R}_{\text{pre}}, \mathfrak{i}, \mathfrak{k}, \mathcal{L}, \text{Reach}, \text{Cl}, \text{Couple}\Big). \tag{253}$$

The primitive carrier set is

$$\mathcal{A} = \{a_1, a_2, a_3\}. \tag{254}$$

The elements a_1, a_2, a_3 are primitive distinguishable carriers. They are not spatial axes, not physical dimensions, not coordinates, not fields, and not already algebraic basis elements.

Primitive carrier distinguishability is

$$a_i \not\equiv a_j \qquad (i \neq j). \tag{255}$$

Primitive non-derivability is

$$a_k \notin \text{Gen}(a_i, a_j) \qquad \text{for every distinct } i, j, k \in \{1, 2, 3\}. \tag{256}$$

The primitive relational roots generated by the three carriers are

$$\rho_1 = \{a_1, a_2\}, \qquad \rho_2 = \{a_2, a_3\}, \qquad \rho_3 = \{a_3, a_1\}. \tag{257}$$

Each primitive relational root carries the minimal four-state closure cycle:

$$\rho_i^{(m)} \qquad i \in \{1, 2, 3\}, \quad m \in \mathbb{Z}_4. \tag{258}$$

The primitive relational state-space is

$$\mathcal{R}_{\text{pre}} = \Big\{ \rho_i^{(m)} : i \in \{1, 2, 3\},\ m \in \mathbb{Z}_4 \Big\}. \tag{259}$$

Therefore

$$|\mathcal{R}_{\text{pre}}| = 3 \cdot 4 = 12. \tag{260}$$

The direct-readability condition is

$$\mathfrak{i}. \tag{261}$$

The mediated-closure condition is

$$\mathfrak{k}. \tag{262}$$

They are not primitive carriers and not relational states:

$$\mathfrak{i}, \mathfrak{k} \notin \mathcal{A} \cup \mathcal{R}_{\text{pre}}. \tag{263}$$

Mediated closure returns to direct readability:

$$\mathfrak{k} \circ \mathfrak{k} = \mathfrak{i}. \tag{264}$$

The primitive non-central class is

$$\mathcal{N}_{\text{pre}} = \mathcal{A} \cup \mathcal{R}_{\text{pre}}. \tag{265}$$

For every $x \in \mathcal{N}_{\text{pre}}$, primitive non-central self-return is defined and is structurally distinct from mediated closure return:

$$\text{Self}(x) \neq \mathfrak{k} \circ \mathfrak{k} \qquad (x \in \mathcal{N}_{\text{pre}}). \tag{266}$$

The no carrier-offspring coupling principle is

$$\text{Couple}\Big(a_j, \rho_i^{(m)}\Big) = \varnothing \qquad \text{for all } i, j, m. \tag{267}$$

The primitive reach bound is

$$\text{Reach} \leq 4. \tag{268}$$

Primitive multi-step closure is evaluated only through the left-nested closure path:

$$\text{Cl}_L(x_1, x_2, \ldots, x_n) = \text{Cl}\Big(\text{Cl}_L(x_1, x_2, \ldots, x_{n-1}), x_n\Big). \tag{269}$$

A.3 Faithful Effective Realization

A faithful effective realization of $\mathfrak{U}_{\mathrm{pre}}$ is a finite algebraic image

$$\Phi : \mathfrak{U}_{\mathrm{pre}} \longrightarrow \mathfrak{H}_{\mathrm{eff}} \tag{270}$$

satisfying the faithfulness requirements listed below.

First, primitive carriers are represented by distinct effective images:

$$\Phi(a_i) \neq \Phi(a_j) \qquad (i \neq j). \tag{271}$$

Second, primitive carrier non-derivability is preserved:

$$\Phi(a_k) \notin \mathrm{Gen}_{\mathrm{eff}}\Big(\Phi(a_i), \Phi(a_j)\Big) \qquad (i, j, k \text{ distinct}). \tag{272}$$

Third, relational-state images are distinct from carrier images:

$$\Phi(\rho_i^{(m)}) \notin \Phi(\mathcal{A}) \qquad \text{for all } i, m. \tag{273}$$

Fourth, distinct primitive relational states have distinct effective images:

$$(i, m) \neq (j, n) \Longrightarrow \Phi(\rho_i^{(m)}) \neq \Phi(\rho_j^{(n)}). \tag{274}$$

Fifth, central-condition images are distinct from all non-central primitive images:

$$\Phi(\mathfrak{i}), \Phi(\mathfrak{k}) \notin \Phi(\mathcal{N}_{\mathrm{pre}}). \tag{275}$$

Sixth, the two central conditions are represented distinctly:

$$\Phi(\mathfrak{i}) \neq \Phi(\mathfrak{k}). \tag{276}$$

Seventh, no carrier-offspring coupling is preserved:

$$\Phi(\mathcal{A}) \times \Phi(\mathcal{R}_{\mathrm{pre}}) = 0. \tag{277}$$

Eighth, primitive reach admissibility is preserved:

$$\text{Reach} > 4 \Longrightarrow \text{effective product is inadmissible.} \tag{278}$$

Ninth, primitive left-nested closure is preserved:

$$\Phi\Big(\mathrm{Cl}_L(x_1, \ldots, x_n)\Big) = (\cdots((\Phi(x_1)\Phi(x_2))\Phi(x_3))\cdots\Phi(x_n)). \tag{279}$$

Tenth, the realization preserves the distinction between pre-algebraic formation and effective algebraic closure:

$$\Phi(\mathcal{A}) \times \Phi(\mathcal{A}) \not\equiv \Phi(\mathcal{R}_{\mathrm{pre}}) \quad \text{as a formation rule.} \tag{280}$$

Eleventh, the realization is linearly faithful: distinct primitive images remain linearly independent unless an explicit primitive rule forces their identification:

$$\sum_{\ell} \alpha_\ell \Phi(u_\ell) = 0 \Longrightarrow \alpha_\ell = 0 \quad \text{unless forced by a stated primitive closure rule.} \tag{281}$$

A realization satisfying (271)–(281) is called faithful.

A.4 Minimal Faithful Linear Realization

A faithful realization is called a minimal faithful linear realization if the effective image space is generated linearly by the images of the primitive carrier class, the primitive relational-state class, and the two central conditions, with no additional linearly independent images and no linear identifications except those explicitly forced by primitive closure requirements.

Thus, if u and v are images of distinct primitive objects belonging either to distinct primitive classes or to distinct members of the same primitive class, then no linear relation

$$\alpha u + \beta v = 0 \tag{282}$$

is allowed unless it is forced by a stated primitive identification, primitive closure-return rule, or primitive admissibility collapse.

More generally, for distinct primitive images $u_1, \ldots, u_N$, a relation

$$\sum_{\ell=1}^{N} \alpha_\ell u_\ell = 0 \tag{283}$$

is forbidden unless it follows from an explicit primitive rule.

The primitive architecture contains no rule identifying two distinct primitive carriers:

$$a_i \neq a_j \qquad (i \neq j). \tag{284}$$

It contains no rule identifying two distinct relational states:

$$\rho_i^{(m)} \neq \rho_j^{(n)} \qquad ((i,m) \neq (j,n)). \tag{285}$$

It contains no rule identifying a carrier with a relational state:

$$a_i \neq \rho_j^{(m)}. \tag{286}$$

It contains no rule identifying a central condition with a non-central object:

$$\mathfrak{i}, \mathfrak{k} \notin \mathcal{N}_{\text{pre}}. \tag{287}$$

Therefore, in a minimal faithful linear realization, the effective images of these primitive distinctions remain linearly independent.

This is not an additional physical assumption. It is the algebraic expression of faithful primitive distinction: a distinction present in the primitive layer must not disappear as a linear dependence unless the primitive layer itself contains a rule forcing that disappearance.

Minimality also forbids introducing any additional linearly independent effective image not corresponding to a primitive carrier, a primitive relational state, or one of the two central conditions:

$$\text{no extra independent image beyond } \Phi(\mathcal{A}) \cup \Phi(\mathcal{R}_{\text{pre}}) \cup \{\Phi(\mathfrak{i}), \Phi(\mathfrak{k})\}. \tag{288}$$

A.5 Readability-Preserving Effective Image

A faithful effective realization is readability-preserving if the image of the direct-readability condition leaves every already formed effective image unchanged under readability.

Thus, for every effective image x,

$$\Phi(\mathfrak{i})x = x\Phi(\mathfrak{i}) = x. \tag{289}$$

This is the minimal algebraic representation of direct readability. If the readability image changed an effective image, then it would not represent readability alone. It would represent an additional operation not present in the primitive condition $\mathfrak{i}$.

Therefore, in the readability-preserving realization, the image of $\mathfrak{i}$ must be the effective neutral readability element:

$$\Phi(\mathfrak{i}) = e_0, \qquad e_0x = xe_0 = x. \tag{290}$$

This condition does not introduce a physical observer. It only states that the algebraic image of direct readability must not alter what it reads.

A.6 Minimal Non-Degenerate Signed Realization

A signed effective realization is non-degenerate if the non-central self-return class is represented by a nonzero central return class.

Hence, for every non-central effective basis image e_a, $a = 1, \ldots, 15$, one requires

$$e_a^2 \neq 0. \tag{291}$$

The realization is minimal if it introduces no additional central return class beyond the images of $\mathfrak{i}$ and $\mathfrak{k}$. Any such additional central return class would represent a third primitive central condition not present in $\mathfrak{U}_{\text{pre}}$.

The mediated closure return has already occupied the positive readability class:

$$e_{16}^2 = +e_0. \tag{292}$$

Since the primitive layer distinguishes non-central self-return from mediated closure return,

$$\text{Self}(x) \neq \mathfrak{k} \circ \mathfrak{k}, \qquad x \in \mathcal{N}_{\text{pre}}, \tag{293}$$

the non-central self-return cannot be represented by the same $+e_0$ class.

The alternative 0 is excluded by non-degeneracy:

$$e_a^2 = 0 \quad \text{is forbidden for} \quad a = 1, \ldots, 15. \tag{294}$$

A new central return image is excluded by minimality:

$$e_a^2 = \eta_{\text{new}} \quad \text{with} \quad \eta_{\text{new}} \notin \text{span}\{e_0, e_{16}\} \quad \text{is forbidden.} \tag{295}$$

Therefore the only remaining minimal signed central return class is

$$-e_0. \tag{296}$$

Thus, within the first minimal non-degenerate signed realization, the primitive distinction between non-central self-return and mediated closure return is represented as

$$\text{Self}(\mathcal{N}_{\text{pre}}) \longmapsto -e_0, \qquad \mathfrak{k} \circ \mathfrak{k} \longmapsto +e_0. \tag{297}$$

This does not assert that the primitive layer contains a negative sign. It asserts only that once a minimal non-degenerate signed algebraic realization is required, the primitive distinction has the unique minimal signed representation $-e_0$ versus $+e_0$.

A.7 Formation Is Not Effective Multiplication

The primitive generation

$$\mathcal{A} \Longrightarrow \mathcal{R}_{\text{pre}} \tag{298}$$

is not an algebraic product. It is a pre-algebraic formation of relational offspring.

After the effective realization has been introduced, the product of two carrier images is an effective closure operation, not a repetition of the primitive formation operation.

Therefore

$$\Phi(\mathcal{A}) \times \Phi(\mathcal{A}) \neq \Phi(\mathcal{R}_{\text{pre}}) \tag{299}$$

as a formation rule.

If $F \times F$ were allowed to produce R, then the effective algebraic product would identify two different levels:

$$\text{pre-algebraic generation} \quad \text{and} \quad \text{effective algebraic closure.} \tag{300}$$

Such an identification would violate the foundational distinction among primitive carriers, relational offspring, and effective closure.

Thus the generated status of R does not mean that carrier images multiply into R. It means that relational-state images originate pre-algebraically from carrier coexistence. Once the effective algebraic realization exists, closure products must respect the primitive class distinction.

Consequently, carrier-image products cannot close into R. Their admissible closure target is the central sector C.

A.8 Lemma 1: At Least Three Effective Carrier Images Are Necessary

Lemma 1. Any faithful effective realization contains at least three distinct effective carrier images.

Proof. The primitive carrier set contains three pairwise distinguishable elements:

$$\mathcal{A} = \{a_1, a_2, a_3\}. \tag{301}$$

Faithfulness requires distinct primitive carriers to have distinct effective images:

$$\Phi(a_i) \neq \Phi(a_j) \qquad (i \neq j). \tag{302}$$

Therefore

$$|\Phi(\mathcal{A})| \geq 3. \tag{303}$$

If fewer than three effective carrier images were used, then at least two distinct primitive carriers would have the same image, contradicting (271). Hence at least three effective carrier images are necessary:

$$|\Phi(\mathcal{A})|_{\min} \geq 3. \tag{304}$$

□

A.9 Lemma 2: A Fourth Effective Carrier Image Is Non-Minimal

Lemma 2. A faithful minimal effective realization contains exactly three effective carrier images. A fourth effective carrier image is non-minimal.

Proof. By Lemma 1, at least three effective carrier images are necessary.

Suppose a fourth effective carrier image u is added to the carrier sector.

There are two cases.

First, suppose

$$u \in \mathrm{Gen}_{\mathrm{eff}}\Big(\Phi(a_1), \Phi(a_2), \Phi(a_3)\Big). \tag{305}$$

Then u is generated from the three carrier images and is not primitive as a carrier image. It cannot be counted as an additional primitive carrier image.

Second, suppose

$$u \notin \mathrm{Gen}_{\mathrm{eff}}\Big(\Phi(a_1), \Phi(a_2), \Phi(a_3)\Big). \tag{306}$$

Then u represents an additional primitive independence not present in $\mathfrak{U}_{\mathrm{pre}}$. This violates faithfulness to the primitive carrier architecture.

Thus a fourth effective carrier image is either derived and therefore not primitive, or independent and therefore unfaithful to the primitive data. Hence the faithful minimal number of effective carrier images is exactly three:

$$|\Phi(\mathcal{A})| = 3. \tag{307}$$

□

A.10 Lemma 3: Exactly Twelve Effective Relational-State Images Are Necessary

Lemma 3. Any faithful minimal effective realization contains exactly twelve effective relational-state images.

Proof. The primitive relational state-space is

$$\mathcal{R}_{\mathrm{pre}} = \left\{\rho_i^{(m)} : i \in \{1,2,3\},\ m \in \mathbb{Z}_4\right\}. \tag{308}$$

Therefore

$$|\mathcal{R}_{\mathrm{pre}}| = 12. \tag{309}$$

Faithfulness requires distinct primitive relational states to have distinct effective images:

$$(i,m) \neq (j,n) \Longrightarrow \Phi(\rho_i^{(m)}) \neq \Phi(\rho_j^{(n)}). \tag{310}$$

Hence at least twelve effective relational-state images are necessary:

$$|\Phi(\mathcal{R}_{\mathrm{pre}})| \geq 12. \tag{311}$$

If more than twelve effective relational-state images are introduced, then any additional image has one of two statuses.

Either it is generated from the twelve images:

$$r_{\mathrm{extra}} \in \mathrm{Gen}_{\mathrm{eff}}\Big(\Phi(\mathcal{R}_{\mathrm{pre}})\Big), \tag{312}$$

in which case it is not primitive as a relational-state image, or it is not so generated:

$$r_{\text{extra}} \notin \mathrm{Gen}_{\text{eff}}\Big(\Phi(\mathcal{R}_{\text{pre}})\Big), \tag{313}$$

in which case it represents a primitive relational state not present in $\mathfrak{U}_{\text{pre}}$.

Therefore the faithful minimal number of effective relational-state images is exactly twelve:

$$|\Phi(\mathcal{R}_{\text{pre}})| = 12. \tag{314}$$

□

A.11 Lemma 4: Exactly Two Effective Central Images Are Necessary

Lemma 4. Any faithful minimal effective realization contains exactly two effective central-condition images.

Proof. The primitive architecture contains exactly two central conditions:

$$\mathfrak{i}, \qquad \mathfrak{k}. \tag{315}$$

They are distinct:

$$\mathfrak{i} \neq \mathfrak{k}. \tag{316}$$

Faithfulness requires their effective images to be distinct:

$$\Phi(\mathfrak{i}) \neq \Phi(\mathfrak{k}). \tag{317}$$

Thus at least two effective central-condition images are necessary:

$$|\Phi(\{\mathfrak{i}, \mathfrak{k}\})| \geq 2. \tag{318}$$

If a third central image is introduced, then either it is generated by $\Phi(\mathfrak{i})$ and $\Phi(\mathfrak{k})$, in which case it is not primitive as a central-condition image, or it represents a third primitive central condition not present in $\mathfrak{U}_{\text{pre}}$, in which case faithfulness fails.

Therefore the faithful minimal number of effective central-condition images is exactly two:

$$|\Phi(\{\mathfrak{i}, \mathfrak{k}\})| = 2. \tag{319}$$

□

A.12 Theorem 1: The Minimal Faithful Linear Basis Has Seventeen Elements

Theorem 1. Every minimal faithful linear effective realization of $\mathfrak{U}_{\text{pre}}$ has exactly seventeen linearly independent basis images:

$$2 + 3 + 12 = 17. \tag{320}$$

Proof. By Lemma 2, the faithful minimal realization contains exactly three effective carrier images:

$$|\Phi(\mathcal{A})| = 3. \tag{321}$$

By Lemma 3, it contains exactly twelve effective relational-state images:

$$|\Phi(\mathcal{R}_{\text{pre}})| = 12. \tag{322}$$

By Lemma 4, it contains exactly two effective central-condition images:

$$|\Phi(\{\mathfrak{i}, \mathfrak{k}\})| = 2. \tag{323}$$

Faithfulness requires these three image classes to be disjoint:

$$\Phi(\mathcal{A}) \cap \Phi(\mathcal{R}_{\text{pre}}) = \varnothing, \tag{324}$$

$$\Phi(\mathcal{A}) \cap \Phi(\{\mathfrak{i}, \mathfrak{k}\}) = \varnothing, \tag{325}$$

$$\Phi(\mathcal{R}_{\text{pre}}) \cap \Phi(\{\mathfrak{i}, \mathfrak{k}\}) = \varnothing. \tag{326}$$

Therefore there are exactly

$$3 + 12 + 2 = 17 \tag{327}$$

distinct primitive images.

By the minimal faithful linear realization condition, these seventeen distinct primitive images are linearly independent because no primitive rule identifies any two of them and no primitive closure rule imposes a linear dependence among them.

Therefore these seventeen images form the effective basis of the first minimal faithful linear realization:

$$\dim \mathfrak{H}_{\text{eff}} = 17. \tag{328}$$

This proves the theorem. ☐

A.13 Canonical Labeling of the Seventeen Basis Images

By Theorem 1, a faithful minimal linear realization has seventeen basis images. A canonical labeling may be chosen as follows.

The direct-readability condition is labeled by

$$\Phi(\mathfrak{i}) = e_0. \tag{329}$$

The mediated-closure condition is labeled by

$$\Phi(\mathfrak{k}) = e_{16}. \tag{330}$$

The three primitive carriers are labeled by

$$\Phi(a_1) = e_1, \qquad \Phi(a_2) = e_2, \qquad \Phi(a_3) = e_3. \tag{331}$$

The twelve primitive relational states are labeled by

$$\Phi(\mathcal{R}_{\text{pre}}) = \{e_4, e_5, \ldots, e_{15}\}. \tag{332}$$

Define the central sector:

$$C = \mathrm{span}\{e_0, e_{16}\}. \tag{333}$$

Define the carrier sector:

$$F = \mathrm{span}\{e_1, e_2, e_3\}. \tag{334}$$

Define the relational-state sector:

$$R = \mathrm{span}\{e_4, e_5, \ldots, e_{15}\}. \tag{335}$$

The first minimal faithful linear realization therefore decomposes as

$$\mathfrak{H}_{\mathrm{eff}} = C \oplus F \oplus R. \tag{336}$$

The direct sum in (336) follows from linear faithfulness: the central-condition images, carrier images, and relational-state images are linearly independent and belong to distinct primitive classes.

A.14 Theorem 2: Direct Readability Induces the Effective Identity

Theorem 2. In a readability-preserving faithful effective realization, the direct-readability image e_0 acts as the effective identity:

$$e_0 e_a = e_a e_0 = e_a \qquad (a = 0, 1, \ldots, 16). \tag{337}$$

Proof. By definition of the canonical labeling,

$$\Phi(\mathfrak{i}) = e_0. \tag{338}$$

By readability preservation, for every effective image x,

$$\Phi(\mathfrak{i})x = x\Phi(\mathfrak{i}) = x. \tag{339}$$

Substituting $\Phi(\mathfrak{i}) = e_0$, one obtains

$$e_0 x = x e_0 = x. \tag{340}$$

Applying this to all basis images e_a, $a = 0, \ldots, 16$, yields

$$e_0 e_a = e_a e_0 = e_a. \tag{341}$$

Thus e_0 is the effective identity because it is the readability-preserving image of direct readability. □

A.15 Theorem 3: Mediated Closure Induces the Hyperbolic Return Image

Theorem 3. In a faithful effective realization, the mediated-closure image satisfies

$$e_{16}^2 = e_0. \tag{342}$$

Proof. The primitive mediated-closure return condition is

$$\mathfrak{k} \circ \mathfrak{k} = \mathfrak{i}. \tag{343}$$

The canonical labeling gives

$$\Phi(\mathfrak{k}) = e_{16}, \qquad \Phi(\mathfrak{i}) = e_0. \tag{344}$$

Faithfulness of the effective realization requires the closure-return condition to be preserved:

$$\Phi(\mathfrak{k} \circ \mathfrak{k}) = \Phi(\mathfrak{i}). \tag{345}$$

Therefore

$$\Phi(\mathfrak{k})^2 = \Phi(\mathfrak{i}), \tag{346}$$

and hence

$$e_{16}^2 = e_0. \tag{347}$$

Thus the hyperbolic return image $e_{16}^2 = e_0$ is induced by primitive mediated closure. It is not an independent algebraic insertion. ☐

A.16 Theorem 4: Non-Central Self-Return Induces the Negative Effective Return Class

Theorem 4. In the first minimal non-degenerate signed effective realization preserving the distinction between non-central self-return and mediated closure return, every non-central basis image satisfies

$$e_a^2 = -e_0 \qquad (a = 1, \ldots, 15). \tag{348}$$

Proof. The primitive non-central class is

$$\mathcal{N}_{\text{pre}} = \mathcal{A} \cup \mathcal{R}_{\text{pre}}. \tag{349}$$

Its effective image is

$$\Phi(\mathcal{N}_{\text{pre}}) = \{e_1, e_2, \ldots, e_{15}\}. \tag{350}$$

The primitive distinction is

$$\text{Self}(x) \neq \mathfrak{k} \circ \mathfrak{k} \qquad (x \in \mathcal{N}_{\text{pre}}). \tag{351}$$

By Theorem 3,

$$\Phi(\mathfrak{k} \circ \mathfrak{k}) = +e_0. \tag{352}$$

A minimal signed central return representation has only the signed readability classes

$$+e_0 \qquad \text{and} \qquad -e_0 \tag{353}$$

available without introducing a new central return image.

The class $+e_0$ is already occupied by mediated closure return:

$$\mathfrak{k} \circ \mathfrak{k} \longmapsto +e_0. \tag{354}$$

The alternative 0 is excluded by non-degeneracy:

$$e_a^2 = 0 \qquad (a = 1, \dots, 15) \tag{355}$$

is forbidden.

A new central image is excluded by minimality:

$$e_a^2 = \eta_{\text{new}} \qquad \eta_{\text{new}} \notin C \tag{356}$$

is forbidden.

Therefore the only remaining minimal signed central return class for non-central self-return is

$$-e_0. \tag{357}$$

Hence

$$e_a^2 = -e_0 \qquad (a = 1, \dots, 15). \tag{358}$$

This proves the theorem. □

Clarification. The theorem does not assert that the primitive layer contains a negative sign. It asserts only that, once a minimal non-degenerate signed algebraic realization is required, the primitive distinction between non-central self-return and mediated closure return has the unique minimal signed representation $-e_0$ versus $+e_0$.

A.17 Theorem 5: Induced Sector Product Policies

Theorem 5. The faithful minimal effective realization induces

$$F \times R = 0, \tag{359}$$

$$F \times F \subseteq C, \tag{360}$$

and

$$R \times R \subseteq C. \tag{361}$$

Proof. The primitive no carrier-offspring coupling rule is

$$\mathrm{Couple}(\mathcal{A}, \mathcal{R}_{\text{pre}}) = \varnothing. \tag{362}$$

Since

$$\Phi(\mathcal{A}) = F \tag{363}$$

and

$$\Phi(\mathcal{R}_{\text{pre}}) = R, \tag{364}$$

the effective image of no carrier-offspring coupling is

$$F \times R = 0. \tag{365}$$

Next, primitive carrier independence forbids any primitive carrier from being generated by the other two:

$$a_k \notin \mathrm{Gen}(a_i, a_j) \qquad (i, j, k \text{ distinct}). \tag{366}$$

Therefore an effective product of carrier images cannot close inside F. If it did, one carrier image would be obtainable as an effective closure of other carrier images, contradicting faithful preservation of primitive non-derivability.

The product $F \times F$ also cannot close inside R. The reason is not numerical; it is structural. The formation of $\mathcal{R}_{\text{pre}}$ from $\mathcal{A}$ is a pre-algebraic generation process, whereas $F \times F$ is an effective algebraic closure operation. Allowing

$$F \times F \subseteq R \tag{367}$$

would identify pre-algebraic formation with effective multiplication, contradicting the formation/closure distinction in Section A.7.

Hence the only faithful minimal closure target for $F \times F$ is the central sector:

$$F \times F \subseteq C. \tag{368}$$

Finally, relational-state images are offspring images, not primitive carrier images. Closure of relational-state images cannot generate carrier images without violating the carrier/offpsring distinction:

$$R \times R \not\subseteq F. \tag{369}$$

Because relational-state closure must return to direct readability or mediated closure, its faithful effective closure target is central:

$$R \times R \subseteq C. \tag{370}$$

This proves the three induced sector product policies. ☐

A.18 Theorem 6: Oriented Carrier-Pair Closure

Theorem 6. For distinct carrier images $e_a, e_b \in F$, the product closes into the mediated central image:

$$e_a e_b = \sigma_F(a,b) e_{16} \qquad (a, b \in \{1,2,3\},\ a \neq b), \tag{371}$$

with

$$\sigma_F(a,b) = -\sigma_F(b,a). \tag{372}$$

Proof. By Theorem 5,

$$F \times F \subseteq C. \tag{373}$$

For self-return, Theorem 4 gives

$$e_a^2 = -e_0 \qquad (a = 1,2,3). \tag{374}$$

For distinct carrier images, the product must not produce another carrier image because that would contradict carrier non-derivability. It must not produce a relational-state image because that would confuse pre-algebraic relational formation with effective algebraic closure. Hence it must lie in C.

The central sector has two effective central images:

$$C = \operatorname{span}\{e_0, e_{16}\}. \tag{375}$$

The e_0-channel is already used by direct readability and by non-central self-return after signed realization. The distinct-carrier product is not self-return; it is a mediated carrier-pair closure. Therefore the minimal central target is the mediated closure image:

$$e_a e_b \in \operatorname{span}\{e_{16}\} \qquad (a \neq b). \tag{376}$$

Exchange of the ordered carrier pair reverses orientation. Therefore

$$\sigma_F(a,b) = -\sigma_F(b,a). \tag{377}$$

Thus

$$e_a e_b = \sigma_F(a,b) e_{16} \qquad (a \neq b). \tag{378}$$

□

A fixed orientation convention may be chosen by

$$e_1 e_2 = e_{16}, \qquad e_2 e_3 = e_{16}, \qquad e_3 e_1 = e_{16}, \tag{379}$$

with reversed products

$$e_2 e_1 = -e_{16}, \qquad e_3 e_2 = -e_{16}, \qquad e_1 e_3 = -e_{16}. \tag{380}$$

A.19 Theorem 7: Primitive Left-Nested Closure Induces Effective Left-Nesting

Theorem 7. The effective realization admits only left-nested multi-product evaluation:

$$x_1 x_2 \cdots x_n = (\cdots((x_1 x_2) x_3) \cdots x_n). \tag{381}$$

Proof. At the primitive level, multi-step closure is defined only by

$$\mathrm{Cl}_L(x_1, x_2, \ldots, x_n) = \mathrm{Cl}\Big(\mathrm{Cl}_L(x_1, \ldots, x_{n-1}), x_n\Big). \tag{382}$$

Faithfulness requires the effective product to preserve this closure path:

$$\Phi\Big(\mathrm{Cl}_L(x_1, \ldots, x_n)\Big) = (\cdots((\Phi(x_1)\Phi(x_2))\Phi(x_3)) \cdots \Phi(x_n)). \tag{383}$$

If another bracketing were allowed, for example

$$x_1(x_2 x_3), \tag{384}$$

then an additional rule would be required to compare it with

$$(x_1 x_2) x_3. \tag{385}$$

No associativity rule or bracketing-comparison rule exists in the primitive architecture. Therefore the only faithful effective multi-product evaluation path is the left-nested rule (381). □

A.20 Theorem 8: Twelvefold Relational-State Generator

Theorem 8. The effective relational-state sector admits a twelve-step generator satisfying

$$L_{16}^{12}\Big|_R = \mathrm{Id}_R\,. \tag{386}$$

Proof. The primitive relational state-space has twelve elements:

$$|\mathcal{R}_{\mathrm{pre}}| = 12. \tag{387}$$

The primitive relational state-shift $\mathcal{L}$ generates the closure-state progression:

$$\mathcal{L}\rho_i^{(m)} = \rho_i^{(m+1 \bmod 4)}. \tag{388}$$

Under the effective realization, the twelve-state relational ordering is represented by the sector R. Let L_{16} denote the effective image of the induced twelve-state shift.

Since the effective relational-state sector has twelve states, the induced generator returns after twelve steps:

$$L_{16}^{12}\Big|_R = \mathrm{Id}_R\,. \tag{389}$$

This proves the theorem. □

A.21 Theorem 9: Induced Admissibility Window

Theorem 9. The primitive reach bound induces the effective admissibility window

$$\Delta(i,j) \le 4. \tag{390}$$

Pairs with

$$\Delta(i,j) \in \{5,6\} \tag{391}$$

are inadmissible and are represented by domain-as-zero:

$$\Delta(i,j) \in \{5,6\} \Longrightarrow e_i e_j = 0. \tag{392}$$

Proof. The primitive reach rule is

$$\mathrm{Reach} \le 4. \tag{393}$$

In the effective rim, let $p(e_i) \in \mathbb{Z}_{12}$ denote the induced cyclic position. Define

$$\Delta(i,j) = \min\left(|p(e_i) - p(e_j)|, 12 - |p(e_i) - p(e_j)|\right). \tag{394}$$

Faithfulness requires the effective admissibility rule to preserve primitive reach. Hence admissible effective products must satisfy

$$\Delta(i,j) \le 4. \tag{395}$$

On a twelve-cycle, the distances exceeding 4 are

$$5 \quad \text{and} \quad 6. \tag{396}$$

These exceed the primitive four-state closure reach. Therefore they are inadmissible. The effective algebraic image of inadmissibility is domain-as-zero:

$$e_i e_j = 0. \tag{397}$$

Therefore

$$\Delta(i,j) \in \{5,6\} \Longrightarrow e_i e_j = 0. \tag{398}$$

□

A.22 Theorem 10: Induced Four-Class Channel Policy

Theorem 10. The primitive four-state relational closure cycle induces the effective channel policy

$$d = 0 \Longrightarrow +e_0, \qquad d = 2 \Longrightarrow +e_{16}, \tag{399}$$

while the transition-like classes

$$d \in \{1, 3\} \tag{400}$$

require exchange orientation.

Proof. The primitive four-state cycle is

$$\mathbb{Z}_4 = \{0, 1, 2, 3\}. \tag{401}$$

The state $m = 0$ represents direct readability:

$$m = 0 \Longrightarrow \mathfrak{i}. \tag{402}$$

The state $m = 2$ represents mediated closure:

$$m = 2 \Longrightarrow \mathfrak{k}. \tag{403}$$

Under the effective realization,

$$\mathfrak{i} \mapsto e_0, \qquad \mathfrak{k} \mapsto e_{16}. \tag{404}$$

Therefore the even closure classes are represented by

$$d = 0 \Longrightarrow +e_0, \qquad d = 2 \Longrightarrow +e_{16}. \tag{405}$$

The classes 1 and 3 are transition-like. They are neither direct readability nor mediated closure. They require exchange orientation when an ordered comparison is made:

$$d \in \{1, 3\} \Longrightarrow \sigma(i, j) = -\sigma(j, i). \tag{406}$$

This proves the theorem. □

A.23 Corollary 1: A Fourth Carrier Changes the Architecture

Corollary 1. Adding a fourth primitive carrier does not refine the same minimal architecture. It changes the architecture.

Proof. If one begins with four primitive carriers,

$$\mathcal{A}_4 = \{a_1, a_2, a_3, a_4\}, \tag{407}$$

then the number of unordered carrier-pair roots becomes

$$\binom{4}{2} = 6. \tag{408}$$

With the same minimal four-state closure cycle, the relational-state count becomes

$$6 \cdot 4 = 24. \tag{409}$$

Including two central conditions, the corresponding minimal count would be

$$4 + 24 + 2 = 30. \tag{410}$$

Thus a fourth primitive carrier does not produce the same 17-basis architecture. It produces a different primitive architecture with a different minimal effective realization.

If the fourth carrier is derived from the first three, it is not primitive. If it is independent, it changes the primitive data. Therefore a fourth carrier either fails primitivity or breaks the minimal architecture. □

A.24 Corollary 2: Fewer Than Twelve Relational-State Images Fails Faithfulness

Corollary 2. A faithful realization with fewer than twelve relational-state images cannot preserve $\mathcal{R}_{\text{pre}}$.

Proof. The primitive relational state-space has cardinality

$$|\mathcal{R}_{\text{pre}}| = 12. \tag{411}$$

If fewer than twelve relational-state images are used, then by the pigeonhole principle two distinct primitive relational states must be identified:

$$\Phi(\rho_i^{(m)}) = \Phi(\rho_j^{(n)}) \qquad \text{for some } (i, m) \neq (j, n). \tag{412}$$

This contradicts faithfulness condition (274). Hence fewer than twelve relational-state images fails faithfulness. □

A.25 Corollary 3: More Than Twelve Relational-State Images Fails Minimality

Corollary 3. A realization with more than twelve relational-state images is non-minimal unless it represents a different primitive architecture.

Proof. The primitive architecture generates exactly twelve relational states:

$$|\mathcal{R}_{\text{pre}}| = 12. \tag{413}$$

An additional relational-state image r_{extra} must either be generated from the twelve images:

$$r_{\text{extra}} \in \text{Gen}_{\text{eff}}\Big(\Phi(\mathcal{R}_{\text{pre}})\Big), \tag{414}$$

or not:

$$r_{\text{extra}} \notin \text{Gen}_{\text{eff}}\Big(\Phi(\mathcal{R}_{\text{pre}})\Big). \tag{415}$$

In the first case, it is not primitive as a relational-state image. In the second case, it represents a relational state not present in $\mathfrak{U}_{\text{pre}}$. Therefore more than twelve relational-state images either violates minimality or changes the primitive architecture. □

A.26 Final Minimality Statement

Combining the previous lemmas, theorems, and corollaries, the first minimal faithful linear, readability-preserving, non-degenerate signed effective algebraic realization of the primitive relational architecture has the form

$$\mathfrak{H}_{\mathrm{eff}} = C \oplus F \oplus R. \tag{416}$$

The sectors are

$$C = \mathrm{span}\{e_0, e_{16}\}, \tag{417}$$

$$F = \mathrm{span}\{e_1, e_2, e_3\}, \tag{418}$$

$$R = \mathrm{span}\{e_4, e_5, \ldots, e_{15}\}. \tag{419}$$

The dimension is

$$\dim \mathfrak{H}_{\mathrm{eff}} = 2 + 3 + 12 = 17. \tag{420}$$

The induced direct-readability rule is

$$e_0 e_a = e_a e_0 = e_a \qquad (a = 0, 1, \ldots, 16). \tag{421}$$

The induced mediated-closure return is

$$e_{16}^2 = e_0. \tag{422}$$

The induced non-central self-return rule is

$$e_a^2 = -e_0 \qquad (a = 1, \ldots, 15). \tag{423}$$

The induced sector policies are

$$F \times F \subseteq C, \qquad R \times R \subseteq C, \qquad F \times R = 0. \tag{424}$$

The induced twelvefold generator is

$$L_{16}^{12}\Big|_R = \mathrm{Id}_R\,. \tag{425}$$

The induced admissibility window is

$$\Delta \leq 4, \qquad \Delta \in \{5, 6\} \Longrightarrow 0. \tag{426}$$

The induced effective multi-product rule is

$$x_1 x_2 \cdots x_n = (\cdots((x_1 x_2) x_3) \cdots x_n). \tag{427}$$

Therefore the effective algebraic axioms are not independent postulates detached from the primitive layer. They are the induced minimal faithful algebraic image of the primitive relational architecture under the stated structural requirements.

A.27 Limits of the Result

The result proved in this appendix is conditional.

It is conditional on the primitive architecture $\mathfrak{U}_{\mathrm{pre}}$:

$$\mathfrak{U}_{\mathrm{pre}} = \Big(\mathcal{A}, \mathcal{R}_{\mathrm{pre}}, \mathfrak{i}, \mathfrak{k}, \mathcal{L}, \mathrm{Reach}, \mathrm{Cl}, \mathrm{Couple}\Big). \tag{428}$$

It is conditional on faithfulness:

$$\text{primitive distinctions are preserved.} \tag{429}$$

It is conditional on linear faithfulness:

$$\text{distinct primitive images remain linearly independent unless a primitive rule forces otherwise.} \tag{430}$$

It is conditional on readability preservation:

$$\Phi(\mathfrak{i})x = x\Phi(\mathfrak{i}) = x. \tag{431}$$

It is conditional on non-degenerate signed realization:

$$e_a^2 \neq 0 \qquad (a = 1, \dots, 15). \tag{432}$$

It is conditional on minimality:

$$\text{no additional primitive carrier image, relational-state image, or central image is introduced.} \tag{433}$$

Under these conditions, the minimal effective algebraic realization is uniquely fixed up to relabeling:

$$\mathfrak{U}_{\mathrm{pre}} + \text{faithful minimal realization requirements} \Longrightarrow \mathfrak{H}_{\mathrm{eff}} = C \oplus F \oplus R, \qquad \dim \mathfrak{H}_{\mathrm{eff}} = 17. \tag{434}$$

The theorem does not assert that a different primitive architecture could not be constructed. It asserts that once the primitive architecture and the faithful minimal realization requirements are fixed, the 17-basis effective realization follows.

This is the mathematically defensible form of the minimality claim.