

# CONTRIBUTIONS TO THE STUDY OF THE INDUCTION COIL.

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## § 1. INTRODUCTION.

THE induction coil has been investigated in recent years by Colley, Oberbeck, Walter, Johnson and others, both theoretically and experimentally. A list of the more important recent contributions to the subject is given at the end of this paper.

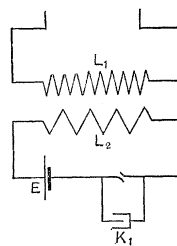


Fig. 1.

The induction coil is an electromagnetic system with two degrees of freedom. It is usually constructed by winding a *secondary* coil of a large number of turns of fine wire upon a primary of a few turns of heavy wire. (See Fig. 1.) The core of the primary may be filled with iron wires to increase its inductance. The terminals of the primary are so arranged that they can be conveniently attached to some source of electromotive force, and the primary circuit is provided

with a make-and-break apparatus. A condenser of suitable capacity is put around the break, so as to diminish as far as possible the sparking at the break.

The usual method of operating the coil is as follows. A battery having been placed in the primary circuit, the circuit is opened and closed periodically. The condenser around the break is then ad-

justed so that the sparking at the break is reduced to a minimum. The secondary circuit can be left open, or can be closed through a vacuum tube, an electrolytic cell, or in any other way that is desired.

The action of the coil may be briefly stated. When the primary circuit is closed, the current begins to grow, and soon reaches what is practically its maximum value. This *make current* in the primary induces an electromotive force in the secondary, which, however is not of any importance. The primary circuit is now broken, and owing to its capacity, that is, to the capacity around the break, dies away in a series of oscillations, whose period is constant, but whose amplitude is rapidly decreasing. In a small induction coil they are so rapid that there are 1,000 to 10,000 of them in a second. These oscillations of the primary current set up an induced electromotive force in the secondary, which depends for its magnitude, as will be shown later, upon the constants of the coil, the initial strength of the primary current, the velocity of breaking the primary, and the capacity of the condenser around the break.

The usual method of using a coil is with a slow break, the primary being broken less than a hundred times a second. In this case the current in the primary rises to approximately its full value before the break occurs, and the oscillations due to the break die away before the next make. The current under these conditions would be represented by Fig. 2. This case will evidently be completely covered if a *single* break is considered.

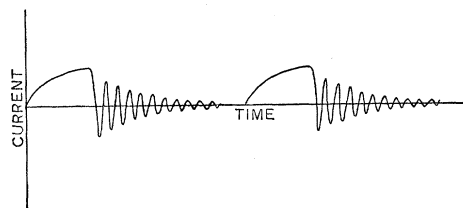


Fig. 2.

If the break is *not* slow, as in the case of the Wehnelt interruptor, the theory becomes more complicated, as the make current does not have time to rise to a steady value before the break occurs, and the break current also overlaps the next make current.

The two simplest cases will be treated in this paper, viz. : first, the case of a single break, and second, the case of a simple harmonic electromotive force in the primary.

## § 2. THEORY OF THE COIL.

In general both the primary and secondary circuits have inductance, resistance and capacity.

It will be seen from Fig. 1, that when the primary circuit is closed, its capacity may be considered infinite, but when it is broken, its capacity is that of the condenser  $K$ .

The capacity of the secondary circuit in the ordinary mode of using the induction coil is the distributed capacity due to the juxtaposition of successive turns of the coil. This, although very small, is not infinitesimal. This question of the distributed capacity of the secondary will be discussed more fully further on in this paper.

*Theory of One Circuit for a Single Break.*

Since the theory of a *single* circuit containing an inductance, resistance, electromotive force, and a condenser around the break is closely related to that of the primary of an induction coil, a brief discussion of the single circuit will be given before the discussion of two circuits. The same conditions will be supposed to hold as for the primary, viz: the breaks and makes will be sufficiently far apart so that the oscillations due to the break have died away before the make occurs.

We will then suppose the current to have risen to its maximum value before the break occurs, and will consider what takes place after breaking it.

The well-known equation for such a circuit is the following:

$$(1) \quad L \frac{dI}{dt} + RI + \frac{1}{K} \int I dt = E,$$

where  $I$  is the current;  $t$ , the time;  $L$ , the inductance;  $R$ , the resistance;  $K$ , the capacity around the break; and  $E$  the steady impressed electromotive force.

This equation may be written in the form

$$(2) \quad L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{K} = E,$$

where  $q$  is the quantity of electricity which has passed any cross-section of the circuit during the time  $t$  or, what is the same thing, the quantity of electricity which has entered the condenser in the time  $t$ . If we bring  $E$  over to the left hand side of the equation, and write

$$q' = q - Q$$

where  $Q = KE$  is the steady charge of the condenser under the electromotive force  $E$ , equation (2) becomes

$$(3) \quad L \frac{d^2 q'}{dt^2} + R \frac{dq'}{dt} + \frac{q'}{K} = 0.$$

This is the well-known equation for a system with one degree of freedom.<sup>1</sup>

It is found that the charge is oscillatory or non-oscillatory according as

$$R^2 < 4 \frac{L}{K} \quad \text{or} \quad R^2 > 4 \frac{L}{K}.$$

In this discussion we shall consider only oscillatory discharges, since the induced electromotive force in the secondary of an induction coil due to a non-oscillatory discharge is so small that it is of no practical importance.

The solution for (3) is given by

$$(4) \quad q' = e^{\mu t} (A \cos \nu t + B \sin \nu t)$$

where

$$\mu = -\frac{R}{2L},$$

$$\nu = \sqrt{\frac{1}{KL} - \frac{R^2}{4L^2}}.$$

The arbitrary constants  $A$  and  $B$  will be determined by the initial conditions, which in the case we are considering are the following :

When  $t = 0$ ,  $q' = -Q$  and  $I = I_0$ .

<sup>1</sup> See Webster, *Elect. Mag.*, pp. 484-488.

Whence it follows that

$$A = -Q$$

$$B = \frac{I_0 + \mu Q}{\nu}.$$

Therefore since  $q' = q - Q$  we have

$$(5) \quad q = Q - e^{\mu t} \left[ Q \cos \nu t - \frac{I_0 + \mu Q}{\nu} \sin \nu t \right],$$

which may be written

$$(6) \quad q = Q - e^{\mu t} \left[ Q^2 + \left( \frac{I_0 + \mu Q}{\nu} \right)^2 \right]^{\frac{1}{2}} \cos (\nu t + \alpha)$$

where

$$\alpha = \tan^{-1} \frac{I_0 + \mu Q}{\nu Q}.$$

For the current we have

$$(7) \quad I = \frac{dq}{dt} = e^{\mu t} \left[ Q^2 + \left( \frac{I_0 + \mu Q}{\nu} \right)^2 \right]^{\frac{1}{2}} [\nu \sin (\nu t + \alpha) - \mu \cos (\nu t + \alpha)]$$

and for the difference of potential between the two sides of the condenser

$$(8) \quad V = \frac{q}{K} = E - \frac{e^{\mu t}}{K} \left[ Q^2 + \left( \frac{I_0 + \mu Q}{\nu} \right)^2 \right]^{\frac{1}{2}} \cos (\nu t + \alpha).$$

In the case which is usually considered,  $E = 0$ , and equations (6), (7), and (8) become

$$(9) \quad q = \frac{I_0 e^{\mu t}}{\nu} \sin \nu t$$

$$(10) \quad I = \frac{I_0 e^{\mu t}}{\nu} [\nu \cos \nu t + \mu \sin \nu t]$$

$$(11) \quad V = \frac{I_0 e^{\mu t}}{K \nu} \sin \nu t.$$

*Theory of one Circuit. Harmonic E.M.F.*

If instead of the circuit being broken, we have impressed upon it a periodic electromotive force  $E_0 \cos \omega t$  we shall have instead of equation (2)

$$(12) \quad L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{q}{K} = E_0 \cos \omega t.$$

The solution of which is given by

$$(13) \quad q = \frac{E_0}{\omega \sqrt{R^2 + \left(L\omega - \frac{1}{K\omega}\right)^2}} \cos(\omega t - \alpha),$$

$$(14) \quad I = \frac{E_0}{\sqrt{R^2 + \left(L\omega - \frac{1}{K\omega}\right)^2}} \sin(\omega t - \alpha),$$

$$(15) \quad V = \frac{E_0}{\omega K \sqrt{R^2 + \left(L\omega - \frac{1}{K\omega}\right)^2}} \cos(\omega t - \alpha).$$

$V$  is the difference of potential between the two sides of the condenser.  $V_m$  the maximum difference of potential is given by

$$(16) \quad V_m = \frac{E_0}{\omega} \frac{1}{K \sqrt{R^2 + \left(L\omega - \frac{1}{K\omega}\right)^2}}.$$

*Theory of two Circuits for a Single Break.*

The theory of the induction coil is the theory of two circuits. The well-known equations for two circuits, are<sup>1</sup>

$$(17) \quad \begin{aligned} L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt} + R_1 I_1 + \frac{1}{K_1} \int_0^t I_1 dt &= E_1, \\ L_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt} + R_2 I_2 + \frac{1}{K_2} \int_0^t I_2 dt &= E_2. \end{aligned}$$

In the case of the induction coil, since there is no impressed electromotive force in the secondary,  $E_2 = 0$ . Also since the impressed electromotive force in the primary is usually small com-

<sup>1</sup> See Webster, *Elect. Mag.*, pp. 491-502.

pared with the induced electromotive force in the primary,  $E_1$  may be neglected. Then equations (17) may be written

$$(18) \quad \begin{aligned} L_1 \frac{d^2 q_1}{dt^2} + M \frac{d^2 q_2}{dt^2} + R_1 \frac{dq_1}{dt} + \frac{q_1}{K_1} &= 0, \\ L_2 \frac{d^2 q_2}{dt^2} + M \frac{d^2 q_1}{dt^2} + R_2 \frac{dq_2}{dt} + \frac{q_2}{K_2} &= 0. \end{aligned}$$

Equations (18) are the equations for a system with two degrees of freedom, and no impressed forces.

It is well known from the theory of such equations<sup>1</sup> that the displacements  $q_1$  and  $q_2$ , which are here quantities of electricity, may either die away without oscillations, or may die away with a series of oscillations. The first will happen if  $R_1$  and  $R_2$  are both very large. The second case is the case of the induction coil, and the one in which we are interested.

The solutions of these equations may be written down immediately, in the following form:

$$(19) \quad \begin{aligned} q_1 &= e^{-\alpha t}(A_1 \cos \beta t + B_1 \sin \beta t) + e^{-\gamma t}(C_1 \cos \delta t + D_1 \sin \delta t), \\ q_2 &= e^{-\alpha t}(A_2 \cos \beta t + B_2 \sin \beta t) + e^{-\gamma t}(C_2 \cos \delta t + D_2 \sin \delta t). \end{aligned}$$

In which  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\delta$  are given by

$$(20) \quad \begin{aligned} \alpha &= \frac{R_1 L_2 + R_2 L_1}{4m} + \frac{R_1 [L_2 n - 2K_1 m] + R_2 [L_1 n - 2K_2 m]}{4m \sqrt{n^2 + LM^2 K_1 K_2}}, \\ \gamma &= \frac{R_1 L_2 + R_2 L_1}{4m} - \frac{R_1 [L_2 n - 2K_1 m] + R_2 [L_1 n - 2K_2 m]}{4m \sqrt{n^2 + LM^2 K_1 K_2}}, \\ \alpha^2 + \beta^2 &= \frac{n + \sqrt{(L_1 K_1 - L_2 K_2)^2 + 4M^2 K_1 K_2}}{2K_1 K_2 m}, \\ \gamma^2 + \delta^2 &= \frac{n - \sqrt{(L_1 K_1 - L_2 K_2)^2 + 4M^2 K_1 K_2}}{2K_1 K_2 m}. \end{aligned}$$

Where

$$m = L_1 L_2 - M^2,$$

$$n = L_1 K_1 + L_2 K_2.$$

<sup>1</sup> Webster, loc. cit., pp. 494-498.

In finding the values of  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$  we have neglected the product  $\alpha\gamma$  as small compared with the product  $\beta\delta$ . This is justified since  $\alpha$  and  $\gamma$  are the reciprocals of the time constants of the two circuits and  $\beta$  and  $\delta$  are the reciprocals of the periods multiplied by  $2\pi$ , and the periods are small compared with the time constants.

Since equations (19) can be written

$$(21) \quad \begin{aligned} q_1 &= e^{-\alpha t} A \cos(\beta t - \theta_1) + e^{-\gamma t} B \cos(\delta t - \theta_2), \\ q_2 &= e^{-\alpha t} C \cos(\beta t - \theta_3) + e^{-\gamma t} D \cos(\delta t - \theta_4), \end{aligned}$$

where

$$A = \sqrt{A_1^2 + B_1^2}, \quad B = \sqrt{C_1^2 + D_1^2}, \quad C = \sqrt{A_2^2 + B_2^2}, \quad D = \sqrt{C_2^2 + D_2^2},$$

and

$$\theta_1 = \tan^{-1} \frac{B_1}{A_1}, \quad \theta_2 = \tan^{-1} \frac{D_1}{C_1}, \quad \theta_3 = \tan^{-1} \frac{B_2}{A_2}, \quad \theta_4 = \tan^{-1} \frac{D_2}{C_2}.$$

we see that  $q_1$  and  $q_2$  and therefore the currents, are each sums of two damped harmonic oscillations. The coefficients,  $A$ ,  $B$ ,  $C$ ,  $D$ , give the amplitudes of the component oscillations in each circuit.

The constants of equations (19) and (21) must be determined from the initial conditions, viz: when

$$t = 0, \quad q_1 = 0, \quad I_1 = \frac{dq_1}{dt} = I_0,$$

$$q_2 = 0, \quad I_2 = \frac{dq_2}{dt} = 0.$$

Boynton<sup>1</sup> has determined these constants for the case of the Tesla coil, and Colley, Oberbeck, Walter and Johnson have found approximate values for the case of the induction coil.

In the induction coil as ordinarily used there is no condenser in the secondary, and the only capacity it has is the distributed capacity due to the juxtaposition of the successive turns of the coil. This has been found to be very small in ordinary coils. If a coil is wound bifilarly the capacity is very much increased, and the inductance decreased.

<sup>1</sup> PHYS. REV., Vol. VII., pp. 35-48, 1898.



Max Wien<sup>1</sup> found for three coils that he examined the following results :

Coil No. 1.  $R = 2921$  siemens,  
 $L = 3.97$  henries,  
Distributed capacity = .00077 microfarads.

Coil No. 2.  $R = 69$  siemens,  
 $L = .199$  henries,  
Distributed capacity = .0019 microfarads.

Coil No. 3 (wound bifilarly).  
 $R = 135.1$  siemens,  
 $L = .767$  henries,  
Distributed capacity = .0139 microfarads.

Walter<sup>2</sup> found for the secondary of an induction coil having an inductance of about 500 henries a capacity of only about 1.1 millionths of a microfarad ; and for the secondary of a 60 cm. induction coil, having an induction of about 9,000 henries, a capacity of about 6.5 millionths of a microfarad.

Oberbeck<sup>3</sup> assumes that the capacity of the secondary of a small induction coil is that of a small Leyden jar, *i. e.*, about .0005 of a microfarad.

The theory of the distributed capacity of a coil has not yet been worked out, but it seems probable that the capacity is very small. The experiments which will be described later seem to indicate this.

Approximate expressions for the difference of potential in the secondary, and for the primary current have been obtained by neglecting small quantities.

Walter<sup>4</sup> and Oberbeck<sup>5</sup> have obtained the following expressions for the case of the open secondary.

<sup>1</sup> Wied. Ann., Bd. 44, p. 712, 1891.

<sup>2</sup> Wied. Ann., Bd. 66, pp. 628-630, 1898.

<sup>3</sup> Wied. Ann., Bd. 64, p. 204, 1898.

<sup>4</sup> Wied. Ann., Bd. 62, pp. 307-322 ; Bd. 66, pp. 623-635.

<sup>5</sup> Wied. Ann., loc. cit.

For the difference of potential between the terminals of the secondary they get

$$(22) \quad V_2 = \frac{I_0 M}{L_2 K_2 - L_1 K_1} \left[ \sqrt{L_2 K_2} e^{-\frac{R_2}{2L_2} t} \sin \frac{t}{\sqrt{L_2 K_2}} - \sqrt{L_1 K_1} e^{-\frac{R_1}{2L_1} t} \sin \frac{t}{\sqrt{L_1 K_1}} \right].$$

The periods of the component oscillations are evidently given by

$$(23) \quad T_1 = \pi \sqrt{L_1 K_1}, \quad T_2 = \pi \sqrt{L_2 K_2}.$$

If we consider only the first few oscillations, the damping may be neglected and (22) becomes

$$(24) \quad V_2 = \frac{I_0 M}{L_2 K_2 - L_1 K_1} \left[ \sqrt{L_2 K_2} \sin \frac{t}{\sqrt{L_2 K_2}} - \sqrt{L_1 K_1} \sin \frac{t}{\sqrt{L_1 K_1}} \right].$$

Three important cases may be distinguished according as  $L_1 K_1$  is large compared with  $L_2 K_2$ ; is equal to  $L_2 K_2$ , or is small compared with  $L_2 K_2$ .

In the first case (24) reduces to

$$(25) \quad V_2 = \frac{I_0 M}{\sqrt{L_1 K_1}} \sin \frac{t}{\sqrt{L_1 K_1}}.$$

In the second, to

$$(26) \quad V_2 = \frac{I_0 M}{2\sqrt{L_1 K_1}} \sin \frac{t}{\sqrt{L_1 K_1}} = \frac{I_0 M}{2\sqrt{L_2 K_2}} \sin \frac{t}{\sqrt{L_2 K_2}}.$$

In the third, to

$$(27) \quad V_2 = \frac{I_0 M}{\sqrt{L_2 K_2}} \sin \frac{t}{\sqrt{L_2 K_2}}.$$

Assuming that there is no magnetic leakage,

$$M = \sqrt{L_1 L_2}$$

and we get for the maximum difference of potential in the three cases

$$25a \quad V_2 \text{ max} = I_0 \sqrt{\frac{L_2}{K_1}},$$

$$26a \quad V_2 \text{ max} = \frac{I_0}{2} \sqrt{\frac{L_2}{K_1}} = \frac{I_0}{2} \sqrt{\frac{L_1}{K_2}},$$

$$27a \quad V_2 \text{ max} = I_0 \sqrt{\frac{L_1}{K_2}}.$$

Formula 25*a* assumes that the capacity of the secondary can be neglected. Walter<sup>1</sup> found by experiment that this was true for small coils. The larger the coil the more will the potential deviate from this value and approach those given by 26*a* and 27*a*.

For the primary current, neglecting the reaction of the secondary, Walter<sup>2</sup> obtains

$$(28) \quad I_1 = I_0 e^{-\frac{R}{2L_1}t} \cos \frac{t}{\sqrt{L_1 K_1}}.$$

The period of the primary current is therefore given by

$$(29) \quad T_1 = 2\pi \sqrt{L_1 K_1}.$$

For the maximum difference of potential between the plates of the primary condenser, during the oscillations of the primary current, Walter<sup>3</sup> gives the following expression:

$$(30) \quad V_1 \text{ max} = I_0 \sqrt{\frac{L_1}{K_1}}.$$

Equation (30) shows that if the capacity of the primary circuit is very small, the induced electromotive force in this circuit is very great.

<sup>1</sup> Loc. cit.

<sup>2</sup> Wied. Ann., Bd. 62, p. 310, 1897.

<sup>3</sup> Loc. cit., p. 322.

Johnson<sup>1</sup> makes a correction for the reaction of the secondary and writes (29) and (30) as follows:

$$29a \quad T_1 = 2\pi \sqrt{L_1 K_1 \left(1 - \frac{M^2}{L_1 L_2}\right)}$$

and

$$30a \quad V_{1 \max} = I_0 \sqrt{\frac{L_1}{K_1} \left(1 - \frac{M^2}{L_1 L_2}\right)}.$$

*Theory of two Circuits. Harmonic E.M.F.*

If, instead of a constant current, which is suddenly broken, we have an alternating current in the primary, equations (17) become

$$(31) \quad \begin{aligned} L_1 \frac{d^2 q_1}{dt^2} + M \frac{d^2 q_2}{dt^2} + R_1 \frac{dq_1}{dt} + \frac{q_1}{K_1} &= E \cos \omega t, \\ L_2 \frac{d^2 q_2}{dt^2} + M \frac{d^2 q_1}{dt^2} + R_2 \frac{dq_2}{dt} + \frac{q_2}{K_2} &= 0. \end{aligned}$$

We will only consider the case in which the primary circuit is a closed circuit containing no condenser, and in which therefore  $K_1$  can be considered infinite.

We obtain for the secondary circuit

$$(32) \quad q_2 = \frac{M\omega^2 E}{\sqrt{D_1^2 + D_2^2}} \cos(\omega t - \alpha)$$

where

$$D_1 = (L_1 L_2 - M^2)\omega^4 - \left(R_1 R_2 + \frac{L_1}{K_2}\right)\omega^2,$$

$$D_2 = (R_2 L_1 + R_1 L_2)\omega^3 - \frac{R_1}{K_2}\omega$$

and

$$\alpha = \cot^{-1} \frac{D_1}{D_2}.$$

To get  $V_2$  the difference of potential of the condenser in the secondary we write

$$V_2 = \frac{q_2}{K_2}.$$

<sup>1</sup> Wied. Ann., Bd. 3, pp. 450 and 458, 1900.

This gives for  $V_m$ , the amplitude of the potential oscillation,

$$(33) \quad V_m = \frac{M\omega^2 E}{K_2 \sqrt{\left[(L_1 L_2 - M^2)\omega^4 - \left(R_1 R_2 + \frac{L_1}{K_2}\right)\omega^2\right]^2 + \left[(R_2 L_1 + R_1 L_2)\omega^3 - \frac{R_1}{K_2}\omega\right]^2}}$$

In the induction coil, as ordinarily used, with an open secondary,  $K_2 = 0$ . In this case (33) becomes

$$(34) \quad V_m = \frac{M\omega E}{\sqrt{L_1^2 \omega^2 + R_1^2}}.$$

To find what capacity in the secondary will give the maximum amplitude for the potential oscillation, we must differentiate the denominator of (33) with respect to  $K_2$  and put the result equal to zero. Calling this value of the capacity  $K_m$ , we get

$$(35) \quad K_m = \frac{L_1(L_1 L_2 - M^2)\omega^2 + R_1^2 L_2}{(L_1 L_2 - M^2)\omega^4 + (R_1^2 L_2^2 + 2R_1 R_2 M^2 + R_2^2 L_1^2)\omega^2 + R_1^2 R_2^2}.$$

Experiments verifying this formula will be described in § 6.

### § 3. INSTRUMENTS AND METHODS.

The induction coil used in the experiments described in the following pages, was made in separate pieces so that each part of it was independent of every other part, and the number of sections in the secondary could be varied at will.

The primary was made of two layers of heavy double-cotton-covered copper wire, No. 12, Brown & Sharpe gauge, and had the following dimensions :

Number of turns in outer layer,	109½
Number of turns in inner layer,	109
Total number of turns,	218½
Mean length,	25.63 cm.
Diameter of the outer coil,	4.47 cm.
Diameter of the inner coil,	4.11 cm.
Diameter of the wire,	.230 cm.

The primary was made by winding the wire upon a wooden cylinder, about which had been previously wrapped several layers

of heavy paper. To the ends of the paper tube were glued two wooden end pieces to keep the coil from spreading. When the coil had been wound, and the ends of the wire attached to one of the end pieces, the wooden cylinder was withdrawn. The inductance of the primary could be changed by putting iron wires in the hollow core. The iron wires used were of soft iron 35 cm. long and .0830 cm. in diameter. Most of the experiments were conducted without any iron in the core. The self inductance of the primary with an air core was found to be .000281 of a henry. Its resistance at 21° Centigrade, was .1666 of an ohm.

TABLE I.

Mark.	No. of Turns.	Inside Diameter in Centimeters.	Outside Diameter in Centimeters.	No. of Turns in a Layer.	No. of Layers.	Self-Inductance in Henries.	Mutual Inductance with Primary in Henries.	Resistance at 21° C. in Ohms.
AI	5469	7.62	13.00	52-60	100	3.551	.00763	1527.0
AII	4600	7.62	13.00	50	92	2.594		1351.7
BI	5558	7.36	12.85	52-59	100	3.595		1516.0
BII	5000	7.36	12.49	50-57	95	2.913		1365.0
BIII	5000	7.37	12.82	50	100	2.951		1391.3
BIV	5000	7.34	12.25	51-60	90	2.934		1425.1
CI	1250	7.39	8.88	50	25	.1867		297.8
CII	"	7.36	8.85	"	"	.1896		298.5
CIII	"	7.37	8.87	"	"	.1880		297.5
CIV	"	7.40	8.95	"	"	.1893		295.8
DI	2500	7.38	10.28	50	50	.684		612.2

The secondary was made in eleven sections. Each section was independent of all the others and could be slipped at will on and off a hard rubber sleeve, which itself was slipped over the primary. The sections were all wound with No. 34 (Brown & Sharpe gauge) single-silk-covered copper wire. The diameter of the wire was .01601 cm. The sections were all of the same width, viz: 1.27 cm. (half an inch). Six of them contained about 5,000 turns each; one contained about 2,500 turns and the other four contained about 1,250 turns each. Their dimensions are given in Table I. Table Ia contains the inductances, self and mutual, of the coils joined up in series.

TABLE Ia.

Coils Used.	Self Inductance in Henries.	Mutual Inductance with Primary in Henries.
BII + BIII.	9.67	
BII + BIII + BIV.	18.92	
BI + BIV.	11.06	.0145
BI + BIV + BII.	20.84	.0213
CI + CII.	.5774	
CI + CII + CIII.	1.076	
CI + CII + CIII + CIV.	1.639	

All the sections were wound on a form in a lathe, a strip of Banker's Bond paper being placed between each two layers. AI and AII were wound on wooden forms, on which they were permanently retained. All the others were wound on a metal form. After the section had been wound the metal form was removed, and the section boiled in paraffin in a vacuum to exclude all moisture and air. Connections were made by soldering heavier wires to the ends of the sections, and inserting the ends in mercury cups.

The condensers used in these experiments had a total capacity of about thirteen microfarads. The smaller condensers were very carefully put together and were made of alternate sheets of mica and tinfoil. After the condenser had been assembled it was clamped by a metal clamp and boiled in beeswax. The metal clamp was retained permanently. The smaller condensers were divided into ten sections ranging from .006 to 0.15 of a microfarad. A set of 16 sections of a mica and tinfoil condenser, kindly loaned by Messrs. Morris E. Leeds & Co., of Philadelphia, was also used. Each section had a capacity of about .2 microfarad. These were boiled in paraffin and held together by a wooden clamp. Three large Stanley, paper and tinfoil, condensers, each having a capacity of about 2.5 microfarads, were also used.

The capacities of the condensers were determined by making an absolute determination of one section by the tuning fork charge and discharge method<sup>1</sup> and then comparing the other sections with this section by the bridge method.

<sup>1</sup> Thomson & Searle, Phil. Trans., 1890 Vol. A, pp. 583-621.

The *measurements of difference of potential* were all made with a modified Kelvin quadrant electrometer, used idiostatically. The deflections were therefore proportional to the squares of the potential.

The instrument used was the one used by Professor Webster, and described by him in his paper on electrical oscillations.<sup>1</sup> The quadrants were made of brass and the needle of aluminum. The needle was suspended by a quartz fiber and connected to the binding post by a platinum wire dipping into sulphuric acid. The instrument worked excellently as long as the acid was fresh. After the acid had been standing for some weeks it was found to be necessary to boil it for several hours, for if not fresh or recently boiled there was a lag when a reading was taken.

The differences of potential measured were usually small, less than 50 volts, since the primary was nearly always used with an air core. The electrometer as adjusted for these experiments was too sensitive to measure directly potentials greater than 50 volts. It could be arranged however to measure any potential by the introduction of an auxiliary air condenser of adjustable capacity in series with it in the electrometer circuit. In Fig. 3 *A* is the auxiliary air condenser, and *V* the electrometer.

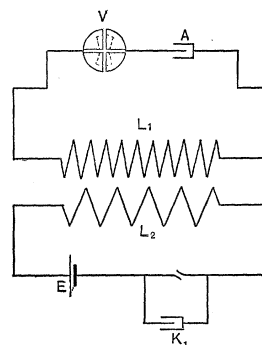


Fig. 3.

The true difference of potential can be calculated from the apparent difference given by the electrometer reading, and the known capacity of the auxiliary condenser.

In all the experiments the electrometer was used idiostatically, one side being connected to earth so as to give a constant zero.

The usual method of using the electrometer was as follows. After taking a series of observations, the instrument was calibrated by noting the deflections due to different known differences of potential. The known differences of potential used were those of a storage battery, determined by a Weston voltmeter. A calibration curve

<sup>1</sup> PHYS. REV., Vol. VI., p. 301, 1898.



was plotted, and the value of the potential corresponding to a given deflection read off directly from the curve.

A curious source of error observed and studied by Hallwachs<sup>1</sup> caused the writer some trouble in the earlier experiments. Since in the idiostatic method of using the electrometer the deflection is proportional to the square of the potential, when the sign of the potential is changed without changing its absolute value the deflection should be unchanged. It was found that this was not quite true, and investigation showed that it was due to the contact difference of potential between the brass quadrants and the aluminum needle, which increased the deflection in one case and decreased it in the other. An approximately correct value for the difference of potential is found by taking the mean between the two deflections. This was done in calibrating the instrument. Hallwachs has shown that an electrometer in which the quadrants and the needle are made of different metals can be used to find the contact difference of potential between the two metals. The difference of potential between the brass quadrants and the aluminum needle was determined in this way by the writer, and found to be equal to .42 of a volt.

In these experiments the electrometer was used idiostatically in two ways, first, with a steady deflection with an alternating current, and second, ballistically with a transient current. In both cases the deflection was proportional to the mean square of the potential. In both cases the potential performed a simple harmonic oscillation, the difference being that in the first case the oscillation was undamped, and in the second case it was strongly damped, the time constant being about a thousandth of a second.

The *measurements of inductance* were made by comparison with a standard coil, the inductance of which had been previously calculated by Maxwell's formula<sup>2</sup> for the self-inductance of a circular coil of rectangular cross-section.

If (see Fig. 4)

$n$  = number of turns in coil.

$a$  = mean radius of coil.

$r$  = diagonal of the cross-section.

$\theta$  = angle made by diagonal with lower edge of the cross-section.

<sup>1</sup> Wied. Ann., Bd. XXIX., pp. 1-47, 1886.

<sup>2</sup> See Rayleigh, Phil. Trans., 1882, pt. II., p. 675.

We have, using the first two terms of the series

$$\begin{aligned}
 L = 4\pi an^2 \left[ \log_e \frac{8a}{r} + \frac{1}{12} - \frac{4}{3} \left( \theta - \frac{\pi}{4} \right) \cot 2\theta - \frac{1}{3}\pi \operatorname{cosec} 2\theta \right. \\
 \left. - \frac{1}{6} \cot^2 \theta \log_e \cos \theta - \frac{1}{6} \tan^2 \theta \log_e \sin \theta \right] \\
 + \frac{\pi n^2 r^2}{24a} \left[ \log_e \frac{8a}{r} (2 \sin^2 \theta + 1) + 3.45 + 27.475 \cos^2 \theta \right. \\
 \left. - 3.2 \left( \frac{\pi}{2} - \theta \right) \frac{\sin^3 \theta}{\cos \theta} + \frac{1}{5} \frac{\cos^4 \theta}{\sin^2 \theta} \log_e \cos \theta + \frac{1}{3} \frac{\sin^4 \theta}{\cos^2 \theta} \log_e \sin \theta \right]
 \end{aligned}$$

In calculating the value of  $L$  for the standard coil it was found that neglecting the second term would introduce a minus error of .4 of one per cent.

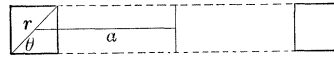


Fig. 4.

The coils used as the standard were two coils of the same dimensions, used by Taylor,<sup>1</sup> in his work on standard cells, as the fixed coils of an electro-dynamometer. They are fully described by him in the paper referred to.

The mean value of the self inductance of each coil by calculation from the formula was found to be .001881 of a henry.

The comparison of the inductances to be measured with the standard was made by Maxwell's bridge method using the alternating current and a telephone<sup>2</sup> instead of a galvanometer. The comparison was usually made by comparing the unknown inductance first with the upper coil and then with the lower, and taking the mean of the two values obtained. The two values so obtained usually agreed to about a tenth of one per cent. It was found that with the alternating current used, the inductances could be measured to about a tenth of one per cent. The self inductances of the secondary segments, both separately and combined together are given in Tables I. and Ia.

The mutual inductances of the secondary segments with the primary were measured by Maxwell's<sup>3</sup> bridge method for comparing

<sup>1</sup> *PHYS. REV.*, Vol. VII., pp. 156-159, 1898.

<sup>2</sup> See Max Wien, *Wied. Ann.*, Bd. 44, pp. 681-688, 1891.

<sup>3</sup> See Gray, *Absolute Measurements*, Vol. II., pp. 465-469.

the mutual inductance of two coils with the known self inductance of one of the coils. In this case also, the alternating current and the telephone were used instead of a steady current and a galvanometer. The telephone method is much more rapid than the galvanometer method. The results are given in Table I. The inductances were all measured without any iron in the core, that is with an air core.

The *measurements of resistance* were made by the usual zero method of the Wheatstone bridge with a steady current and a galvanometer. The resistance box used was one made by Elliott Bros., of London. The resistances of the box were examined by Mr. Frank K. Bailey, of the university, and found to be accurate to a tenth of one per cent. The resistances are given in Table I. reduced to 21° centigrade.

The *measurements of currents* were made for direct currents with a Weston ammeter, and for alternating currents with a hot wire ammeter, which was calibrated by comparison with the Weston ammeter. The currents were measured to about one per cent.

To conduct the experiments described in § 6, it was necessary to obtain a harmonic current that should be free from overtones. This was done by Pupin's<sup>1</sup> method, which consists in inserting inductance and capacity in series in the circuit, and then adjusting the relative amounts of inductance and capacity until the circuit is in resonance for the particular frequency which is desired. When this is so, if the resistance of the circuit is relatively small, the frequency which is desired will so far predominate that the other frequencies cannot be detected.

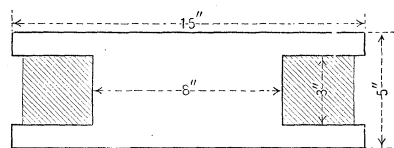


Fig. 5.

In order to use this method it is necessary to have either a large inductance or a large capacity. Since it was easier to construct a large inductance

than a large capacity, a coil was built which gave the required inductance. The dimensions of a coil which would give the desired inductance were calculated from Maxwell's formula on p. 297. The cross-section of the whole coil, drawn to scale, is shown in Fig. 5.

<sup>1</sup>Am. Jour. Sci., Vol. XLVIII., pp. 379-389, 473-485, 1894.

The portion occupied by the wire is shaded. This shaded portion was intended to be 3 inches on a side. In the coil as constructed it was 3 inches from top to bottom, and  $2\frac{3}{4}$  inches from side to side. The coil was wound with about two miles of No. 18 (Brown & Sharpe gauge) double cotton insulated copper wire. Diameter, .0403 of an inch. The wire was wound in 60 layers of 60 turns each, making 3,600 turns in all. At the beginning and end of every ten layers, the wire was led off to a binding post on the top of the coil, so that the coil was divided into six independent sections.

The dimensions of the wooden form on which the coil was wound are those shown in Fig. 9, viz :

Outside diameter,	15 inches.
Inside diameter,	8 "
Height of groove,	3 "
Depth of groove,	$3\frac{1}{2}$ "
Height of whole coil,	5 "

The dimensions of the coil proper were

Outside diameter,	$13\frac{1}{2}$ inches
Inside diameter,	8 "
Height of cross-section,	3 "
Depth of cross-section,	$2\frac{3}{4}$ "

The coil was designed to give the maximum inductance for the length of wire used.<sup>1</sup> Its inductance calculated before construction was 3.33 henries. After construction its inductance was found by comparison with the standard inductance to be 3.01 henries. This is probably partly due to the fact that the designed geometrical conditions, viz.: a rectangular cross-section was not quite fulfilled.

#### § 4. THE IRON CORE.

To determine the effect of increasing the amount of iron in the core, the number of iron wires inserted in the core was varied, the current for any given experiment being kept constant.

An alternating current of about 60 complete periods per second was used in the primary. The difference of potential between the terminals of the secondary was measured with the electrometer, used idiostatically.

<sup>1</sup> See Maxwell, *Elect. Mag.*, Vol. II., p. 316.

Tables II.-V. and Figs. 6-9 show the results obtained. In the curves the differences of potential in the secondary are plotted as ordinates, and the number of wires as abscissas.

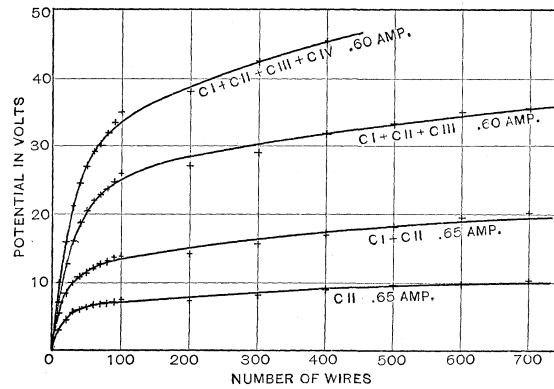


Fig. 6.

Table II. and Fig. 6 show how the form of the curve varies as the inductance in the secondary is increased by adding coils of the same number of turns and of the same diameter placed side by side.

TABLE II.

Number of Wires.	C II .65 Amp.	C I + C II .65 Amp.	C I + C II + C III .60 Amp.	C I + C II + C III + C IV .60 Amp.
	Volts.	Volts.	Volts.	Volts.
0	.0	.0	.0	.0
10	2.5	5.5	8.0	10.0
20	4.2	8.3	12.8	16.0
30	5.5	10.0	16.1	21.1
40	6.1	10.9	18.8	24.6
50	6.4	11.4	20.5	27.2
60	6.6	12.0	22.0	29.4
70	6.9	12.7	22.7	30.0
80	7.0	13.0	23.8	32.0
90	7.2	13.7	24.8	33.5
100	7.5	13.8	26.0	35.0
200	7.6	14.2	27.1	38.0
300	8.5	15.8	29.0	42.5
400	9.2	17.1	31.8	45.5
500	9.6	18.4	33.5	54.0
600	9.9	19.6	35.1	57.0
700	10.3	20.3	35.8	58.5

Table III. and Fig. 7 also show how the form of the curve varies as the inductance is increased, but in this case the increase

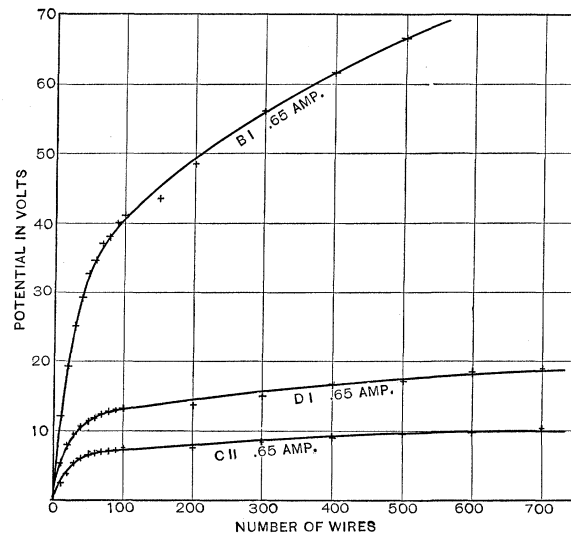


Fig. 7.

is made by replacing the original secondary by a coil of the same width, but of greater diameter and therefore of a greater number of turns.

TABLE III.

Number of Wires.	CII .65 Amp.	DI .65 Amp.	BI .65 Amp.
	Volts.	Volts. <sup>1</sup>	Volts.
0	.0	.0	.0
10	2.5	5.3	12.2
20	4.2	8.0	19.2
30	5.5	9.7	25.2
40	6.1	10.5	29.3
50	6.4	11.4	32.6
60	6.6	11.8	34.5
70	6.9	12.3	37.0
80	7.0	12.7	37.8
90	7.2	12.9	40.0
100	7.5	13.2	40.7
200	7.6	13.7	48.4
300	8.5	15.1	56.0
400	9.2	16.6	62.0
500	9.6	17.3	66.8
600	9.9	18.5	68.0
700	10.3	19.0	

TABLE IV.

Number of Wires.	CII .65 Amp.	CII 1.05 Amp.	CII 2.3 Amp.
	Volts.	Volts.	Volts.
0	.0	.0	2.5
10	2.5	4.4	6.3
20	4.2	7.2	9.7
30	5.5	9.4	12.6
40	6.1	11.5	15.5
50	6.4	13.7	17.8
60	6.6	15.4	20.0
70	6.9	16.4	22.6
80	7.0	17.5	24.3
90	7.2	18.4	26.8
100	7.5	19.2	29.0
200	7.6	23.2	43.0
300	8.5	26.0	50.0
400	9.2	26.5	52.3
500	9.6	28.2	53.8
600	9.9	29.2	54.5
700	10.3	29.6	54.5

Tables IV., V. and Figs. 8, 9 show the change in the form of the curve for the same secondary coil as the current is increased.

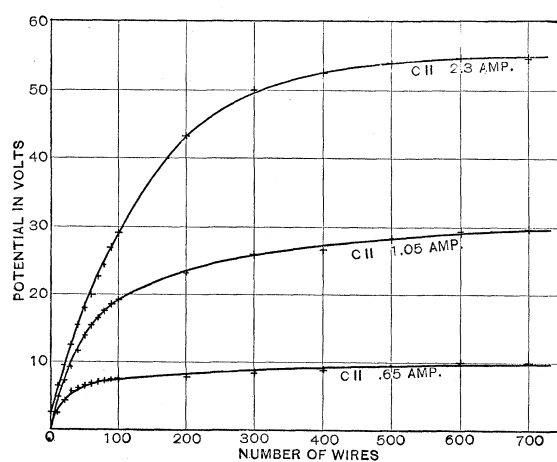


Fig. 8.

TABLE V.

Number of Wires.	DI .65 Amp. Volts.	DI 1.05 Amp. Volts.	Number of Wires.	DI .65 Amp. Volts.	DI 1.05 Amp. Volts.
0	.0	.0	90	12.9	25.2
10	5.3	7.0	100	13.2	26.3
20	8.0	11.1	200	13.7	27.7
30	9.7	14.7	300	15.1	30.8
40	10.5	17.4	400	16.6	32.0
50	11.4	19.8	500	17.3	34.2
60	11.8	21.7	600	18.5	36.0
70	12.3	23.1	700	19.0	
80	12.7	24.3			

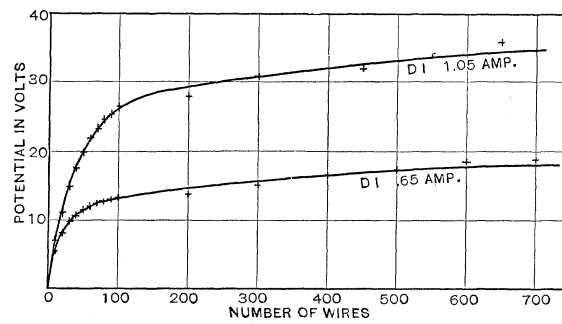


Fig. 9.

The form of these curves is given by equation (34).

Since  $M$  and  $L_1$  are linear functions of the permeability of the core, and the permeability of air is very small compared with that of iron, we may assume that they are proportional to the number of iron wires in the core, and we may write

$$M = k_1 x,$$

$$L_1 = k_2 x,$$

where  $x$  is the number of iron wires in the core. Equation (34) then becomes

$$V_m = \frac{k_1 \omega E x}{\sqrt{k_2^2 \omega^2 x^2 + R_1^2}},$$



which is an equation of the fourth degree in  $V_m$  and  $x$ . The graph of  $V$  as a function of  $x$  gives a curve of the fourth order symmetrical with respect to both the  $x$  and  $y$  axes, and having points of inflexion at the origin. The portion of the curve plotted in the figures is for  $x$  positive and  $V$  positive. It is evident by inspection of equation (34) that when the number of wires is infinite  $V_m$  approaches a finite maximum value,  $\frac{ME}{L_1}$ . Also, that the potential increases very rapidly as the number of wires is increased up to a certain point after which an increase in the number of wires only makes a small difference in the potential. This is a fact of practical importance in the construction of coils.

The question arose as to whether it would make any difference whether the wires were placed together in the axis of the core, or placed loosely anywhere within the core. It was found by experiment that, within the limits of accuracy of the experiments, it made no difference whether the iron wires were placed in a bundle in the middle of the core or placed loosely anywhere within it.

#### § 5. DISTRIBUTION OF THE INDUCTION.

A matter of considerable importance in the construction of coils is the knowledge of the flux of induction through the primary at different points of its length. It is important to know how the flux varies as we move along the coil from one end to the other, in order to know where to place the secondary coils so that the greatest flux shall pass through them.

To determine this, experiments were made, by placing upon the primary one of the secondary coils, and sliding it along from one end to the other. The primary was divided into ten equal parts, and the secondary was made to take eleven different positions at equal distances apart, including the ends. The terminals of the secondary coil were attached to the electrometer. The current used in the primary was the commercial alternating current, with a frequency of about 60 oscillations a second. Three different sized secondary coils were used, viz.: 5,000 turns, 2,500 turns and 1,250 turns. The coils used were BIII, DI, and CIV.

The results are given in Table VI. and Fig. 10.

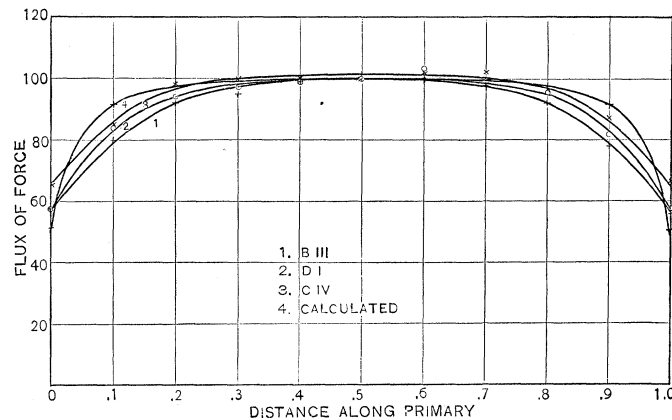


Fig. 10.

The method of performing the experiment was to move the secondary from left to right, noting the deflections for each of the eleven positions, and then move it back again from right to left noting the deflections, and finally take the mean of the two readings for the same position.

For BIII the mean primary current was 5.1 amp.; for DI, 5.6 amp.; for CIV, 8.4 amp.

TABLE VI.

Fraction of Total Length of Primary.	BIII		DI		CIV	
	Diff. of Pot. in Volts.	Reduced to 100.	Diff. of Pot. in Volts.	Reduced to 100.	Diff. of Pot. in Volts.	Reduced to 100.
.0	6.3	57	4.5	57	3.0	64
.1	8.8	80	6.6	84	4.0	85
.2	10.1	92	7.4	94	4.6	98
.3	10.5	95	7.7	97	4.7	100
.4	10.9	99	7.8	99	4.7	100
.5	11.1	101	7.9	100	4.7	100
.6	11.0	100	8.1	103	4.8	102
.7	10.8	98	7.8	99	4.8	102
.8	10.1	92	7.5	95	4.5	96
.9	8.6	78	6.5	82	4.1	87
1.0	6.2	56	4.6	58	3.1	66

The theory of this experiment is given by equation (34).

Since when we move the exploring coil along the primary everything is unchanged except  $M$ , we have  $V_m$  directly proportional to  $M$ . But  $M$  is directly proportional to the flux of induction through the solenoid at that point of its length. Therefore  $V_m$  is a measure of the flux of induction through the solenoid, and is directly proportional to it.

The flux of induction through the primary was also calculated. The variation of the field from that of an infinitely long solenoid is due to the free ends of the solenoid. The correction due to the ends was calculated from Maxwell's<sup>1</sup> formula for the force at any point due to a charged disk. The force was then integrated over a spherical cap, bounded by the solenoid, at different distances from the end of the solenoid. This gives the flux through the solenoid at different distances from its end. Using the first three terms of the series the following formula was obtained :

$$\text{Flux} = 4\pi^2 r^2 \left\{ \frac{1}{2} \frac{a^2}{r^2} \left( \frac{z}{r} - 1 \right) - \frac{3}{2 \cdot 2 \cdot 4} \frac{a^4}{r^4} \left[ \left( \frac{z^3}{r^3} - 1 \right) - \left( \frac{z}{r} - 1 \right) \right] \right. \\ \left. + \frac{3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8} \frac{a^6}{r^6} \left[ 7 \left( \frac{z^5}{r^5} - 1 \right) - 10 \left( \frac{z^3}{r^3} - 1 \right) + 3 \left( \frac{z}{r} - 1 \right) \right] \right\},$$

Where  $a$  is the radius of the solenoid.

$r$  the distance from the center of the end to the spherical cap.

$z$  the distance from the end of that turn of the solenoid through which the cap passes.

This formula gives the flux through a turn of the solenoid at a distance  $z$  from the left hand end, due to a disc of unit density placed at the left hand end. To find the flux when unit current passes through the solenoid we must multiply by the number of turns in unit length of the coil. To get the total effect due to both ends, we must add the effects of the two ends, which are in the same direction. This gives the correction to be subtracted from the flux for an infinitely long solenoid. The solenoid used in these experiments was made of two layers, an inner and an outer layer. The flux was calculated for each separately and the total flux taken as the sum of the two. The results are given in Table VII. The results are only given for half the length of the coil, as the two

<sup>1</sup> Maxwell, *Elect. & Mag.*, Vol. II., p. 284.

halves are, of course, symmetrical. The corrected flux through the solenoid is plotted as curve 4 in Fig. 10.

If the flux were calculated on the assumption that the primary was infinitely long, we should have 1,551 cm. for every point along the primary, instead of the values given in the column last but one of Table VII.

TABLE VII.

Fractional distance from left hand end of coil.	Outer Coil.				Inner Coil.				Both Coils Together.		
	Flux due to disk of unit density at left hand end.	Flux due to disk of unit density at right hand end.	Sum of both = $S_1$	$S_1 \times 4.273 = F_1$	Flux due to disk of unit density at left hand end.	Flux due to disk of unit density at right hand end.	Sum of both = $S_2$	$S_2 \times 4.253 = F_2$	Total flux for unit current due to ends = $F_1 + F_2$	Total flux through primary.	Reduced to 100.
0	98.59	.39	98.98	422.9	83.35	.26	83.61	355.7	778.6	772	50.5
.1	19.54	.47	20.01	85.5	14.97	.34	15.31	65.1	150.6	1400	91.6
.2	7.48	.57	8.05	34.4	5.47	.40	5.87	25.0	59.4	1492	97.6
.3	3.72	.76	4.48	19.1	2.62	.56	3.18	13.6	32.7	1518	99.3
.4	2.21	1.00	3.21	13.7	1.54	.75	2.29	9.8	23.5	1527	99.9
.5	1.47	1.47	2.94	12.6	1.07	1.07	2.14	9.1	21.7	1529	100.0

The flux of induction through the primary at any point of its length is of course measured by the number of lines passing through a single turn wound tightly upon it at that point. This is the quantity calculated in Table VII. Since the coils BIII, CIV and DI, used in the experiment, consist of a large number of turns, most of the turns being situated at an appreciable distance from the primary, the flux through these coils will not be directly proportional to that for a single turn. This difference in the conditions may perhaps account for the difference between the curves for the observed and calculated values in Fig 10.

#### § 6. CAPACITY IN THE SECONDARY.

Experiments were made to determine the effect of inserting localized capacity in series in the secondary.

The current used in the primary was the commercial alternating current. It was analyzed by Pupin's<sup>1</sup> method, and was found to contain the third and fifth harmonics.

<sup>1</sup> See Pupin, loc. cit.

For a given frequency,  $\omega$ , the amplitude of the potential oscillation in the secondary condenser, is given by equation (33). If more than one frequency is present in the current, the potential is the sum of the potentials due to the different frequencies.

Experiments were made with each of the three frequencies present in the commercial current. The frequency desired was brought out by Pupin's method which has been described in § 3. To test the accuracy of Pupin's method the large resonance coil described in § 3 was placed in series with a condenser in a circuit which had impressed upon it the electromotive force of the commercial alternating current. The fundamental of the alternating current was about 60 oscillations a second. The potential of the condenser was observed as its capacity was varied in the neighborhood of that capacity which would give resonance for the fundamental. These observations are given in Table VIII. The theoretical values were then calculated from equation (16). The results are given in Table IX. The observed potentials were relative, not absolute. The results are plotted in Fig. 11. The crosses represent the calculated values, and the circles the observed values. It is hard to say why the two curves do not coincide. If we should assume that the

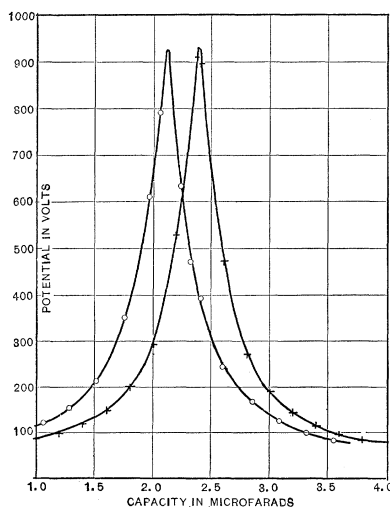


Fig. 11.

TABLE VIII.

Capacity in Microfarads.	Square Root of De- flection $\times 112.4$ .	Capacity in Microfarads.	Square Root of De- flection $\times 112.4$ .
1.06	120	2.32	470
1.26	151	2.40	393
1.51	212	2.60	248
1.76	352	2.86	167
1.97	612	3.09	126
2.06	793	3.30	101
2.23	634	3.55	82

TABLE IX.

Capacity in Microfarads.	Potential in Volts.	Capacity in Microfarads.	Potential in Volts.
1.00	86	2.60	473
1.20	101	2.80	271
1.40	121	3.00	191
1.60	148	3.20	143
1.80	200	3.40	117
2.00	292	3.60	97
2.20	530	3.80	84
2.37	910	4.00	74
2.40	897		

large resonance coil has a distributed capacity of about .3 microfarad, the two curves would be very nearly coincident.

TABLE X.

Capacity in Microfarads.	Potential in Volts.	Capacity in Microfarads.	Potential in Volts.	Capacity in Microfarads.	Potential in Volts.
.000	17.1	.210	26.2	1.26	47.7
.032	17.7	.224	32.4	1.51	62.6
.038	18.0	.232	36.7	1.76	104.0
.052	18.3	.235	37.1	1.97	181.0
.059	18.4	.244	34.9	2.23	187.0
.071	19.2	.255	29.0	2.40	116.0
.080	21.9	.283	22.4	2.32	139.0
.083	24.1	.306	21.1	2.06	234.0
.090	25.0	.319	20.7	2.60	73.4
.095	22.1	.421	20.7	2.86	49.4
.107	19.5	.621	25.2	3.09	37.2
.127	19.0	.884	29.9	3.30	29.9
.155	19.4	1.060	35.5	3.55	24.3
.178	20.6				
.190	22.0				

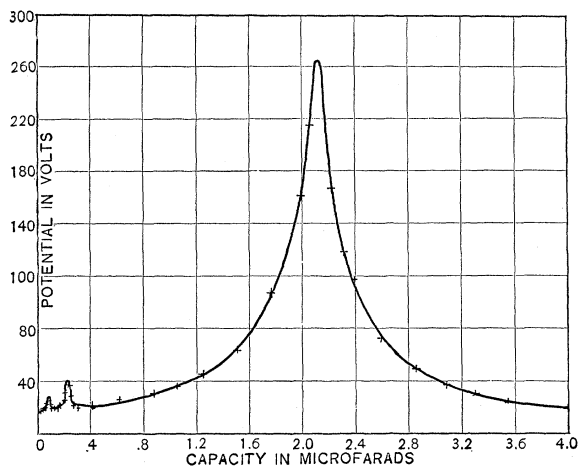


Fig. 12.

Table X. shows the results of the analysis of the commercial alternating current, made in the way described. The capacity of the circuit was gradually increased from nothing up to 3.5 microfarads. The observations are plotted in Fig. 12. The three peaks reading from right to left represent the fundamental, the third harmonic and the fifth harmonic, respectively.

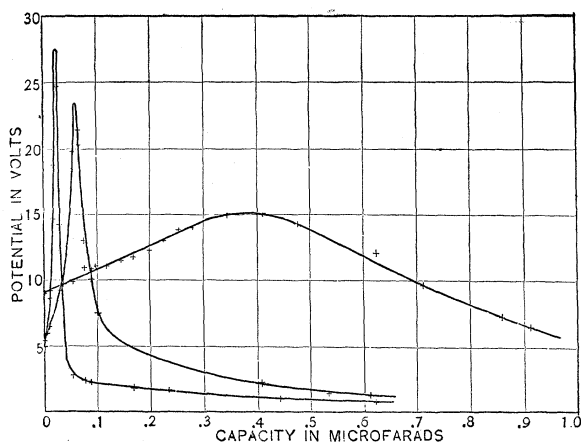


Fig. 13.

Having made these preliminary observations, the main problem was attacked, with the results given in Tables XI.-XIV., and Figs. 13-15.

TABLE XI.

BI + BIV.

Capacity in Microfarads.	Potential in Volts.		
	" = 60 2.2 Amp. in Primary.	" = 180 .3 Amp. in Primary.	" = 300 .3 Amp. in Primary.
.000	9.1	5.0	4.6
.007	9.0	6.2	6.5
.008	9.2	6.0	6.7
.012	9.2	6.5	8.6
.019			14.7
.020			18.7
.027			24.7
.032	9.7	9.2	14.2
.060	9.9	19.7	2.8
.069		21.4	
.071		20.3	
.082	10.9	13.0	2.4
.095	10.8	10.0	2.2
.107	11.0	7.5	
.127	11.1		
.155	11.5		
.178	11.8		1.8
.210	12.3		
.238	13.0		
.250			1.6
.269	13.7		
.297	14.0		
.367	15.0		
.436	15.0	2.2	
.462			1.0
.506	14.2	1.7	
.659	12.0	1.2	.7
.755	9.5		
.909	7.1		
.968	6.3		

Tables XI. and XII. give the results obtained by changing the capacity of the secondary condenser, keeping the primary conditions constant. Three frequencies were used, viz.: those of the fundamental, the third overtone, and the fifth overtone. The results given in Table XI. were obtained by using BI and BIV in the



TABLE XII.  
BI + BIV + BII.

Capacity in Microfarads.	Potential in Volts.		
	$n = 60$ 2.6 Amp. in Primary.	$n = 180$ .3 Amp. in Primary.	$n = 300$ .2 Amp. in Primary.
.000	12.0	6.8	7.0
.004		8.3	10.5
.007	12.0	9.7	15.0
.008		10.2	20.4
.012	12.4	13.0	32.5
.013			65.0
.016			27.3
.019			17.3
.020		15.4	13.0
.027		20.7	
.032	13.0	26.6	5.2
.038		28.1	
.040		26.0	
.043		22.5	
.060	14.3	9.9	1.5
.082	15.5	5.5	
.095	16.0	4.5	
.127	17.5		
.186	20.7		
.238	22.0		
.250	21.9		
.269	21.5		
.281	21.5		
.318	19.0		
.372	16.9		
.532	10.1		
.777	1.8		

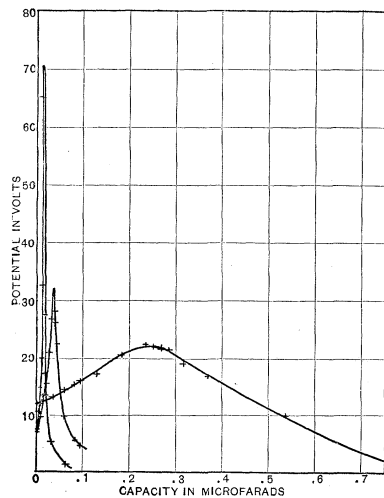


Fig. 14.

secondary, and those given in Table XII. by using BI, BIV and BII in the secondary. This gave a variation of the secondary inductance. BI + BIV and BI + BIV + BII have self inductances of 11.1 and 20.8 henries respectively. The observations are plotted in Figs. 13 and 14. The theory of these curves is given in the latter part of § 2, and the form of the resonance curves is given by equation (33).

Fig. 15 represents the "resonance curve" for the commercial current, without the weeding out of the overtones. The observations from which the curve was plotted are given in Table XIII. BI and BIV were used in the secondary, and the current in the primary was 2 amp. This curve shows, as already stated, that the commercial current contained two harmonics beside the fundamental. It leads to the same conclusion as Fig. 12, the difference being that in Fig. 12 the potential of the primary condenser is plotted, whereas in Fig. 15 the potential of the secondary condenser is plotted. Fig. 15 shows that a small induction coil and an electrometer form an excellent detector of harmonics.

TABLE XIII.

Capacity in Microfarads.	Potential in Volts.	Capacity in Microfarads.	Potential in Volts.
.000	16.1	.084	15.8
.026	43.8	.090	15.0
.030	30.9	.105	14.6
.031	27.8	.141	13.1
.033	25.5	.156	14.0
.038	19.7	.183	14.6
.039	18.6	.247	17.0
.044	17.4	.315	18.6
.046	17.2	.390	19.3
.050	16.3	.456	19.1
.053	18.0	.543	18.2
.057	18.1	.611	16.0
.063	18.8	.652	15.7
.068	18.8	.727	14.5
.074	18.0	.794	11.8

Figs. 13-15 show that the potential of the secondary in a small induction coil may in some cases be increased by the insertion of a condenser in series in the secondary.

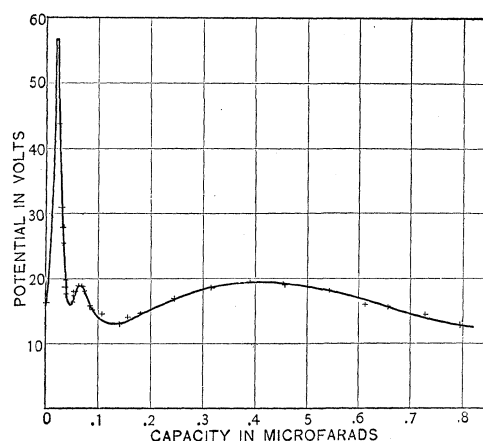


Fig. 15.

TABLE XIV.

Frequency.	Observed Value of $K_m$ in Microfarads.	Calculated Value of $K_m$ in Microfarads.
$L_2 = 11.1$ henries.		
60	.403	.421
180	.066	.071
300	.024	.027
$L_2 = 20.8$ henries.		
60	.237	.257
180	.036	.039
300	.014	.015

From these tables and curves can be found the value of the secondary capacity which will give the greatest difference of potential in the secondary for a given frequency. In Table XIV. are given the values read off from the curves, side by side with the values calculated from equation (35).

*To be continued.*