

the first year of the high school. A large amount of drill work is necessary during this first year. How much time shall we give up to other things?

Up to this time geometry has been little disturbed. For this there have been two reasons. The colleges, technical schools and the physics teachers have not complained that the student is deficient in geometry. Why? The student's mind is more often geometric than algebraic and the student needs no geometry in his work. A little mensuration, mostly learned in the grades, and a few additional theorems picked up in the secondary school amply serve his purpose. Some attempt is being made, and rightly, too, to reach geometry in a less formal way, to take away some of the interdependence of theorems and to leave the student more to his own resources in the development of theorems. This has two effects, partially good, partially bad: The beauty and continuity of the subject are impaired, the student knows fewer theorems, has more thought power and can use his theorems to greater advantage.

The tendency to favor algebra at the expense of geometry will probably have a marked effect on our courses in mathematics. Some go so far as to declare that the year and a half each now devoted to algebra and geometry will soon give way to a course in secondary mathematics, similar to those now offered in Germany and Italy. Such course would doubtless give us students better prepared for college, better prepared for business and with less dislike for mathematics. It should produce more mathematics teachers, of which there is now a dearth, and should consequently improve mathematical conditions in both college and secondary school.

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### FACTORING THE TYPE $px^2+qx+r$ .

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Of all types this is the most important, not alone because of its frequent occurrence but also because it includes those common forms  $x^2+px+q$ ,  $x^2+2xy+y^2$  and  $x^2-y^2$ . In fact, it is the general form of the product of two binomials, each of which has a term of the first degree in  $x$ . Because it is the most important type met in factoring, its proper presentation to beginning classes

should be a matter of considerable care and thought on the part of the teacher.

Factoring is taking the back track from the product to the factors that produced the product. It is *unmultiplying* the product. In multiplying two binomials,  $ax+b$  and  $cx+d$ , four terms,  $acx^2+bcx+adx+bd$ , are obtained which combine, by algebraic addition, into three terms,  $acx^2+(bc+ad)x+bd$ . Inspection of this general product and comparison with the type-form, show that  $q$ , the coefficient of  $x$ , is the sum of two terms,  $bc$  and  $ad$ , whose product,  $abcd$ , is also the product of the coefficient  $p$  and the third term  $r$ , for  $p=ac$  and  $r=bd$ . Now this product,  $pr$ , is a known quantity and if it can be resolved into two factors (in this case  $bc$  and  $ad$ ) whose sum is  $q$ , the trinomial is factorable; otherwise it is not factorable. These two factors  $bc$  and  $ad$ , are the coefficients of  $x$  which, in the multiplication were added to give  $q$  and, when found, enable one to change the trinomial back to the quadrinomial from which it came. This change is the first and all-important step on the back track that leads to the factors. The quadrinomial, when determined, is easily resolved into its binomial factors.

An example: Factor  $12x^2-11x-5$ .

$$12 \times -5 = -60$$

Two factors of  $-60$  whose sum is  $-11$ , are  $-15$  and  $+4$ .

$$12x^2-11x-5=$$

$$12x^2-15x+4x-5=$$

$$3x(4x-5)+1(4x-5)=$$

$$(4x-5)(3x+1).$$

This is, step by step, the reverse of multiplication for:

$$(4x-5)(3x+1)=$$

$$3x(4x-5)+1(4x-5)=$$

$$12x^2-15x+4x-5=$$

$$12x^2-11x-5.$$

NOTE:—This method is given as Case VI, § 117 p. 85 of *Algebra for Secondary Schools*, by Webster Wells. Being the edition of 1906, it is not strange that the author presents this paper here.—MATHEMATICAL EDITOR.