

Luray, Va. The hill referred to has extensive caverns beneath it, and, as appears evident, has been left in relief owing to the more rapid denudation of the surrounding country; the reason being that rain falling on the area where the rock is cavernous percolated downward and was prevented from forming surface streams and in consequence lost its ability to mechanically erode, while the surrounding country where the existence of surface streams was possible was degraded more rapidly.

The influence of subterranean drainage, as must be well known although seldom mentioned, is frequently indicated by minor elevations, especially in limestone regions where joints and other openings permit of the ready descent of surface water. Similar conditions on a larger scale, as just stated, may reasonably be held accountable for the origin of the hill above the caverns at Luray, and seemingly furnish the basis for an hypothesis which meets the conditions present at Mackinac Island and Gibraltar. If this hypothesis is sustained by future tests, it not only furnishes an explanation of the origin of the elevations just mentioned, but embodies a principle which is widely applicable. For example, it is frequently stated in modern text-books of physical geography, that residual hills standing on plains of subaerial denudation or 'monadnocks,' owe their prominence to the greater resistance of the rocks of which they are composed, mainly because of their hardness, in comparison with the rocks about them; or have been spared on account of their geographical position, that is, they occur at localities where streams originated and flowed away in various directions, and in consequence were left in relief after the country about them had been conspicuously degraded. To these explanations of the origin of monadnocks a third may now be added, namely: If the rocks of a given area are more open and porous, or traversed by fissures or caverns to a conspicuously greater degree than the rocks beneath the surrounding region—the general elevation being sufficient to favor subterranean drainage—they may be left in relief because the water reaching them will be conducted away by means of underground channels and

thus in a great measure and in general almost entirely deprived of its power to mechanically erode, while adjacent areas are not favored in this manner.

A consideration of all the known facts relating to the rocky heights forming Mackinac Island and Gibraltar indicates that at each of these localities a residual of the nature of a monadnock has been left as the region about it was lowered by erosion; the controlling condition being that the rocks left in relief are fissured and cavernous, thus facilitating subterranean drainage, while the country about them was denuded at a more rapid rate through the agency of surface streams.

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A NOTABLE ADVANCE IN THE THEORY OF CORRELATION.

To Professor Karl Pearson the new science of biometry is indebted not only for its name, but also for those refinements and extensions of the methods of statistical analysis without which it would be far from occupying the position which it holds to-day. In the remarkable series of memoirs which have appeared under the general title 'Mathematical Contributions to the Theory of Evolution,' Pearson and his assistants have laid a foundation on which a superstructure of great import to biology can, and will be, reared. The most recent of the memoirs in this series* brings forth a very interesting extension of the theory of correlation which at once greatly widens the range of problems and material which can be effectively handled by biometric methods.

In the development of the method of determining the degree of correlation between characters not admitting of quantitative measurement,† it was thought necessary in forming the correlation table to arrange the classes

* 'Mathematical Contributions to the Theory of Evolution,' XIII. 'On the Theory of Contingency and its Relation to Association and Normal Correlation.' Drapers' Company Research Memoirs, Biometric Series, I., pp. 1-35, 2 pl., 1904.

† *Phil. Trans.*, Vol. 195 A, pp. 1-47, and pp. 79-150.

or subgroups of the characters in a definite order corresponding to the real (though in detail undeterminable) quantitative scale in the character or attribute itself. The order of the classes appeared to be the important thing, and consequently the method was assumed to be limited to such attributes as could be arranged in a definite scale order. In recent work, however, by varying the order of the classes Pearson has found that so far as the value of the correlation coefficient is concerned this group order has practically no influence. For the new conception of correlation which arose from a consideration of this fact Pearson proposes the term *contingency*.

As a measure of the *contingency* of any classification of characters, it is proposed to use some measure of the 'total deviation of the classification from independent probability.' The practical method of making such a measure Pearson develops in the following way.

"Let A be any attribute or character and let it be classified into the groups A_1, A_2, \dots, A_s , and let the total number of individuals examined be N , and let the numbers which fall into these groups be n_1, n_2, \dots, n_s , respectively. Then the probability of an individual falling into one or the other of these groups is given by $n_1/N, n_2/N, \dots, n_s/N$, respectively. Now suppose the same population to be classified by another attribute into the groups B_1, B_2, \dots, B_t , and the group frequencies of the N individuals to be m_1, m_2, \dots, m_t , respectively. The probability of an individual falling into these groups will be respectively $m_1/N, m_2/N, m_3/N, \dots, m_t/N$. Accordingly the number of combinations of B_v with A_u to be expected on the theory of independent probability if N pairs of attributes are examined is

$$N \times \frac{n_u}{N} \times \frac{m_v}{N} = \frac{n_u \cdot m_v}{N} = \nu_{uv}, \text{ say.}$$

"Let the number actually observed be n_{uv} . Then, allowing for the errors of random sampling,

$$n_{uv} - \frac{n_u m_v}{N} = n_{uv} - \nu_{uv}$$

is the deviation from independent probability

in the occurrence of the groups A_u, B_v . Clearly the total deviation of the whole classification system from independent probability must be some function of the $n_{uv} - \nu_{uv}$ quantities for the whole table." The value of any function of these quantities will clearly be independent of the order of classification.

The following functions of the $n_{uv} - \nu_{uv}$ quantities were chosen for practical use.

(a) $1 - P$; the *contingency grade*, where P is determined from χ^2 by the use of Elderton's tables.* The quantity χ^2 is a measure of the deviation of the observed results from independent probability, depending on the $n_{uv} - \nu_{uv}$ quantities as shown by the equation

$$\chi^2 = S \left\{ \frac{(n_{uv} - \nu_{uv})^2}{\nu_{uv}} \right\},$$

where S indicates summation of like quantities over the whole table. A large value for $1 - P$ indicates that there is association between the attributes, while with a small value of this function the chances are that the system arose from independent probability.

(b) The function

$$\phi^2 = \frac{\chi^2}{N};$$

termed the *mean square contingency*.

(c) The function

$$\psi = \Sigma \frac{(n_{uv} - \nu_{uv})}{N};$$

where Σ denotes summation of all $n_{uv} - \nu_{uv}$ quantities *having the same sign*. This function ψ is called the *mean contingency*.

In determining the functions of ϕ^2 and ψ which shall be used practically, Pearson considers the relation of these quantities in the case of normal correlation. After some analysis the result is reached that

$$\phi^2 = \frac{r^2}{1 - r^2},$$

or

$$r = \pm \sqrt{\frac{\phi^2}{1 + \phi^2}}$$

in the case of normal correlation. This result proves at once that 'the coefficient of correlation is * * * entirely independent of the arrangement of our classes on the basis of any assumed order or scale.'

* *Biometrika*, Vol. I., p. 155.

This function

$$\sqrt{\frac{\phi^2}{1+\phi^2}}$$

is called the *first coefficient of contingency* and is denoted by C_1 .

The analysis of the relation of function ψ in the case of normal correlation leads to the practical result that the value of r may be obtained if ψ is given, which, of course, is the case, the latter function being obtained from the observations. A table and plotted curve from which values of r correct to two places may be read off directly, are given. If the coefficient so obtained from ψ be designated as C_2 the *second coefficient of contingency*, we have as a *limiting case*

$$C_1 = C_2 = r$$

when the correlation is normal and the grouping is sufficiently fine. The approach of C_1 and C_2 to equality may be taken as a measure of the approach of the system to normality and of the correctness of the grouping.

An investigation into the problem of the probable errors of contingency coefficients leads to the result that the probable error of any contingency coefficient C may, for rough judgments, safely be taken to be less than

$$2 \times .67449 \frac{1 - C^2}{\sqrt{n}}.$$

The percentage probable error of

$$\phi^2 = \frac{1.34898}{\sqrt{N}} \sqrt{\frac{1 + \phi^2}{\phi^2}}.$$

After considering the subject of multiple contingency and its relation to multiple normal correlation the author proceeds to give some illustrative examples showing something of the sort of problems to which the method may be applied, and also how it is to be used in practise. The examples include (a) the correlation between father and son in respect to stature, (b) color inheritance in greyhounds, (c) fraternal resemblance in hair color in man, and (d) the correlation between father and son in respect to occupation or profession.

The net results brought out by the analysis and confirmed by the numerical illustrations may best be stated in the author's own words:

"With normal frequency distributions both contingency coefficients pass with sufficiently fine grouping into the well-known correlation coefficient. Since, however, the contingency is independent of the order of grouping, we conclude that, when we are dealing with alternative and exclusive sub-attributes, we need not insist on the importance of any particular order or scale for the arrangement of the subgroups. This conception can be extended from normal correlation to any distribution with linear regression; small changes (*i. e.*, such that the sum of their squares may be neglected as compared with the squares of mean or standard deviation) may be made in the order of grouping without affecting the correlation coefficient." These results "are not so fruitful for practical working as might at first sight appear, for they depend in practise on the legitimacy of replacing finite integrals by sums over a series of varying areas, where no quadrature formula is available. If we, to meet the difficulty, make a very great number of small classes, the calculation, especially of the mean square contingency, becomes excessively laborious. Further, since in observation individuals go by units, casual individuals, which may fairly represent the frequency of a considerable area, will be found on some one or other isolated small area, and thus increase out of all proportion the contingency. The like difficulty occurs when we deal with outlying individuals in the case of frequency curves, only it is immensely exaggerated in the case of frequency surfaces. It is thus not desirable in actual practise to take too many or too fine subgroupings. It is found, under these conditions, that the correlation coefficient as determined by the product moment or fourfold division methods is approximated to more closely in the case of the contingency coefficient found from mean square contingency than in the case of that found from mean contingency. Probably 16 to 25 contingency subgroups will give fairly good results in the case of mean square contingency, but for each particular type of investigation it appears desirable to check the number of groups proper for the purpose by comparing with the results of test fourfold

division correlations. Under such conditions it appears likely that very steady and consistent results will be obtained from mean square contingency.”

In the calculation of contingency coefficients the present writer has found that the following procedure saves much time and labor. The value of the independent probability ν_{uv} for each compartment of the table is obtained by the use of a Thatcher calculating instrument (Keuffel and Esser). With this instrument one can read directly to four or five figures the values of any expression which can be put into the form ax/b , where a and b are constants and x is a variable. Since ν_{uv} for any compartment equals $(n_u \cdot m_v)/N$ for that compartment, it is evident that by taking either n_u or m_v as the constant, it will only be necessary to make as many settings of the instrument as there are rows or columns in the table. Having obtained the ν_{uv} quantities, the sub-contingencies $(n_{uv} - \nu_{uv})$ may be written down directly, squared from Barlow's tables, and divided by ν_{uv} with an arithmometer or with Zimmermann's or Crelle's multiplication tables. The remainder of the calculations necessary to obtain the mean square contingency and the whole of the calculations for the mean contingency, and their respective coefficients are, of course, easily performed. Proceeding in this way, the calculation of contingency coefficients, even though several experimental groupings are made, has been found to take but comparatively little time.

The noteworthy features of this method of contingency are found in that it, in the first place, broadens and illumines the whole theory of correlation, and in the second place, brings within the range of biometrical investigation a large series of problems to which it has hitherto been impossible to apply exact methods. One can but feel that this memoir, like so many of the others which have preceded it in the series, marks a definite and fundamental step in advance in the steady progress of the science of biometry.

RAYMOND PEARL.

‘GLUCINUM’ OR ‘BERYLLIUM.’

SOME years ago the question of choice between the two names ‘glucinum’ and ‘beryl-

lium’ was gone into quite carefully by Professor F. W. Clarke and also by the committee appointed by the American Association on the Spelling and Pronunciation of Chemical Terms, and the conclusion was arrived at that the name ‘glucinum’ should be used on the ground of priority. In SCIENCE for December 9 Dr. Charles Lathrop Parsons has stated his grounds for preferring the name ‘beryllium.’ Dr. Parsons is, thanks to his bibliographical work on the element in question, thoroughly informed in its literature, but the arguments adduced by him would seem to lead to a conclusion diametrically opposed to that which he has drawn.

It was obviously the privilege of Vauquelin, the discoverer of the element, or rather its oxid, to name it. This he never did, but contented himself by speaking of it at first as ‘la terre du Béryl,’ that is, the earth in beryl. At the close of Vauquelin's first paper the editors of the *Annales* added a note signed ‘Redacteur’ in which they propose the name ‘glucine.’ It was of course well known that Guyton and Fourcroy were the editors. Vauquelin's second paper in the *Annales* was evidently prepared at the same time as the first, or at least before the second was in print. In his third paper, some weeks later, as Dr. Parsons admits, Vauquelin actually adopted the term ‘glucine,’ prefacing its use with ‘on a donné le nom de glucine.’ The paper in the *Journal des Mines* was apparently prepared at the same time as the first two papers in the *Annales* and before the appearance of the suggestion of Guyton and Fourcroy, but at its close occurs the note which Dr. Parsons has quoted. In this he states that Guyton and Fourcroy have advised him to call the new earth ‘glucine’ and while he evidently does not think the name the best that could have been chosen, he clearly acquiesces in the suggestion of the two great authorities and says ‘Cette denomination sera assez significative pour aide le mémoire.’ Finally, as seen above, in his third paper, he adopts the name. As far as priority goes, the argument in favor of ‘beryllium’ would seem to be that probably Vauquelin would have given the earth some other name had he ventured to dissent from