On Correlation in Space.

By Dr. T. A. Hirst, F.R.S.

[*Read Nov. 12th*, 1874.]

On quitting the Chair, Dr. Hirst made a communication, the object of which was to indicate how the method described in his recent paper "On the Correlation of two Planes" (see Vol. V., pp. 40—70) may be extended and applied to the corresponding, but more general and complicated question of the Correlation of two Spaces.

1. Fifteen conditions are in general necessary and sufficient to determine such a correlation. The number of solutions in each case may be most readily determined by the prior consideration of the properties of the *series* of correlations, simply infinite in number, which satisfy fourteen conditions.

2. Such a series possesses three *characteristics*, and includes a finite number of *exceptional correlations* of three distinct types.

3. The three characteristics of a series are :

μ, the number of correlations, relative to each of which two arbitrary points (one in each space) are *conjugate;* each corresponding to a plane, which passes through the other.

ρ, the number of correlations, relative to each of which two arbitrary planes (one in each space) are conjugate; each corresponding to a point which is situated in the other.

ν, the number of correlations, relative to each of which two arbitrary right lines (one in each space) are conjugate; each corresponding to a line which meets the other.

4. Of the exceptional correlations in a series,

π possess a pair of *singular points* (one in each space), whose corresponding planes are wholly indeterminate;

ω possess a pair of *singular planes* (one in each space), whose corresponding points are wholly indeterminate; and

χ possess a pair of *singular axes* (one in each space), whose corresponding lines are wholly indeterminate.

5. Between the *characteristics* and *singularities* of every series, the following three simple relations exist:

$$4\mu = \pi + 3\omega + 2\chi,$$
$$4\rho = 3\pi + \omega + 2\chi,$$
$$2\nu = \pi + \omega + 2\chi,$$

whereby μ, ρ, ν can be at once deduced, whenever, π, ω, χ have been directly determined. The conditions being of an elementary character,* this determination can often be made with great facility; more frequently, however, it gives rise to problems of intrinsic interest, for whose solution Professor Sturm has, to some extent, prepared the way by his researches on Projectivity in Space ("Mathematische Annalen," Bd. vi., p. 513.)

6. The characteristics of a series of correlations satisfying fourteen conditions may also be thus defined:

μ indicates the class of the developable surface generated by the planes, in either space, which correspond (in the several correlations of the series) to an arbitrary point in the other space;

ρ indicates the order of the skew curve, in either space, generated by the points which correspond to an arbitrary plane in the other space; and

ν indicates the order of the scroll, in either space, generated by the lines which correspond to an arbitrary line in the other space.

7. The singular planes of each space are common to all the above *developable representatives* of points in the other space; the singular points of each space are also common to all the *skew representatives* of planes in the other space; and, lastly, the singular axes of each space are common to all the *scroll representatives* of right lines in the other space.

8. The properties of these representatives of points, planes, and lines being known, the determination of the number of correlations, in a series, which satisfy any fifteenth condition, presents little difficulty.

9. A knowledge of this number enables us, moreover, to investigate the properties of the *system* of correlations (doubly infinite in number) which satisfy thirteen given conditions.

10. Here the planes which correspond, in the several correlations of the system, to an arbitrary point in one of the two spaces, envelope a surface of known class; the points corresponding to an arbitrary plane generate another surface of known order; and the lines corresponding to an arbitrary line constitute a congruence of known order and class.

11. The exceptional correlations included in such a system are simply infinite in number. The singular planes in each space generate, in fact, a developable, which is circumscribed to every surface that represents a point in the other space; the singular points generate a skew curve, which lies on every surface that represents a plane; and

* The following are examples of such conditions:—Two given points, planes, or lines shall be conjugate (each one condition). The line corresponding to a given line shall pass through a given point, or lie in a given plane (each two conditions). To a given point (plane) shall correspond a given plane (point), (three conditions). To a given line shall correspond a given line (four conditions.)

lastly, the singular lines generate a scroll, which is common to every congruence that represents a line.

12. The special cases where the thirteen conditions are such that the surface representing an arbitrary point in either space is of the *first class*, are particularly interesting; since they furnish us with examples of the rational transformation of the two spaces, and consequently, also, of the point to point representation of surfaces on planes.

13. A fuller exposition of the method pursued, and a more complete statement of the results obtained thereby, are reserved for a future communication.

A New View of the Porism of the In- and Circum-scribed Triangle.

By J. WOLSTENHOLME, M.A.

[*Read November 12th*, 1874.]

Starting with the system of equations

$$\frac{\tan\dfrac{\beta+\gamma}{2}}{\tan\alpha}=\frac{\tan\dfrac{\gamma+\alpha}{2}}{\tan\beta}=\frac{\tan\dfrac{\alpha+\beta}{2}}{\tan\gamma}=p \quad \text{............} \quad (1)$$

(which is equivalent to only a two-fold relation between α, β, γ), I propose to investigate the different forms of equations equivalent to these; and afterwards to give a geometrical interpretation, which gives a complete account of the porism of the in- and circum-scribed triangle, to a pair of coaxal conics.

The angles α, β, γ are throughout supposed unequal and less than 2π.

From (1) we get at once

$$\tan\frac{\beta-\gamma}{2}=\tan\left(\frac{\alpha+\beta}{2}-\frac{\alpha+\gamma}{2}\right)=\frac{p(\tan\gamma-\tan\beta)}{1+p^2\tan\beta\tan\gamma}$$

$$=\frac{p\sin(\gamma-\beta)}{\cos\beta\cos\gamma+p^2\sin\beta\sin\gamma},$$

or $$\cos\beta\cos\gamma+p^2\sin\beta\sin\gamma=-p\{1+\cos(\beta-\gamma)\},$$

or $$\cos\beta\cos\gamma+p\sin\beta\sin\gamma+\frac{p}{1+p}=0 \quad \text{...............} \quad (2),$$

and the two like equations.

From (2) we get, since β, γ are the two roots of the equation,

$$\cos\alpha\cos\theta+p\sin\alpha\sin\theta+\frac{p}{1+p}=0,$$

$$\frac{\cos\dfrac{\beta+\gamma}{2}}{\cos\alpha}=\frac{\sin\dfrac{\beta+\gamma}{2}}{p\sin\alpha}=\frac{\cos\dfrac{\beta-\gamma}{2}}{-\dfrac{p}{1+p}};$$