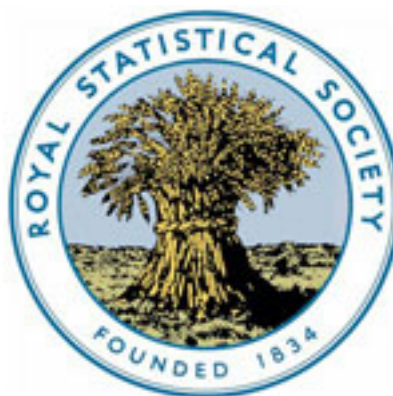


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STATISTICS *of* UNPROGRESSIVE COMMUNITIES.*By* PROFESSOR F. Y. EDGEWORTH.

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STATISTICS, according to some authorities, must be defined so as to relate to society; but must the society be human? It is rather in the unprogressive communities of the lower gregarious animals that we may look for the realisation of that statistical ideal which Quetelet called *Physique Sociale*: the stable averages obtained from large numbers of statistics being compared to the dimensions of a body ascertained by repeated measurements. I have attempted to determine some statistical constants for one of the most orderly societies, that of wasps.¹

I. The first inquiry which presents itself is one initiated by Sir John Lubbock,² How long does a wasp take in loading herself with sweets? What is the average interval of time between the moment at which she sets to work at honey or jam or fruit, and the moment when she flies off with her fragrant load? Some answers to this question obtained from Sir John Lubbock's observations are given in Table I.

¹ My observations have been made at intervals in the course of the last eleven years. They all, as far as I know, relate to the same species, which may be popularly denominated the common or ground wasp. The observations were made in three localities: Edgeworthstown in Ireland, Winchfield in Hampshire, and Hampstead Heath. Of the nests referred to by letters of the alphabet, A, B, C, D were at Edgeworthstown; E, F, G, H at Hampstead.

² "Ants, Bees and Wasps," second edition and fifth edition.

TABLE I.—*Showing the Mean Time Occupied by a Wasp in taking in a Load of Honey; according to some of Sir John Lubbock's Observations.*

1 Chronological Order.	2 Reference* and Description.	3 Number of Loads.	4 Average Time of Lading.
1	P. 415 9.50 A.M.—6.25 P.M. 6.55 A.M.—11.36 A.M.	64	3'5
2	P. 421 1.38 A.M.—5.4 P.M.	18	3'5
3	P. 422 12.6 P.M.—5.36 P.M.	21	3'2

* The references are to Sir John Lubbock's "Ants, Bees, and Wasps."

These averages agree with those which I have obtained by observing wasps at work on liquid marmalade. The average time of lading obtained by me from fourteen observations relating to two or three different wasps was 2'8 minutes. But my agreement with Sir John Lubbock ended when I substituted dry hard marmalade for the liquid sweet. The time spent in getting together a load of this material, sometimes hewn into blocks or blobs and carried outside the bearer, not inside, as usually,³ averaged 10 minutes.⁴ The average obtained from thirty-nine observations made upon several wasps with marmalade of different degrees of solidity was 6 minutes. This cannot be considered a very good average, since the proportion in which the juicy and the hard materials were offered was accidental, having no correspondence with the distribution which prevails in the regions where the average wasp may forage.

II. Let us next inquire what is the interval of time between the moment when a wasp flies off with a load of sweets to the moment when she returns to the scene of work for another load. Table II gives the averages obtained from the statistics of this interval which Sir John Lubbock has given for the wasps and periods specified in our first table.

TABLE II.—*Showing the Mean Interval between a Wasp's Departure from and Return to Honey; according to some of Sir John Lubbock's Observations.*

Chronological Order.	Number of Observations.	Mean Interval.
1	62	9'3
2	18	8'5
3	21	9'5

³ Réaumur says, universally.

⁴ This is the average of five perfect observations. I have made many more imperfect, interrupted observations, pointing in the same direction.

These averages agree closely with that which I have obtained from forty-four observations made upon several wasps⁵ with marmalade of different degrees of solidity, namely, 9 minutes. We seem therefore to have here a remarkably trustworthy average. It should be remarked, however, that a somewhat smaller figure was obtained by Sir John Lubbock for two wasps which he has mentioned in his second and fifth editions respectively. He has not indeed in the latter case given the figures for the interval in question, but only those for the whole cycle which consists of the time of lading together with the interval of absence. The wasp observed by him on 7th August, 1882, of whom he gives particulars at p. 423 of his fifth edition, visiting the honey for the first time at 4.13 A.M. and for the last time at 7.46 P.M., executed 115 cycles in 15 hours 33 minutes, that is at the rate of 8.1 minutes to the cycle. Even supposing that the time of lading was in this case particularly short, the mean interval of absence from the honey must have been less than that which the other observations establish. As for the wasp who, as mentioned in the corresponding passage of the first and earlier editions, was observed on 11th September, 1875, I find for the mean of the first twenty-five intervals of absence recorded 5.24 minutes, and for the mean of the last twenty-five intervals 4.7.⁶

III. Let us next endeavour to determine the time occupied by an average wasp in the operations which she goes through from the moment when she issues from the nest to the moment when—after having obtained and disposed of a load⁷—she re-issues. We are to imagine a continual stream of workers spouting from the aperture in the nest, flowing through a round of operations—through air and woods, among flowers and in the haunts of man—re-entering the cavity from which it issued, permeating the subterranean regions where it enlarges the cells and nourishes their denizens by its deposits, and once more re-issuing into the upper air. What is the mean periodic time of this revolution?

At first sight it might seem that an answer to this question is obtained by adding the constants which have been determined in our first and second sections, making some 15 minutes for the cycle. But the figures which have been obtained above must not, without verification, be taken as typical of the general case.

⁵ To make these observations it is generally necessary to mark the wasp under observation. Cp. "Lubbock," *loc. cit.*

⁶ Assuming that a wasp flies at the rate of a mile in two minutes, we should expect a difference of a mile in the distance of the nest from the honey to be attended with a difference of four minutes in the time-interval under consideration. But see note on p. 382.

⁷ The general case, I think. But see p. 382.

It has not fallen to the lot of every wasp to meet a statistician tendering sweets. Time must ordinarily be spent in looking for good things. Moreover all wasps are not employed in gathering sweets. Some are hewing wood to be employed in the enlargement or repair of the subterranean city; others are procuring flies and other animal food for the young. In the absence of statistics for these industries I propose to determine the duration of the mean cycle by dividing it, not as above, but into two parts which admit of approximate determination, namely, the interval between exit and entrance, the mean time of a voyage, and the mean time elapsing between entrance and re-issue, the mean interval between two successive voyages.

IV. Dealing now with a flow of wasps going through a cycle of operations, we shall find the following simple lemma respecting fluid motion useful. Suppose a fluid of uniform density flowing steadily through a pipe or channel—not necessarily of uniform bore—so that the quantity of fluid, *e.g.*, the number of gallons of water, which flows past every point per minute is the same. Let the channel be closed suddenly at a point A, and at another point B below A. The quantity of fluid intercepted between A and B is equal to the number of gallons which was flowing past each point per minute multiplied by the number of the minutes which a portion of the fluid (or a body immersed in the fluid) would have taken in moving from A to B. Conversely, given the number of gallons passing each point per minute and the quantity intercepted between A and B, we have the time it took to flow from A to B. The proposition remains true if we suppose the course of the stream to be circular, so that the points A and B, as the crow flies—though not as the stream flows—are close to each other. The points may even be identical, if we may suppose that—

“Streams meander level with their fount,”

as in Robert Montgomery’s infelicitous metaphor.

Now substitute wasps for water; and after observing the number of wasps pouring out of the nest, and pouring into it, per minute—numbers which ought to be approximately equal if the motion is steady—close suddenly the aperture which is at once point A and B of the wasp-stream, and intercept all the arrivals. The number of the wasps which are thus excluded divided by the number of wasps which had been issuing or entering per minute gives the average time of absence from the nest.

TABLE III.—*Showing the Determination of the Mean Duration of Voyage by the Method of Exclusion.*

1 Designation of Nest.	2 Time of Closure.	3 Observations on which Rate of Flow is based.	4 Rate of Flow per Minute.	5 Number Excluded.	6 Figure in Col. 5 ÷ Figure in Col. 4 = Mean Duration of Voyage.	7 Remarks.
G	5.9 P.M. 26th September	Exits and entrances from 4.42 P.M. to 5.9 P.M.	8.02	125	15.6	The nest was not actually closed, but it is ascertained from Table IX that, if it had been closed, 125 would have been excluded. See p. 379.
G	7.1 A.M. 27th September	Exits and entrances from 6.36 A.M. to 7.1 A.M.	5.24	90	17.2	It is ascertained from Table VII that, if the nest had been closed, 90 would have been excluded. See p. 379.
H	4.27 P.M. 6th October ...	Exits and entrances from 4 P.M. to 4.27 P.M.	3.52	51	14.5	51 ascertained from Table XI. See p. 381.
G	2.40 P.M. 1st October ...	Exits only from 2.23 P.M. to 2.40 P.M. ...	4.77	81	17	Nest actually closed. Some observations of entrances were taken showing by their approximate equality to the exits that the flow was fairly steady.
B	? About 7 A.M. 17th August	Partly exits and partly entrances during 19 minutes anterior to closure	5.63	100	17.8	Nest actually closed. Time of closure not noted; it was some hours before the operation on the same nest re- ferred to in Table IV.
F	5.50 P.M. 23rd September	Exits only from 5.45 P.M. to 5.50 P.M. ...	7.2	124	17	Nest actually closed.

Table III embodies the result of some experiments of this sort. The observations on which these results are grounded are not all of equal weight. The first sets only fulfil the conditions of a fairly accurate measurement, namely (i), that the number of observations by which the flow is determined should be sufficient to exclude great error; (ii), that the flow should be ascertained to be steady; (iii), that the number of the excluded should be ascertained with precision. A few words on each of these conditions.

(i.) With regard to the first condition it will be remembered by readers of this *Journal*, that I have determined^{*} the fluctuation in the number of wasps entering and issuing per minute from nests of an average size, having a traffic of above 30 per minute. The *modulus* for this fluctuation was found from a great number of observations to be about 8.5. But this conclusion cannot with safety be extended to cases where the traffic is very much smaller than 30. The presumption is that the modulus for smaller figures is an absolutely smaller number than 8.5, but a relatively greater percentage of the mean traffic than 8.5 is of 30 or 40. From summary observations I find that this presumption is verified; and that for a traffic of from 7 to 10 per minute, we may put 40 or 50 per cent. as the modulus for the single observation. It will be seen accordingly that the determination of the flow in the last instance given in the table has hardly any weight; the result is only right by accident. The determination of the flow in the first instances might be affected with an error of modulus 10 per cent., or probable error of 5 per cent.

(ii.) The necessity that the flow should be steady may be explained by the analogy of a familiar proposition in vital statistics. It is well known that if the population was stationary the mean life-time (or "expectation" at birth) would be equal to the average age of the existing population; and the number of births per year \times the mean life-time would be equal to the number of the existing population. But, if the population is increasing, then the number of births per year for the last year \times the mean life-time, will be greater than the number of the existing population; and the product will be less, if the number of births is declining. To transfer these propositions to the case before us, let us figure the time during which a wasp remains out as a sort of life-time—as we speak of the "life" of a banknote. What corresponds to the number of births per year in the human statistics will be with reference to the present problem the number of individuals issuing per minute. Whence by parity of reasoning it is evident that if

^{*} *Journal of the Statistical Society*, Jubilee Volume, p. 209.

this number is not constant, the formula above given will not be accurate; but will give too small a result if the flow is increasing, too large a result if the flow is declining. The latter error is the one which I have most frequently experienced. Table IV exemplifies some instances in which the method seems to have failed, presumably owing to the condition of steadiness not being fulfilled.

TABLE IV.—*Showing some Inexact Applications of the Method of Exclusion.*

1 Designation of Nest.	2 Time of Closure.	3 Observations on which Rate of Flow is based.	4 Rate of Flow per Minute.	5 Number Ex- cluded.	6 Figure in Col. 5 ÷ Figure in Col. 4 = Mean Duration of Voyage.	7 Remarks.
B {	12.40 P.M. 17th Aug. {	Exits and entrances from 12.30 to 12.38 P.M. {	1.3	14	10.8	The nest had already undergone the operation described in Table III some hours before
B {	10.15 A.M. 19th Aug. {	Exits and entrances from 10.10 to 10.15 A.M. {	2.5	65	26	The nest showed signs of recovery during 18th August. This observation is referred to on p. 365.
E {	3.13 P.M. 17th Sept. {	Exits only from 2.53 to 3.13 P.M. {	6	190	31.7	
E {	12 noon 18th Sept., {	Exits only 11.44 A.M. to 12 noon.... {	4.9	131	26.7	

(iii.) As to the determination of the dividend in the formula, the number of the excluded, the method which first occurs is after the gates have been closed, to knock down the outsiders one by one as they arrive, or after arrival, while they hang about their native spot. But where the number of arrivals is considerable, confusion is apt to seize both the operator and those operated on. The one loses count, and the others, seeing the turn which affairs are taking, keep aloof.

The method of census by slaughter can only be justified when the circumstances of the vespiary are such, that it is possible to intercept each incomer without confusion or concert between the excluded. The conditions most favourable for this operation are a long straight hole in solid earth, and a traffic of moderate dimensions. The hole having been stopped at some distance—say, three inches—from the surface, each returning wasp rushes down the *cul de sac*, never to rise again; followed by fate in the form

of a walking stick. I here only succeeded in performing this operation twice^a with the following results.

Nest B, having been closed at 10.15 A.M., 19th August, 1895.—

Between the times	there returned
10.15—10.25.....	20
10.25—10.35.....	20
10.35—10.45.....	14
10.45—10.55.....	10
10.55—11.05.....	1
11.05—11.10.....	0

The nest was not watched after 11.10.

Nest G, having been closed at 2.40 P.M., 1st October, 1895:—

Between the times	there returned
2.40—2.45.....	25
2.45—2.50.....	11
2.50—2.55.....	16
2.55—3.00.....	15
3.00—3.05.....	3
3.05—3.10.....	0
3.10—3.15.....	2
3.15—3.30.....	1
3.30—3.40.....	5
3.40—4.15.....	2

The nest was not watched continuously after 3.30 P.M.; not at all after 4.15 P.M. It was left closed up till the following morning, when one outsider was found waiting at the closed portal.

These statistics agree sufficiently well with the results which have been given in Table III; if we bear in mind that whatever the average period of absence, there are likely to be considerable deviations on the part of individuals from that average. One, as it may happen, hunts flies, another collects sweets, another hews wood—operations not necessarily of equal average duration. Within each class there may be wide variations, according as, for instance, the sweets are hard or liquid, the flies shy or abundant.

TABLE V.—*Showing the Times at which Wasps Returned to Nest B.*
(Time of Closure = 0.)

Time	Number Returned.	Percentage.	
0—10	20	31·2	—
10—20	20	31·2	—
20—30	14	21·9	—
30—40	10	15·6	} 15·7
40—45	1	0·1	

^a An imperfect operation on nest B warranted the conclusion that the great bulk of outsiders had arrived within half-an-hour.

TABLE VI.—*Showing Times at which Wasps returned to Nest G.*
(*Time of Closure = 0.*)

Time.	Number Returned.	Percentage.	
0—10	36	44.4	—
10—20	31	28.3	—
20—30	3	3.7	—
30—50	3	3.7	} 13.7
50—	8	10.0	

If we assume the mean to be about 17, after Table III, the results given in Tables V and VI are inconsistent with the hypothesis that the mean duration of a voyage fluctuates according to a symmetrical probability curve.¹⁰ If the mean be assumed a little greater, an asymmetrical probability curve would seem to suit the data. But they are unfortunately too scanty to build a curve.

To kill in order to count may be legitimate when we thereby ascertain not only the average period, but also the law of deviation from that average. But for the former purpose only this violence is not necessary. A painless way of working the method which I have proposed for determining the mean period of absence¹¹—the “method of exclusion” it might be called—is incidentally afforded by a quite different method, for which it is sufficient to note the times of going out and coming in. At first sight these data might appear inadequate. To revert to our old metaphor, it is as if from a knowledge of the numbers born and dying each year we could determine the mean duration of life. Or again, it is as if a policeman watching with notebook in hand in the neighbourhood of a cabstand, as you may see him watching at one of our great railway stations, could determine the average time taken up by a “fare” by merely noting the times of departure from and return to the stand, but not the “number” or designation of each particular cab. But the magic of statistics can charm an answer from such apparently insignificant data.

Let the statistician arise betimes, and planting himself long before sunrise in front of a nest, with eye intent upon its gateway, watch for the appearance of the first wasp going forth to her work. Let t_1 be the time of this first exit. Now if we could mark individuals as they rush out, or discern those marked as they rush in, we might observe the time τ_1 , at which the wasp who issued at t_1

¹⁰ Since the modulus cannot well be assumed greater than 10, the curve not extending below zero; and that hypothesis is not adequate to account for the occurrence of some 15 per cent. of the observations (see Tables V and VII) above 30. Whatever hypothesis we adopt as to the form of the frequency-curve for wasp-voyages, it would seem that the fluctuation (modulus-squared. See *Methods*—Jubilee Volume, p. 188) must be at least 100.

¹¹ This method is employed in the first three instances of Table III.

returns. We should thus have one observation, namely, $\tau_1 - t_1$ for the sought quantity, the mean time of absence. Proceeding similarly with the second individual we should obtain a second observation $\tau_2 - t_2$; and so on. As the answer to our problem we should put the mean of a large number, say n , of such observations, thus—

$$\frac{\sum_1^n \tau - \sum_1^n t}{n}.$$

Now we cannot determine the τ 's which correspond to particular t 's. But, if we take for $\sum_1^n \tau$, the sum of the first n times of entrance, we may with safety put for the corresponding $\sum_1^n t$, the sum of the times of the first n exits. That is the most probable value¹² which we in our ignorance of particulars can assign for the sum of t 's corresponding to the sum of the first τ 's; and it probably does not differ seriously from the true sum.¹³

For example, on the morning of 27th September, 1895, at the nest G, on Hampstead Heath, I observed the first wasp to issue at 5^h.22^m. A.M. (more than half an hour before sunrise); and I noted the times of departure and arrival for the next 100 minutes, till 7^h.1^m. A.M. During that time there occurred in all 362 entrances. And the time within which the first 362 exits occurred proved to be 85^m.6^s. counting from 5^h.21^m. A.M. as zero. Adding together the times of these 362 entrances (= 23000), and subtracting the sum of the times of the first 362 exits (= 17012), we have 5,988; which being divided by 362 gives 16^m.5^m. as the mean duration of voyage. The original observations and the apparatus for working them up are given in Table VII:—

¹² I do not know that this proposition has been enunciated before. It is deduced from the presumption that the varying lengths of voyages obey the law of error, a proposition which may not be quite accurate. (See above, p. 365.)

¹³ If a certain number, say r , of the n last exits do not correspond to entrances which are included among the n first entrances, then we substitute for our fictitious set of observations, $\sum_1^n \tau - \sum_1^n t$, a real set; either (1) by retaining $\sum_1^n t$, and substituting for the m fictitious τ 's in $\sum_1^n \tau$ the dates of entrances really corresponding to our exits, or (2) by retaining $\sum_1^n \tau$ and substituting for the n fictitious t 's in $\sum_1^n t$, the dates of exit really corresponding to our entrances. The first set of real observations would yield a larger value (for the mean duration of a voyage), the second set of real observations a smaller value than that which is yielded by the fictitious set of observations which we have adopted. The result of the latter then, being intermediate between the results of two real sets of observations, may be presumed to be at least as good as either of them.

TABLE VII.—*Dealing with Observations made on Nest G, 5.21—7.1 A.M., 27th September.*

1 Figure in Col. 2 × corresponding Figure in Col. 3.	2 Number of Entrances at each Minute.	3 Number of Minutes after 5.21 A.M.	4 Number of Exits at each Minute.	5 Figure in Col. 4 × corresponding Figure in Col. 3.
0	0	1	1	1
0	0	2	0	0
0	0	3	1	3
0	0	4	0	0
5	1	5	0	0
0	0	6	0	0
0	0	7	0	0
8	1	8	0	0
0	0	9	4	36
0	0	10	6	60
0	0	11	6	66
0	0	12	4	48
0	0	13	6	78
0	0	14	3	42
15	1	15	4	60
16	1	16	7	112
17	1	17	1	17
36	2	18	2	36
0	0	19	3	57
20	1	20	2	40
0	0	21	6	126
44	2	22	2	44
0	0	23	7	161
72	3	24	14	336
100	4	25	4	100
26	1	26	2	52
162	6	27	5	135
84	3	28	3	84
58	2	29	9	261
90	3	30	12	360
124	4	31	0	0
224	7	32	3	96
99	3	33	3	99
238	7	34	4	136
105	3	35	6	210
180	5	36	2	72
222	6	37	3	111
114	3	38	7	266
195	5	39	8	312
0	0	40	9	360
287	7	41	4	164
294	7	42	5	210
258	6	43	7	301
176	4	44	2	88
180	4	45	6	270
276	6	46	1	46
47	1	47	3	141
336	7	48	3	144
147	3	49	7	343
250	5	50	6	300
306	6	51	1	51
104	2	52	2	104
424	8	53	0	0

TABLE VII.—*Dealing with Observations made on Nest G—Contd.*

1 Figure in Col. 2 × corresponding Figure in Col. 3.	2 Number of Entrances at Each Minute.	3 Number of Minutes after 5.21 A.M.	4 Number of Exits at each Minute.	5 Figure in Col. 4 × corresponding Figure in Col. 3.
0	0	54	1	54
110	2	55	3	165
112	2	56	6	336
171	3	57	5	285
348	6	58	7	406
118	2	59	7	413
240	4	60	3	180
183	3	61	4	244
186	3	62	4	248
252	4	63	2	126
128	2	64	5	320
325	5	65	8	520
462	7	66	7	462
402	6	67	3	201
204	3	68	9	612
276	4	69	4	276
770	11	70	1	70
71	1	71	3	213
360	5	72	8	576
146	2	73	8	584
370	5	74	10	740
375	5	75	12	900
532	7	76	2	152
462	6	77	0	0
468	6	78	3	234
711	9	79	2	158
160	2	80	5	400
486	6	81	7	567
164	2	82	2	164
664	8	83	2	166
504	6	84	5	420
340	4	85	7	595
172	2	86	10	860
261	3	87	10	870
352	4	88	6	528
267	3	89	5	445
450	5	90	8	720
364	4	91	2	182
552	6	92	4	368
279	3	93	8	744
376	4	94	4	376
570	6	95	7	665
864	9	96	4	384
679	7	97	4	388
784	8	98	7	686
891	9	99	5	495
700	7	100	7	700

Here the central column, numbered 3, specifies successive dates, minutes measured from 5.21 A.M. as zero. On the left in Col. 2 are the number of entrances, and on the right in Col. 4 the number of exits which occurred at each date; an event which occurred within 30" after or before a certain minute being treated as

2 B 2

occurring at that minute. Thus the date 24 with 14 on the right and 3 on the left, means that between the times 5^h. 44^m. 30^s. and 5^h. 45^m. 30^s. there issued 14 wasps, and there entered 3. Col. 1 gives the times of entrance, obtained by multiplying each date with the number of entrances occurring at that date. Col. 5 gives the similarly obtained times of exit.

Of course it is open to us to utilise only part of our data; for instance, only the observations of the first 25 minutes, or only those of the first 50 minutes, as is done in the successive rows of Table VIII. The averages obtained from these sets of observations are given in Col. 6 of Table VIII.

The comparison of these averages brings out the curious circumstance that the earlier voyages are, on an average, shorter than the later ones. This peculiarity becomes clearer when we separately consider the experience relating to successive short periods, 0—25, 25—50, 50—75, 75—100. This separation is effected in Cols. 8—10 of Table VIII. Col. 8 is obtained by taking the sum of the dates of the entrance in each period of 25 minutes, and subtracting therefrom the sum of the dates of exit which most probably correspond thereto. The first figure in Col. 8 is repeated from Col. 6. To obtain the second figure we have to separate the sums of dates of entrances occurring in the period 0—25, the first figure in Col. 4, from the sum occurring in the period 0—50, the second figure in Col. 4. This gives the sum of entrances for period 25—50. The sum of exits most probably corresponding to the entrances in period 25—50 is similarly to be obtained from Col. 5. The difference between these two sums gives the $S\tau - St$ proper to the period 25—50. It comes to the same to subtract from each figure in Col. 6 the preceding figure in the same column. Thus, $(4505 - 333) - (2531 - 166) = (4505 - 2531) - (333 - 166) = 1974 - 167 = 1807$ (the second figure in Col. 8). The figures thus obtained for Col. 10 seem to confirm the indication that the twilight voyages are abnormally short.

It is not absolutely necessary—though it is desirable—that the statistician should rise with the wasp in the morning. The closing time in the evening, as well as the going forth to work in the morning affords a $\rho\sigma\theta$ $\sigma\tau\omega$ for the statistician, a fixed point from which to reckon sums of t 's and τ 's. Table IX embodies observations made by me on Nest G on the evening of 26th September. I began to register the exits and entrances at 4.42 P.M., and continued up to the date of the last load home, which proved to be at 6.30.¹⁴

¹⁴ One cannot in the evening as in the morning experiment, determine to have just so many and no more observations. That is why the number of minutes in Tables VII, IX, and XI is not the same.

TABLE VIII.—*Showing results Deducible from Table VII.*

1	2	3	4	5	6	7	8	9	10	11
Periods Measured in Minutes (5.21 A.M. = 0)	Number of Entrances in Periods of Col. 1.	Period in which the Number of Exits = the Number of Entrances in Col. 2.	Sum of Times of Entrance in Periods of Col. 1.	Sum of Times of Exits in Periods of Col. 3.	Figure in Col. 4 Minus corresponding Figure in Col. 5.	Figure in Col. 6 ÷ corresponding Figure in Col. 2 = Mean Duration of Voyage in Periods of Col. 1.	Figure in Col. 6 Minus preceding Figure in same Column.	Figure in Col. 2 Minus preceding Figure in same Column.	Figure in Col. 3 ÷ corresponding Figure in Col. 9 = Mean Duration of Voyage in partial Periods.	End of Periods in Col. 1 Minus End of Periods in Col. 3.
0—25	17	0—11	333	166	167	9.8	167	17	9.8	14
0—50	125	0—34½	4,505	2,681	1,824	14.6	1,657	108	15.35	15½
0—75	226	0—57½	10,948	7,969	3,679	16.3	1,855	101	18.4	17½
0—100	362	0—85½ 10	23,000	17,012	5,988	16.5	2,309	136	16.9	14.9

TABLE IX.—*Dealing with Observations made on Nest G, 4.42—6.30 P.M.,
26th September.*

1 Figure in Col. 3 × corresponding Figure in Col. 1.	2 Number of Exits at each Minute.	3 Number of Minutes after 4.42 P.M.	4 Number of Entrances at each Minute.	5 Figure in Col. 3 × corresponding Figure in Col. 4.
9	9	1	7	—
18	9	2	8	—
27	9	3	6	—
36	9	4	12	—
30	6	5	6	—
84	14	6	11	—
56	8	7	14	—
56	7	8	6	—
45	5	9	8	—
70	7	10	7	—
44	4	11	12	—
108	9	12	9	—
143	11	13	10	—
84	6	14	10	—
210	14	15	5	—
96	6	16	4	64
85	5	17	8	136
162	9	18	2	36
190	10	19	5	95
60	3	20	12	240
210	10	21	9	189
154	7	22	10	220
115	5	23	5	115
144	6	24	10	240
150	6	25	9	225
286	11	26	7	182
216	8	27	9	243
196	7	28	9	252
261	9	29	3	87
150	5	30	6	180
93	3	31	8	248
320	10	32	3	96
264	8	33	4	132
68	2	34	11	374
210	6	35	7	245
108	3	36	7	252
259	7	37	6	222
152	4	38	8	304
312	8	39	6	234
240	6	40	12	480
410	10	41	3	123
126	3	42	10	420
86	2	43	11	473
352	8	44	13	572
315	7	45	10	450
276	6	46	7	322
282	6	47	7	329
283	6	48	7	336
196	4	49	6	294
450	9	50	5	250
306	6	51	9	459
312	6	52	6	312

TABLE IX.—*Dealing with Observations made on Nest G—Contd.*

1 Figure in Col. 3 × corresponding Figure in Col. 1.	2 Number of Exits at each Minute.	3 Number of Minutes after 4.42 r.m.	4 Number of Entrances at each Minute.	5 Figure in Col. 3 × corresponding Figure in Col. 4
318	6	53	5	265
486	9	54	5	270
440	8	55	7	385
112	2	56	9	504
285	5	57	3	171
290	5	58	9	522
295	5	59	12	708
600	10	60	8	480
488	8	61	4	244
434	7	62	9	558
378	6	63	8	504
384	6	64	15	960
195	3	65	8	520
330	5	66	4	264
335	5	67	4	268
136	2	68	8	544
138	2	69	6	414
490	7	70	3	210
576	8	72	4	288
292	4	73	8	584
370	5	74	7	518
300	4	75	8	600
532	7	76	6	456
0	0	77	1	77
234	3	78	5	390
158	2	79	3	237
320	4	80	9	720
81	1	81	8	648
328	4	82	6	492
332	4	83	7	581
336	4	84	5	420
170	2	85	5	425
344	4	86	4	344
261	3	87	6	522
176	2	88	3	264
450	5	90	4	360
182	2	91	3	273
92	1	92	5	460
186	2	93	5	465
94	1	94	3	282
285	3	95	6	570
0	0	96	4	384
0	0	97	2	194
98	1	98	5	490
0	0	99	4	396
0	0	100	2	200
363	3	101	2	202
102	1	102	0	0
0	0	103	0	0
0	0	104	3	312
0	0	105	0	0
0	0	106	2	212
0	0	107	0	0
0	0	108	1	108

TABLE X.—*Showing results Deducible from Table IX.*

1	2	3	4	5	6	7	8	9	10	11
Period Measured in Minutes (44.2 P.M.=0)	Number of Exits in Period of Col. 1.	Period in which Number of Entrances = Number of Exits in Col. 2.	Sum of Times of Exit in Period of Col. 1.	Sum of Times of Entrance in Period of Col. 3.	Figure in Col. 5 Minus corresponding Figure in Col. 4.	Figure in Col. 6 ÷ corresponding Figure in Col. 2 = Mean Duration of Voyage for Period of Col. 1.	Figure in Col. 6 Minus preceding Figure in same Column.	Figure in Col. 2 Minus preceding Figure in same Column.	Figure in Col. 8 ÷ corresponding Figure in Col. 9 = Mean Duration of Voyage for partial Periods.	Beginning of Period of Col. 3 Minus Beginning of corresponding Period of Col. 1.
81—103	42	91½—108	3,739	4,091	352	8.4	352	42	8.4	10½
54—108	166	66½—108	11,932	13,843	1,911	11.5	1,559	124	12.6	12½
27—103	332	43½—108	18,768	23,105	4,337	13.1	2,426	166	14.6	16½
0—108	545	15.5—108	21,656	29,684	8,008	14.7	3,671	213	17.2	15.5

TABLE XI.—*Dealing with Observations on Nest H, 4—5.47 P.M.,
7th October, 1895.*

1 Figure in Col. 3 × Corresponding Figure in Col. 1.	2 Number of Exits at each Minute.	3 Number of Minutes after.	4 Number of Entrances at each Minute.	5 Figure in Col. 3 × Corresponding Figure in Col. 4.
2	2	1	6	6
2	1	2	5	10
21	7	3	3	9
12	3	4	7	28
20	4	5	3	15
18	3	6	0	0
28	4	7	4	28
8	1	8	5	40
27	3	9	2	18
60	6	10	3	30
55	5	11	4	44
36	3	12	3	36
65	5	13	3	39
42	3	14	6	84
105	7	15	4	60
96	6	16	4	64
51	3	17	1	17
36	2	18	6	108
19	1	19	6	114
40	2	20	5	100
84	4	21	6	126
66	3	22	4	88
69	3	23	2	46
24	1	24	0	0
75	3	25	3	75
130	5	26	3	78
27	1	27	1	27
56	2	28	5	140
23	1	29	3	87
30	1	30	6	180
62	2	31	4	124
160	5	32	5	160
165	5	33	2	66
272	8	34	0	0
140	4	35	7	245
180	5	36	3	108
185	5	37	2	74
76	2	38	2	76
156	4	39	2	78
40	1	40	3	120
205	5	41	3	123
210	5	42	4	168
172	4	43	2	86
88	2	44	3	132
45	1	45	6	270
0	0	46	3	138
141	3	47	4	188
48	1	48	3	144
49	1	49	4	196
50	1	50	5	250
204	4	51	3	153
52	1	52	4	208
106	2	53	3	159

TABLE XI.—*Dealing with Observations on Nest H—Contd.*

1 Figure in Col. 3 × Corresponding Figure in Col. 1.	2 Number of Exits at each Minute.	3 Number of Minutes after.	4 Number of Entrances at each Minute.	5 Figure in Col. 3 × Corresponding Figure in Col. 4.
54	1	54	0	0
220	4	55	4	220
56	1	56	1	56
114	2	57	2	114
174	3	58	2	116
59	1	59	1	59
180	3	60	5	300
241	4	61	6	366
186	3	62	3	186
189	3	63	3	189
192	3	64	2	128
195	3	65	1	65
330	5	66	3	198
201	3	67	1	67
204	3	68	3	204
276	4	69	1	69
140	2	70	5	350
355	5	71	4	284
360	5	72	2	144
0	0	73	4	292
148	2	74	3	222
75	1	75	4	300
76	1	76	4	304
77	1	77	5	385
78	1	78	4	312
237	3	79	2	158
80	1	80	3	240
486	6	81	3	243
82	1	82	1	82
249	3	83	1	83
0	0	84	1	84
170	2	85	4	340
172	2	86	2	172
0	0	87	2	174
0	0	88	3	264
0	0	89	4	356
270	3	90	3	270
182	2	91	2	182
184	2	92	2	184
93	1	93	1	93
0	0	94	0	0
95	1	95	1	95
96	1	96	3	288
97	1	97	0	0
0	0	98	3	294
—	—	99	2	198
—	—	100	2	200
—	—	101	1	202
—	—	102	1	102
—	—	103	3	309
—	—	104	3	312
—	—	105	0	0
—	—	106	1	106
—	—	107	1	107

Beginning now with the exits, we have by observation $S_1^n t$, the sum of the dates of the last n exits (Col. 4 of Table IX); and we assume, as the sum of the dates of the corresponding entrances $S_1^n \tau$, the sum of the last n entrances (Col. 5, Table IX). After the explanation which has been given of Tables VII and VIII it will not be necessary to explain the parallel Tables IX and X.

The agreement between the morning and evening experience will be found sufficiently close. Considering in Col. 10 of both Tables VIII and X the last two periods only, as they alone are clear in both tables of the influence of twilight, we have for the mean duration of a voyage in the morning 4164 (the difference between the fourth and the second figure in Col. 6 of Table VIII) $\div 237$ (the difference between the corresponding figures in Col. 2), that is 17.6 ; and for the mean duration of a voyage in the evening by parity, from Table X, $6129 \div 379$, that is 16.2 . The difference between the two means 16.9 and 16.2 is not very considerable.¹⁵ The concurrence of the two sets of observations is mutually confirmatory.¹⁶

¹⁵ It is scarcely within the limit of accidental deviation, if we assume as above (p. 366, Note) that the modulus for a single voyage is about 10 minutes. The modulus for the difference of the two means would be about $\sqrt{\frac{100}{237} + \frac{100}{379}}$, that is $.8$, while the actual difference between the means is 1.4 . (See the present writer's "*Methods of Statistics*," *Jubilee Volume* of this *Journal*.)

¹⁶ It is to be regretted that the morning and evening operations are not mutually corrective with respect to one or two scruples which may occur as to the correctness of the result. It will be instructive to introduce these difficulties by the analogy of vital statistics. As already remarked, the returns of the numbers born and dying each year are not of themselves sufficient to determine the mean lifetime. They become sufficient however when supplemented by a datum which the antediluvian statistician enjoyed—the date at which the first man came into existence. With this fixed point for his t (above p. 367) he might find the sum of the first n t 's, and put as the corresponding sum of τ 's the sum of the dates of the first n deaths. The difference of the two sums divided by n gives the mean lifetime of an antediluvian patriarch. But the following scruple may be suggested. Supposing that the birth of Methuselah and some others of exceptional longevity occurred among the t 's, while their deaths do not figure among the τ 's; the average obtained by our morning method would be unduly small. Nor would it be corrected by the use of the evening method for the periods preceding the flood. The occurrence of the death of a Methuselah in one of those periods would indeed slightly lengthen the resulting mean lifetime, by substituting among the n last τ 's, one later than the τ , which would otherwise have been employed. But this effect would be insignificant, out of all proportion to the possible longevity of Methuselah.

To drop the metaphor, our methods afford no guarantee against the possibility that a certain proportion of the wasps who go out in the morning do not return till the evening; as Virgil seems to have thought about the bees—

“ Mane ruunt portis . . . rursus easdem
Vesper ubi e pastu tandem decedere campis
Admonuit, tum tecta petunt . . . ”

It may be remarked that we have not utilised all the materials in Tables VII and IX; there remains over a tract of exits at the

And in fact we know that there are a certain number of *détenus* in human habitations.

The nature of the error differs according as we do, or do not, wish to include monster voyages in our average.

In the former case of course the results which we have obtained are too small. The error might be corrected, in the method of inspection, if the times of exit and entry were recorded for an entire working day!

But, secondly, let it be supposed that the determination of the mean duration of a voyage, exclusive of exceptionally long ones, is a legitimate operation; considering the latter class to belong to a separate type, to constitute heterogeneous material, as Professor Karl Pearson would say. In this case the results which we have obtained are too large. For, in the morning, corresponding to the first n entrances we have put the first n exits. But, if r of those exits pertain to individuals who have gone on long voyages, the n exits which we require are not the first n , but certain n out of say the first $n + r$. Accordingly our St is *pro tanto* too small; our $(Sr - St) \div n$ too large. The Sr which we have been employing may have been too large. Some of these entrances may have corresponded to exits made long before the period under observation. The individuals who issued in the last n exits must then have returned sooner than we have assumed (unless they spent the night out). Thus again our result is too large. The errors of the morning and evening do not compensate each other.

Such mutual compensation does indeed occur in one species of long voyage, that infinite one from which no traveller returns. Those who have finished their course are said to deposit their corpses outside the city walls—with a posthumous utilitarianism worthy of Bentham. Others are cut off untimely by enemies, in particular man. When we observe the mountains of the dead accumulated in bottles which the gardener or housewife has baited with beer or sugar, a doubt may arise as to the accuracy of a calculation which is based on the assumption that those who go out come in again. It will be found, however, that these fatalities derange the morning and the evening observations in opposite directions. The morning average is too large in the case of infinite voyages for the same reason (explained in the preceding paragraph) as in the case of merely long voyages. But the evening average is now too small. For the exits which really correspond to the last n entrances are not, as we have assumed, the last n exits, but a somewhat earlier set: certain n out of the last $(n - r)$ if, out of that number, r individuals have gone out to return no more. Accordingly the real $(Sr - St) \div n$ is larger than we have put it. Thus the two results are mutually compensatory. Their close agreement seems to show that the mortality among the workers is not very great.

There is another scruple which may also be expressed metaphorically. Our evening method applied to the periods preceding the flood to determine the mean lifetime would be slightly vitiated by the survival of Noah and his family. Their births would figure among the t 's but not their deaths among the τ 's; and accordingly eight earlier τ 's would be included among the nt 's than there ought to have been. The resulting mean lifetime would be *pro tanto* too small. It might be hoped that this error would be counteracted by the use of the morning method in the period after the flood. Not so, however. The deaths of the eight persons now figure among the τ 's, but not their births among the t 's; and accordingly eight later births are included among the nt 's than there ought to have been. Thus the resulting mean lifetime will be again too small.

Analogously it is conceivable that the twilight voyages, both in the evening and in the morning, are not really shorter, but are made to appear so by the circumstance that a certain number of those who go forth in the evening do not come home till morning. The fact that the observations recorded in Table VII were performed on the morning (27th September) following the evening (26th

end of Table VII—after minute $85\frac{1}{2}$ —and a tract of entrances at the beginning of Table IX—before minute 15.5. These observations need not be wasted; they are capable of yielding a sort of bye-product, a collateral proof, by way of the “method of exclusion” which was explained at the beginning of this section.

Suppose that at 7.1 A.M. on the morning to which Table VII refers the gate had been closed. There would have been excluded a number equal to the number of recorded exits not cancelled by entrances: that is the sum of the items in Col. 4 from $85\frac{1}{2}$ to the end; that is 90. Also the *flow* of wasps appears tolerably steady at that time; only a slight tendency to increase is shown by the preponderance of exits during the preceding half-hour or so. For instance, from 70 (exclusive) to 100 (inclusive), that is between 6.31 A.M. and 7.1 A.M., there occurred 167 exits and 154 entrances. Let us take the mean of these so as to get rid of the influence of unsteadiness as far as possible. Then we have for the flow per minute $\frac{1}{2} (156 + 161) \div 38$. Whence mean duration of a voyage = number excluded \div flow = $\frac{90 \times 60}{156 + 167} = 16.7$. Had we based the flow on 20 minutes observations, the result would have been 16.4.

Likewise from Table IX we find that if the gates had been closed at 4.42 P.M., there would have been excluded a number equal to the sum of the items in Col. 4, from 0 to 15.5, that is 133. The flow also at that period appears to be fairly steady, only a very slight tendency to decrease being shown by the preponderance of entrances over exits. Taking the average for the first 20 minutes, we have $\frac{1}{2} (160 + 164) \div 20 = 8.1$. Had we taken the average for 30 minutes the result would have been 7.9. Putting the mean of these two results, 8, for the flow, we have for the mean duration of a voyage $133 \div 8 = 16.6$. The consilience of these independent methods is mutually confirmatory. The statistics of wasps—like their nests—are coherent and compact, held together by a plurality of parallel supports.

It is a vital question whether the constants determined for one vespiary can be extended to another. The presumption raised by previous experience in favour of this induction is confirmed by the observations embodied in Table XI relating to nest H, which was situate at a distance of about 100 yards from nest G with which we have been dealing. The observations were made on the after-September) on which were performed the observations of Table IX, forms no guarantee against this kind of error. To settle this doubt I corked up nest G one night after closing time, and found next morning, about two hours after sunrise, only three individuals hanging about the closed gate. Presumably, unless the number of those who spend the night out is considerable, the results obtained for the daylight voyages would not be affected by the incident in question.

TABLE XII.—*Showing Results Deducible from Table XI.*

1	2	3	4	5	6	7	8	9	10	11
Periods Measured in Minutes (4 p.m. = 0).	Number of Exits in Periods.	Periods in which Number of Entrances = Number of Exits in Col. 2.	Sum of Times of Exit in Periods of Col. 1.	Sum of Times of Entrance in Periods of Col. 3.	Figure in Col. 5 Minus corresponding Figure in Col. 4.	Figure in Col. 6 ÷ corresponding Figure in Col. 2 = Mean Duration of Voyage for Periods of Col. 1.	Figure in Col. 6 Minus preceding Figure in same Column.	Figure in Col. 2 Minus preceding Figure in same Column.	Figure in Col. 8 ÷ corresponding Figure in Col. 9 = Mean Duration of Voyage for successive Periods.	Beginning of Period of Col. 3 Minus Beginning of corresponding Period of Col. 1.
81-107	19	93½-107	1,690	2,022	332	17·5	332	19	17·5	14·3
54-107	92	67½-107	6,622	7,936	1,314	14·3	982	73	13·5	12·6
27-107	168	42-107	9,591	11,992	2,401	14·3	1,087	76	14·3	14
0-107	259	15¼-107	10,809	14,568	3,759	14·5	1,358	91	14·9	14·25

noon of 7th October, 1895, from 4 P.M. up to closing time, which occurred at 5.47 P.M. The deductions from these observations are given in Table XII, according to a plan with which the reader is by this time familiar. It will be seen that the agreement between the evening experience of nest G and nest H is very close; as close as could be expected considering the small numbers and large modulus. The chief difference is that in nest H there is no indication of the voyages being shortened as night approaches. Indeed, they rather appear to be lengthened, the first figure in Col. 10 of Table XII being 17.5 larger than any one, and than the mean, of the remaining figures in the column. That appearance may well be accidental, considering the small number of observations which go to the figure in question, 19, with a modulus perhaps of 2 or 2.5 for the difference between this figure and the mean of the remaining ones. But the difference between 17.5 in Table XII and the corresponding figure in Table X may well be significant; especially as the hypothesis that in nest H the twilight voyages are of full length derives some confirmation from a set of observations—unfortunately rendered imperfect by interruption—which were made on nest H another evening, during the last 40 minutes before closing time.¹⁷

We may apply the method of exclusion by supposing the gate to have been closed at or soon after 4 P.M., *e.g.* 4.27. Then 51 (219 entrances — 168 exits, subsequent to 4.27 P.M.) would have been excluded. For the flow we may put $\frac{1}{2}$ (99 + 91) ÷ 27, (99 being the number of entrances, 91 the number of exits in the first 27 minutes) = 3.52. $51 \div 3.52 = 14.5$. If the gates had been closed at 4.15 P.M. the result of the calculation would be 15.1.

As the result of this long section, I think that we may with some confidence put about a quarter of an hour as the mean duration of a wasp's voyage.

V. We have next to determine that part of the cycle which is spent in the nest.¹⁸ That it occupies some minutes may be inferred from the observation that in small nests some minutes often elapse between an entrance and an exit. For example, in a nest observed by me on 1st August, 1895, the whole movement during half-an-hour, was as follows:—

¹⁷ It may be worth recording that G worked at least 17 minutes longer after twilight on the evening of 27th September than did H on the evenings 4th October and 7th October—both very gloomy evenings.

¹⁸ *Cp.* above, p. 361, par. 1.

10.19 A.M.	Exit	One Wasp
10.20 "	"	"
10.21 "	"	"
10.30 "	"	"
10.30 "	Enter	"
10.31 "	"	"
10.35 "	"	"
10.37 "	Exit	"
10.39 "	"	"
10.45 "	"	"
10.46 "	Enter	"
10.47 "	"	"
10.48 "	"	"

It is clear that the wasps who went at 10.30 A.M. must have been at least 7 minutes in the nest. The three wasps who issued at 37, 39, and 45 minutes past 10, were either identical with the wasps who entered at 30, 31, and 35, in which case the mean time spent in the nest was 8.3 minutes; or, they were not identical, in which case the mean time was greater than 8.3 minutes. The wasps who issued at 10.45 must have been at least 10 minutes in the nest.

This inference is confirmed by observations relating to some of the principal operations which are carried on in a vespiary. Importers of sweets, it may be inferred, from the observations adduced in our first section, do not on an average spend more than 8 minutes in the nest; and must spend a good part of that time at home, unless the way is long, or they loiter on the way.¹⁹ Importers of wood have been observed by me, in a nest from which I removed part of the surrounding earth, to spend some minutes in laying on that material over a part of the communal roof; how many minutes I have not noted. Also exporters of earth have often been observed by me, in the same nest, to spend more than 5 minutes in getting ready a cargo. But I do not know whether these exporters are absent for an average time from the nest.²⁰

Altogether it would be safe I think to assign 4 or 5 minutes for the subterraneous part of the mean cycle.

VI. One advantage of knowing the duration of a cycle is that we have thus a rough and ready method of determining the number—or rather an inferior limit to the number—of the working

¹⁹ I am not sure that they fly direct to the nest. I have repeatedly placed sweets near a wasp's nest and observed that only a small proportion when charged go straight to the nest. The rest fly right off, perhaps to clean themselves after alighting on a tree—as I have observed in one instance—before going into the nest.

²⁰ They seem to carry the earth unnecessarily far.

population. For by the lemma above stated,²¹ the number of wasps required to maintain the traffic must be equal to the flow of wasps per minute multiplied by the number of minutes in a cycle. To vary the enunciation, if we regard each individual as a sort of cog in the great wheel of trade, which revolves uniformly at such a rate that n cogs pass a fixed point per minute, and a complete revolution is performed in 20 (or 25) minutes, then the number of cogs say $N = 20 \times n$ (or $25 \times n$). But the number of wasps engaged in driving the wheel of trade N is not the same as M , the number of adult workers. For N is less than M by the number of adult workers who are not engaged in external trade.²² Also M is less than the total population, by the number, often considerable, of queens and idle males.

We shall fall into error if, in order to compare the population of different nests, or the same nest at different times, we assume that the proportion between the number of those engaged in external trade to the total population is constant. It is thus that some exponents of the quantity theory of money talk glibly of the ratio between credit payments and cash payments, oblivious of the possibility that the ratio is not constant, but may vary with the greater or less difficulty of procuring gold.

The variations in the proportion engaged in external trade may be estimated by comparing the mean traffic (exits + entrances) per minute, for the same nest at times not widely different. But before making this comparison, we must be sure that the difference between the means of average is not accidental, but indicative of a change in the set of causes influencing the traffic at the respective periods. This assurance is afforded by the theory of errors which I have explained in a former number of this *Journal*. The difference between two averages founded respectively on m and n observations is not accidental if it exceed what may be called "the limit of accidental difference," namely:—

$$2 \times \sqrt{\frac{1}{m} + \frac{1}{n}} \times \text{Modulus};$$

the "modulus" being a coefficient, which for wasps' nests of an ordinary size I have found to be 8.5.²³ Table XIII shows some comparisons verified by this theory.

²¹ p. 361.

²² Including those who by reason of their tender age are not fit for out-of-door work; not a great number if it be true, as Réaumur says, that a wasp goes out to work on the very day on which she attains maturity.

²³ Jubilee Vol., *loc. cit.* Cp. above p. 363, showing that for a small traffic the modulus is relatively greater.

TABLE XIII.

1 Designation and Date.	2 Number of Observations.	3 Mean Traffic.	4 Difference between Figure in Col. 3 and preceding Figure in same Column.	5 Limit of Accidental Difference in round Numbers.
A. 4th September, 1884, } 6.30 A.M.	16	25	—	—
Ditto ditto, } 8.15 A.M.	5	42	17	9
A. 6th September, 1884, } 6.45 A.M.	12	25	17	9
Ditto ditto, } 10.30 A.M.	6	44	19	8.5
A. 7th September, 1884, } 10.30 A.M.	22	29	15	8
Ditto ditto, } 4.30 P.M.	—	15	14	—
A. 8th September, 1884, } 3.30 P.M.	8	44	29	—
Ditto ditto, } 5.15 P.M.	24	31	13	7
D. 23rd August, 1895, } 7 A.M.	6	21	—	—
D. 26th August, 1895, } 7 A.M.	6	60	39	10
D. 29th August, 1895, } 8 A.M.	9	23	37	8.5
Ditto ditto, } 3 P.M.	12	8.5	16.5	8
C. 26th August, 1895, } 7 A.M.	6	37.5	—	—
C. 29th August, 1895, } 2.45 A.M.	9	15.5	15.5	8.5

Of the causes of difference in the traffic, one of the most obvious is the coming on of evening. For example, consider the record of the traffic of nest A after 6.19 P.M., 6th September (1884), presented in Table XIV.

TABLE XIV.—*Showing the Decline of Traffic with the Coming on of Evening.*

Period.	Total Traffic (Entrances + Exits) Per Minute.
6.18—6.30 P.M.	29
6.30—6.42 P.M.	19
6.42—6.54 P.M.	11'5

The influence of Hesperus has been more fully exhibited in Tables IX and XI.

VII. Owing to the variations in the traffic which have just been proved to exist, the formula for the working population, which was proposed as rough and ready, is found to be rough indeed, since it affords only an inferior limit to the population, but not ready, since in order to secure an approximate limit, we must endeavour to observe the largest possible flow. The numbers given by the formula and by an actual census²⁴ in certain instances are shown in Table XV.

²⁴ I saw no way of taking the number without taking the lives of the inhabitants. I effected my purpose partly by intercepting individuals, usually with a second object in view (see p. 362 *et seq.*), either as they returned to or as they issued from the nest; partly by pouring tar oil into the nest overnight. The terms "counted outside" and "counted inside" in Table XV designate these two methods of taking the census. The second operation was not always completely successful; hence the necessity of resorting to estimates in two of the instances in Table XV.

I should add that queens have not been included among the observed numbers in Col. 3 of Table XV; but that males have not been excluded, partly because it is difficult to distinguish them from workers, partly because I believe they may form part of the flow on which the calculated numbers in Col. 3 are based.

TABLE XV.—*Showing the Correspondence between the Magnitude of the Population and that of the Traffic.*

Designation of Nest.	Largest Observed Flow.	Inferior Limit Calculated.	Actual Number of Population.	Remarks
B	2.5	50	233	65 counted outside; 168 counted inside.
B	11.0	220	588	420 counted outside; 168 counted inside.
G	14.0	280	(?) 500	221 counted outside; 156 counted inside; 100 estimated to have deserted (exclusive of queens) as the nest was being dug up; 50 estimated to have otherwise escaped count (?).
E	16.0	320	640	120 counted outside; 519 counted inside.
C	25.0	500	1,350	850 counted outside; an immense remaining swarm estimated as 500.
D	30.0	600	1,610	104 counted outside; 1,506 counted inside.

It will be seen that the inferior limit given by the formula is very inferior. There is a rough correspondence between the magnitude of the flow and that of the population.

The proposition that the circulation is proportioned to the numbers in a wasps' nest, is about as true or as false as the proposition that the level of prices varies as the quantity of metallic money.

I trust that the analogy of these studies with human statistics may justify their insertion in this *Journal*. The trade and industry which we have investigated may be petty interests, but by their regularity and orderliness they are particularly well calculated to furnish specimens of statistical methods.

“Admiranda levium spectacula rerum”—

What the poet says of bees, the statistician finds true of wasps.