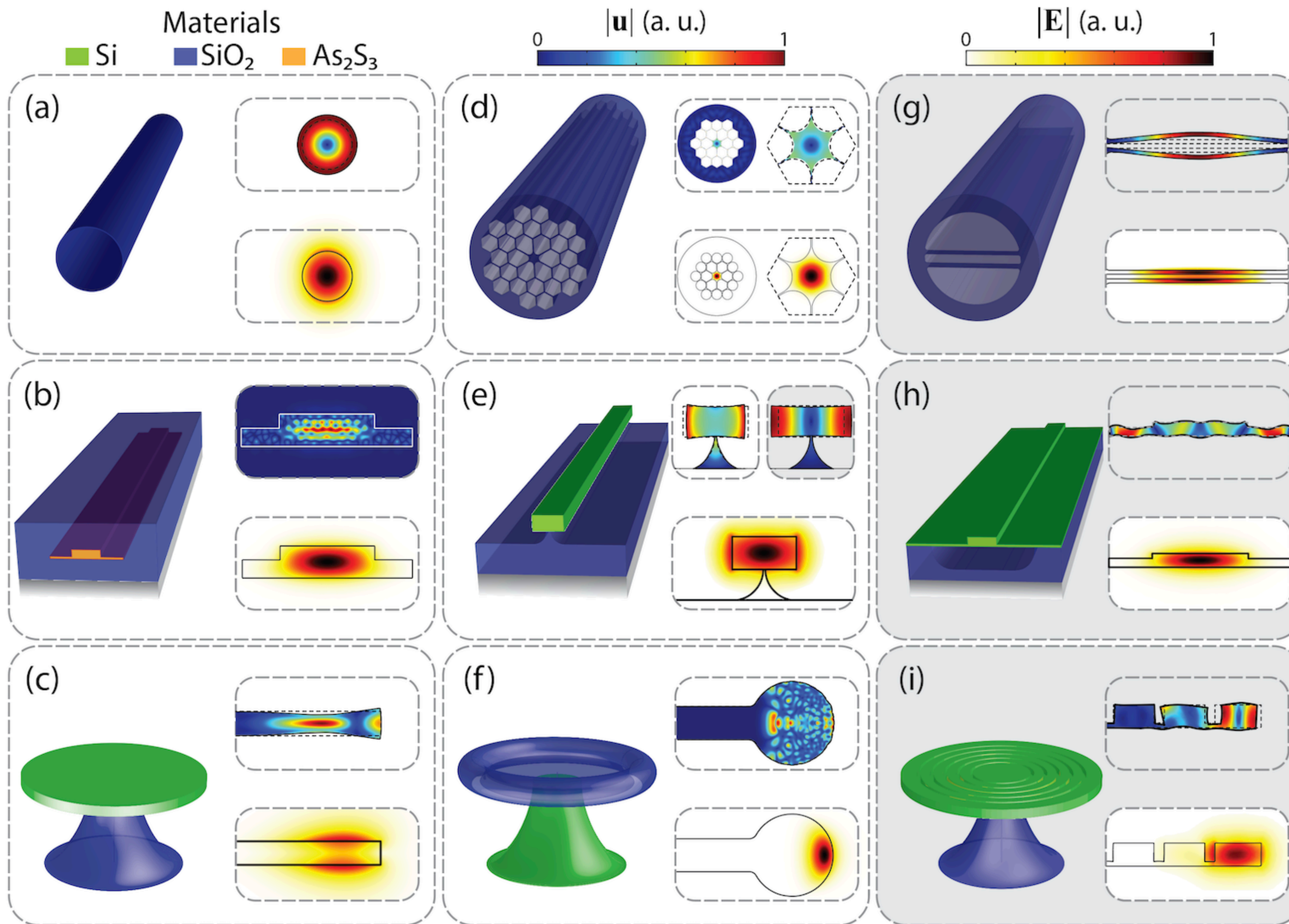


Brillouin Optomechanics in Nanophotonic Structures



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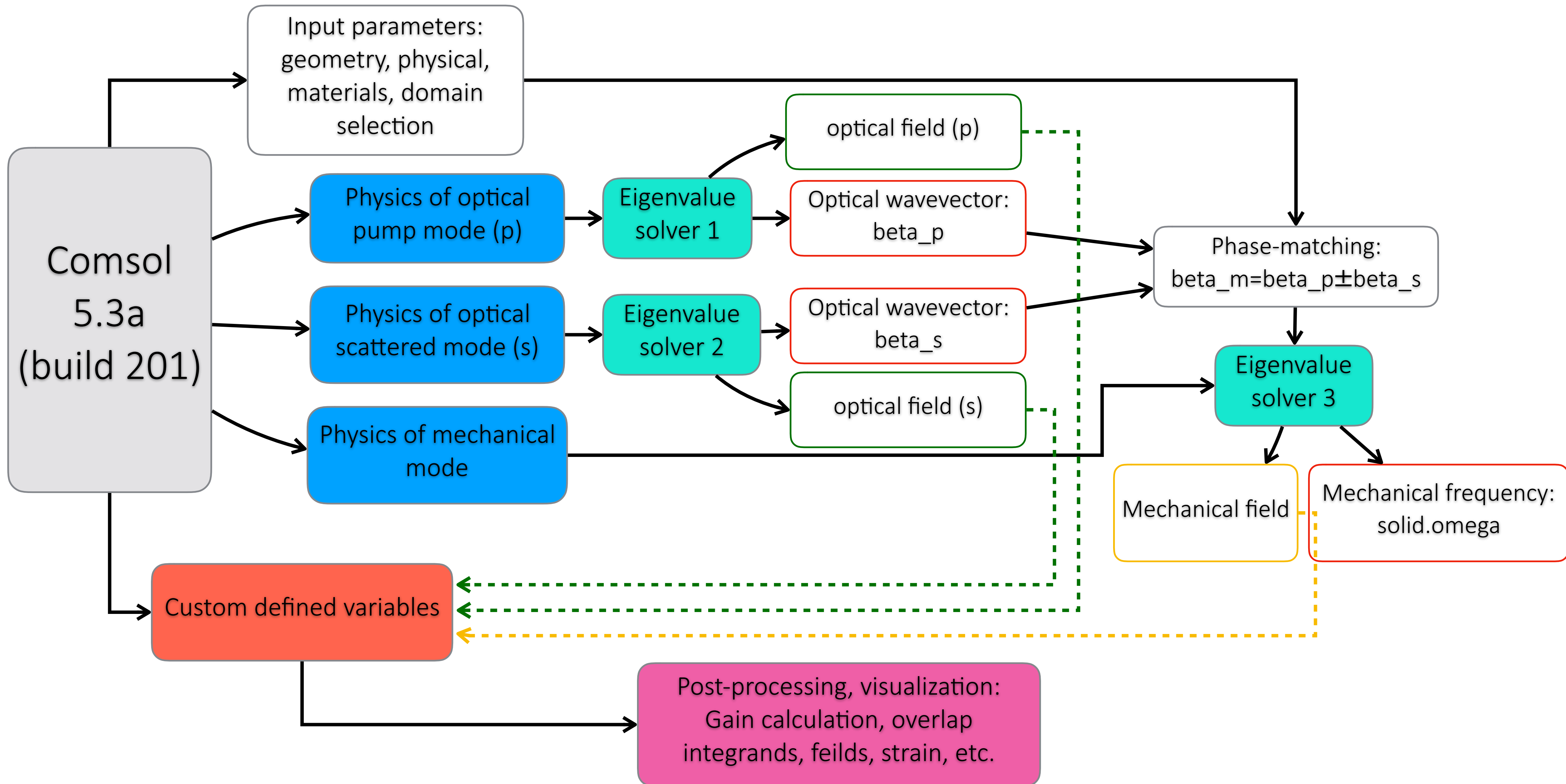
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“Optoacoustics—Advances in
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Simulation structure



COMSOL dependencies




















All the examples provided in this tutorial relies on two modules that are separately sold by COMSOL:

- Electromagnetic Waves
- Structural Mechanics

It is assumed that the user has some experience in using COMSOL Multiphysics. Tutorials on this software are widely available at the vendor website.

Comsol version 5.3a is required. The accompanying simulation files were not tested on other versions.

COMSOL file structure (waveguide)

- ▼  comsol_simulation_file_fig1a_taper_example_tutorial_quarter_domain.mph (root)
 - ▼  Global Definitions
 -  Parameters
 -  Materials
 - ▼  Component 1 (comp1)
 - ▶  Definitions
 - ▶  Geometry 1
 - ▶  Materials
 - ▶  Electromagnetic Waves, Frequency Domain (emw_p)
 - ▶  Electromagnetic Waves, Frequency Domain 2 (emw_s)
 - ▶  Solid Mechanics (solid)
 - ▶  Mesh 1
 - ▼  Study 1
 -  Step 1: Mode Analysis (p)
 -  Step 2: Mode Analysis (s)
 -  Step 3: Eigenfrequency
 - ▶  Solver Configurations
 -  Job Configurations
 - ▶  Results

Parameters

Parameters			
Name	Expression	Value	Description
domain_radius	15e-6 [m]	1.5E-5 m	domain height
radius	300e-9[m]	3E-7 m	waveguide radius
lambda0	1550e-9 [m]	1.55E-6 m	Vacuum wavelength
mass_factor	4	4	Mass factor: 2 for half domain symmetry
Lwg	2*pi*10e-6 [m]	6.2832E-5 m	corresponding cavity length (for g0 calculation)
sol_opt_p	'sol2'		solver label for pump solution (emw_p)
sol_opt_s	'sol3'		solver label for stokes solution (emw_s)
sol_mech	'sol1'		solver label for mechanical solution (emw_p)
signBS	-1	-1	sign of Stokes z-component of E-field (BSBS: -1, FSBS: 1)
Qmech	1	1	Mechanical quality factor - only matters for gain calculation
i_opt_p	1	1	eigenvalue index for pump solution mode
i_opt_s	1	1	eigenvalue index for stokes solution mode

- The variables **sol_opt_p**, **sol_opt_s**, **sol_mech**, are strings and must match the outputs from each solver. The ordering on their labels is the standard one COMSOL creates when they are sought in the order (**sol_opt_p**, **sol_opt_s**, **sol_mech**).
- Tip: One should not need to change these.

- **i_opt_p** and **i_opt_s** are the indexes of the (p) and (s) optical solvers when more the one solution solution is sought for each.
- Tip: The optical modes are ordered in increasing effective refractive index.

- **signBS**: This will set whether the mechanical wave vector will be set as $\beta_p - \beta_s$ (signBS=1, forward) or $\beta_p + \beta_s$ (signBS=-1, backward)

- **mass_factor**: When domain symmetry is employed, the effective mass **must be corrected** by an integer factor; for instance, if only half of the domain is used, mass_factor=2, if only quarter is used mass_factor=4, and so forth.

Custom variable definitions

Component 1 (*comp1*)

Definitions

a= Basic - optical
 a= Basic - mechanical
 a= MB - optical
 a= MB - mechanical
 a= MB - mixed
 a= PE - mechanical
 a= PE - mixed
 a= Total - mixed
 a= Basic - global

\int_{du} surface integral - mechanical domain (*intop_mech*)

\int_{du} surface integral - optical domain (*intop_opt*)

\int_{du} surface integral - PE domain (*intop_pe*)

\int_{du} line integral - MB boundaries (*intop_mb*)

MAX Maximum - mechanical domain (*maxop_mech*)

MAX Maximum - optical domain (*maxop_opt*)

Mechanical domain

Optical domain

MB boundaries

PE domains

Groups of variables are defined to enable a simpler calculation of the Brillouin gain:

- Basic
- MB
- PE
- Total

Variables are defined over “Entire Model” under “Geometric entity level”

Several “Component Coupling” operators are defined to enable easy integration (*intop_XX*) or normalization (*maxop_YY*) of the optical and mechanical fields. Each of these operators must be defined in their proper spatial domains using selection groups.

Four selection groups are defined to simplify the evaluation of each of the “Component coupling” operators above. These group domain are:

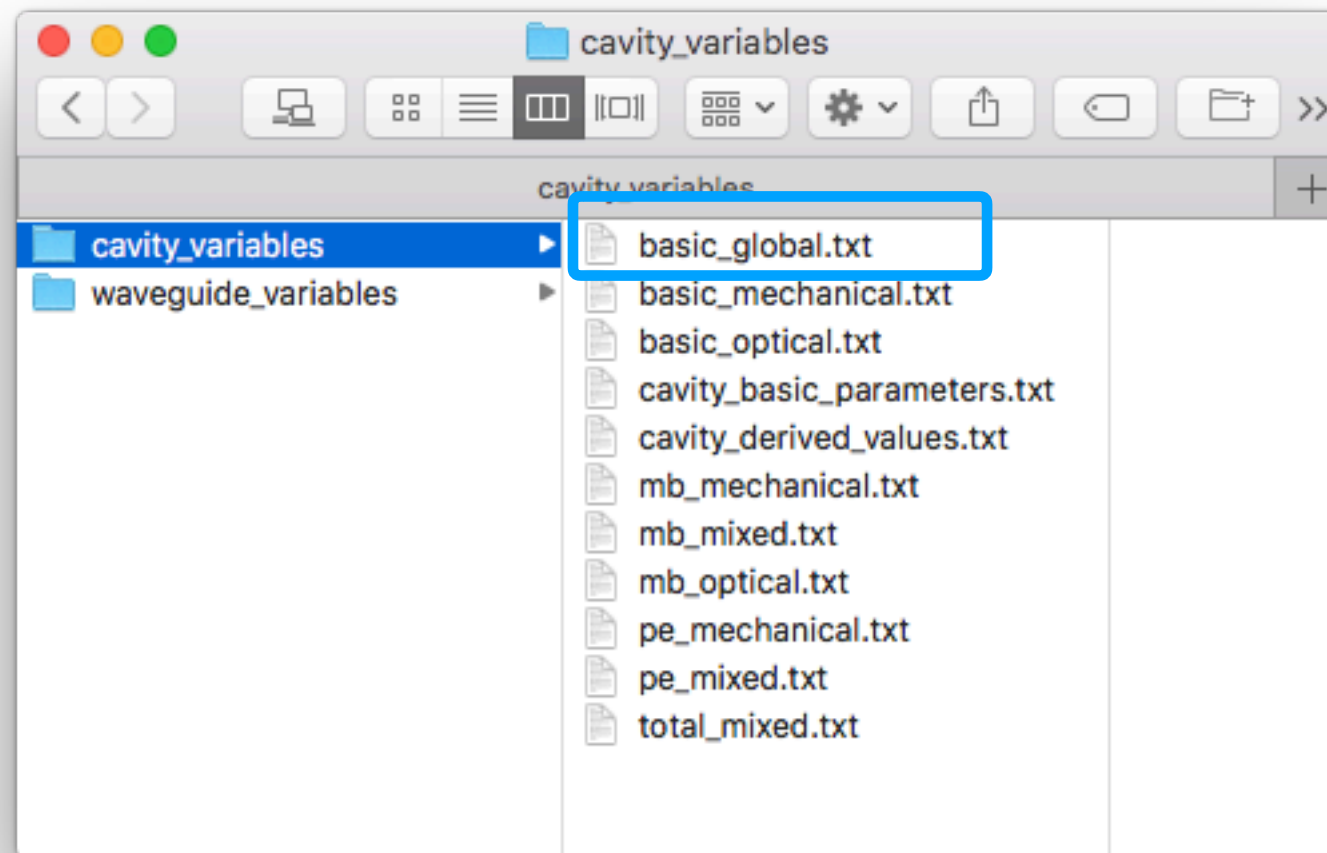
- 2D domain that belongs to the mechanical parts (Mechanical domain)
- 2D domains that belongs to the optical parts (may overlap with Mechanical Domain)
- 1D domains defining the boundaries between materials: **this is very important** for proper evaluation of the Moving boundary (radiation pressure) force. Symmetry boundary should not be selected for MB calculations
- 2D domain automatically defined as the intersect between Mechanical and Optical domains for Photo-Elastic effect.

One should avoid changing the selections directly in the “Components coupling” settings and always use these “Selection” for defining them.

Custom variable definitions: Loading Cavity and Waveguide Variables

A list of variables is provided in the cavity_variables.zip and waveguide_variables.zip files.

Cavity Variables

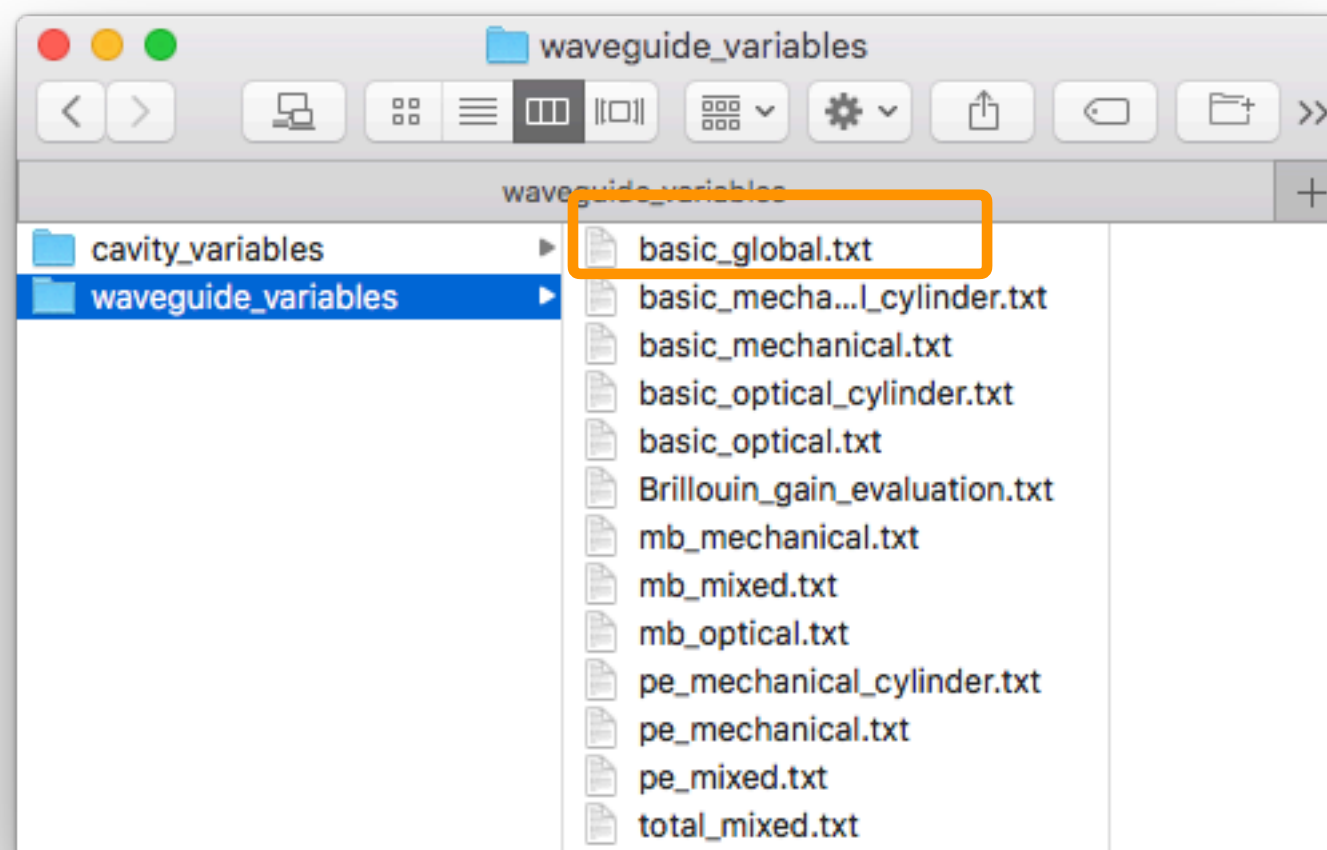


Each folder have multiple .txt files that allows for creating the custom variables at COMSOL

```

omega0 withsol(sol_opt_p,2*pi*emw_p.freq,setind(lambda,i_opt_p)) "Pump frequency"
m_p withsol(sol_opt_p,abs(emw_p.beta),setind(lambda,i_opt_p)) "Pump mode azimuthal number"
m_s withsol(sol_opt_s,abs(emw_s.beta),setind(lambda,i_opt_s)) "Stokes mode azimuthal number"
m_ac (m_p-signBS*m_s) "Phase-matched Mechanical mode azimuthal number"
  
```

Waveguide Variables



```

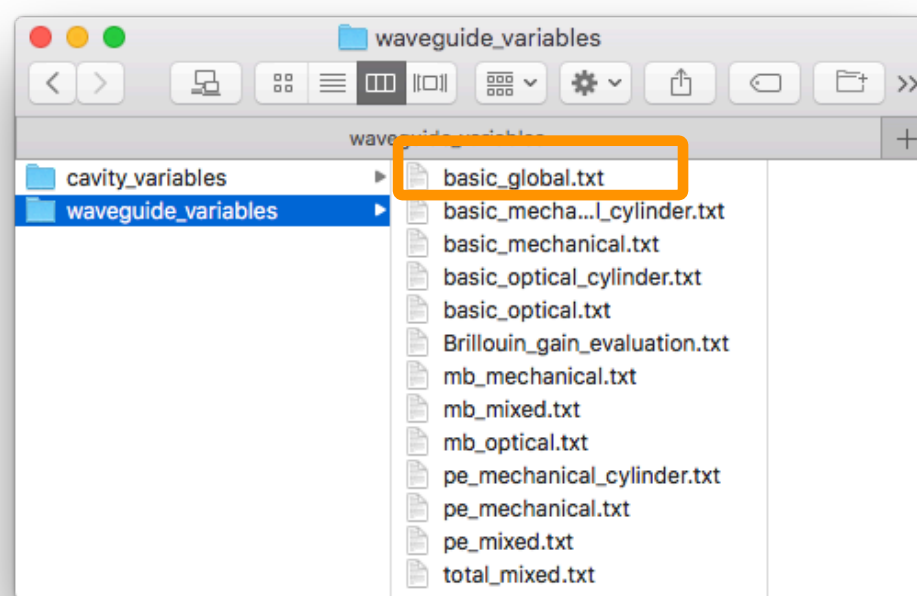
omega0 withsol(sol_opt_p,2*pi*emw_p.freq,setind(lambda,i_opt_p)) "Pump frequency"
beta_p withsol(sol_opt_p,abs(emw_p.beta),setind(lambda,i_opt_p)) "Pump mode wavenumber"
beta_s withsol(sol_opt_s,abs(emw_s.beta),setind(lambda,i_opt_s)) "Stokes mode wavenumber"
beta_ac (beta_p-signBS*beta_s) "Phase-matched Mechanical mode wavenumber"
  
```

Custom variable definitions: Loading Cavity and Waveguide Variables

A list of variables is provided in the cavity_variables.zip and waveguide_variables.zip files.

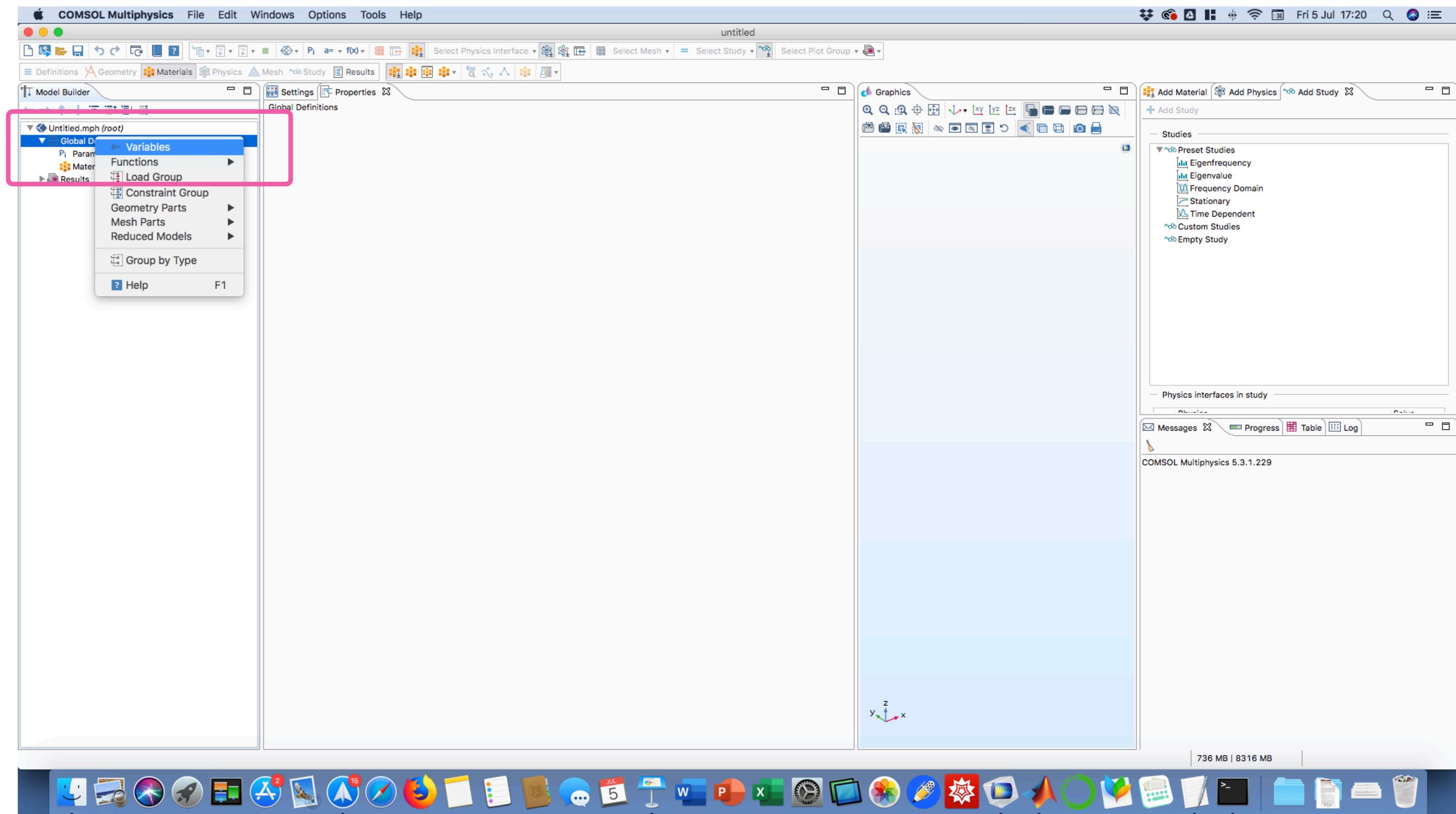
- 1) Open COMSOL
- 2) click on "Global Definitions" —> "Variables"
- 3) Click on Load from file
- 4) All variables, equations and description will be loaded on COMSOL.

Waveguide Variables



```

omega0 withsol(sol_opt_p,2*pi*emw_p.freq,setind(lambda,i_opt_p)) "Pump frequency"
beta_p withsol(sol_opt_p,abs(emw_p.beta),setind(lambda,i_opt_p)) "Pump mode wavenumber"
beta_s withsol(sol_opt_s,abs(emw_s.beta),setind(lambda,i_opt_s)) "Stokes mode wavenumber"
beta_ac (beta_p-signBS*beta_s) "Phase-matched Mechanical mode wavenumber"
    
```

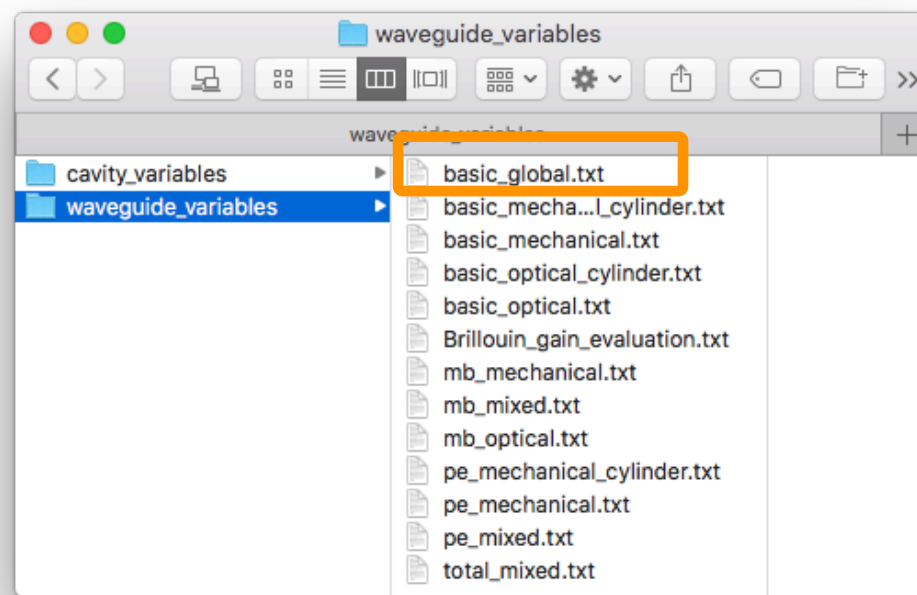


Custom variable definitions: Loading Cavity and Waveguide Variables

A list of variables is provided in the cavity_variables.zip and waveguide_variables.zip files.

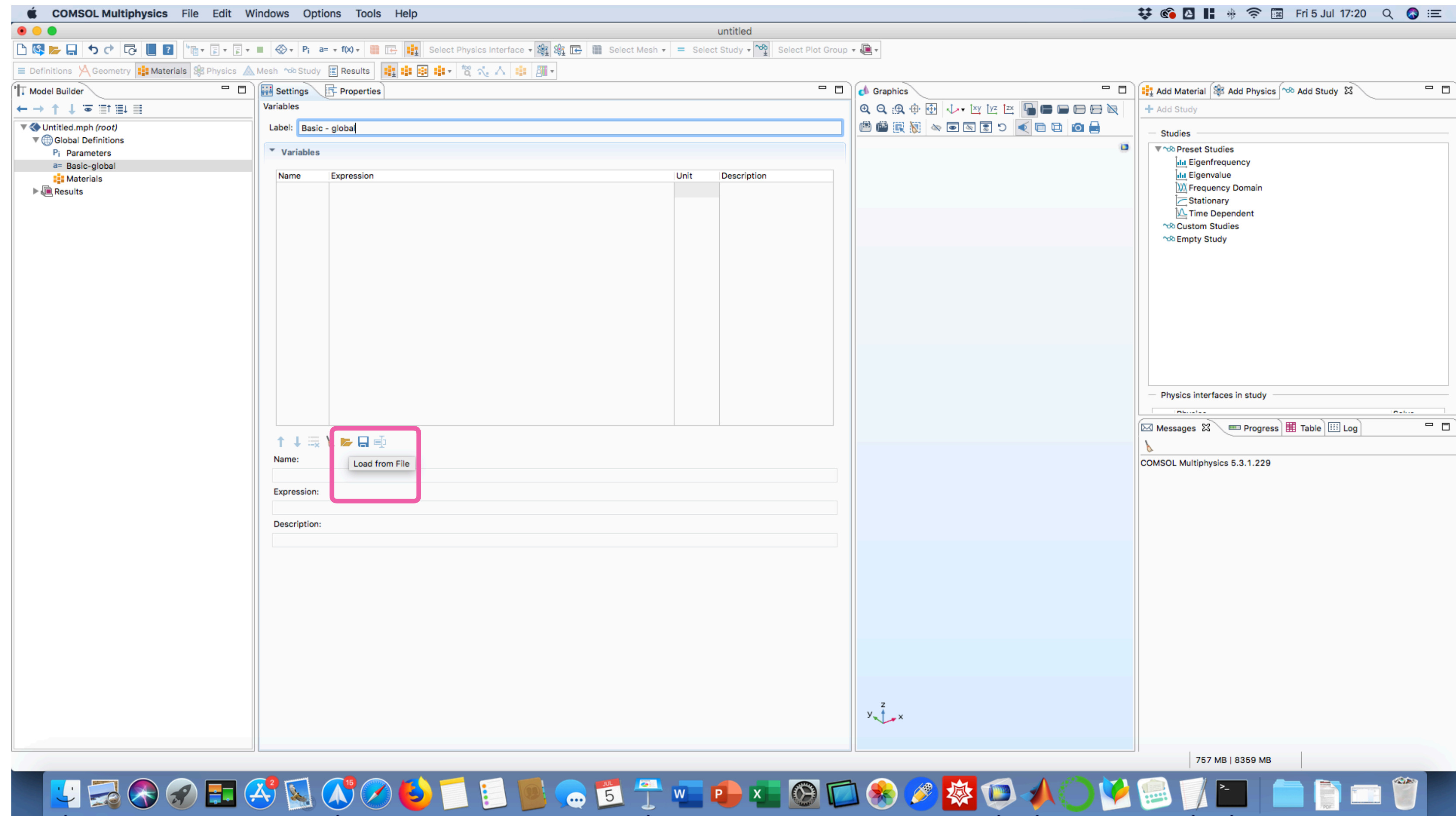
- 1) Open COMSOL
- 2) click on "Global Definitions" —> "Variables"
- 3) Click on Load from file
- 4) All variables, equations and description will be loaded on COMSOL.

Waveguide Variables



```

omega0 withsol(sol_opt_p,2*pi*emw_p.freq,setind(lambda,i_opt_p)) "Pump frequency"
beta_p withsol(sol_opt_p,abs(emw_p.beta),setind(lambda,i_opt_p)) "Pump mode wavenumber"
beta_s withsol(sol_opt_s,abs(emw_s.beta),setind(lambda,i_opt_s)) "Stokes mode wavenumber"
beta_ac (beta_p-signBS*beta_s) "Phase-matched Mechanical mode wavenumber"
    
```

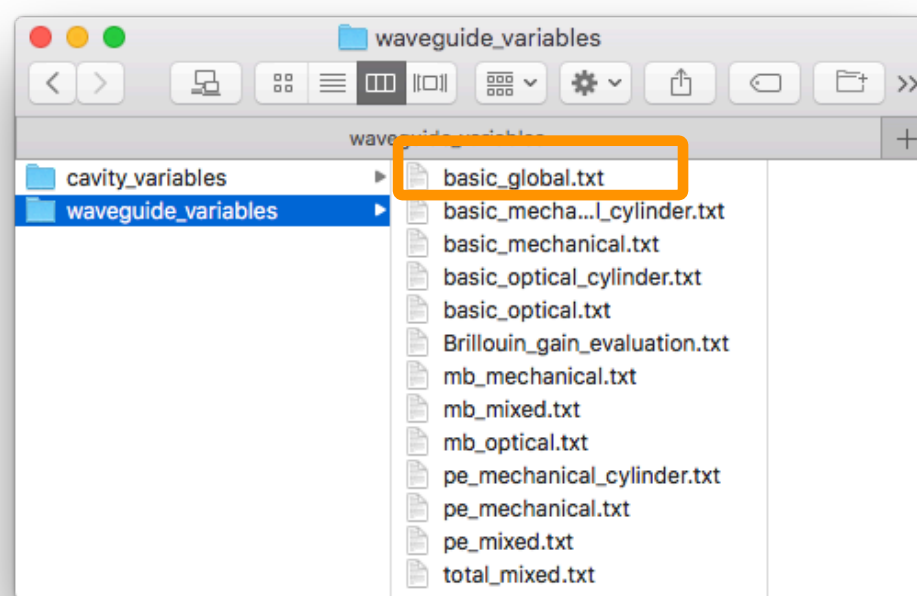


Custom variable definitions: Loading Cavity and Waveguide Variables

A list of variables is provided in the cavity_variables.zip and waveguide_variables.zip files.

- 1) Open COMSOL
- 2) click on "Global Definitions" —> "Variables"
- 3) Click on Load from file
- 4) All variables, equations and description will be loaded on COMSOL and ready to use.

Waveguide Variables

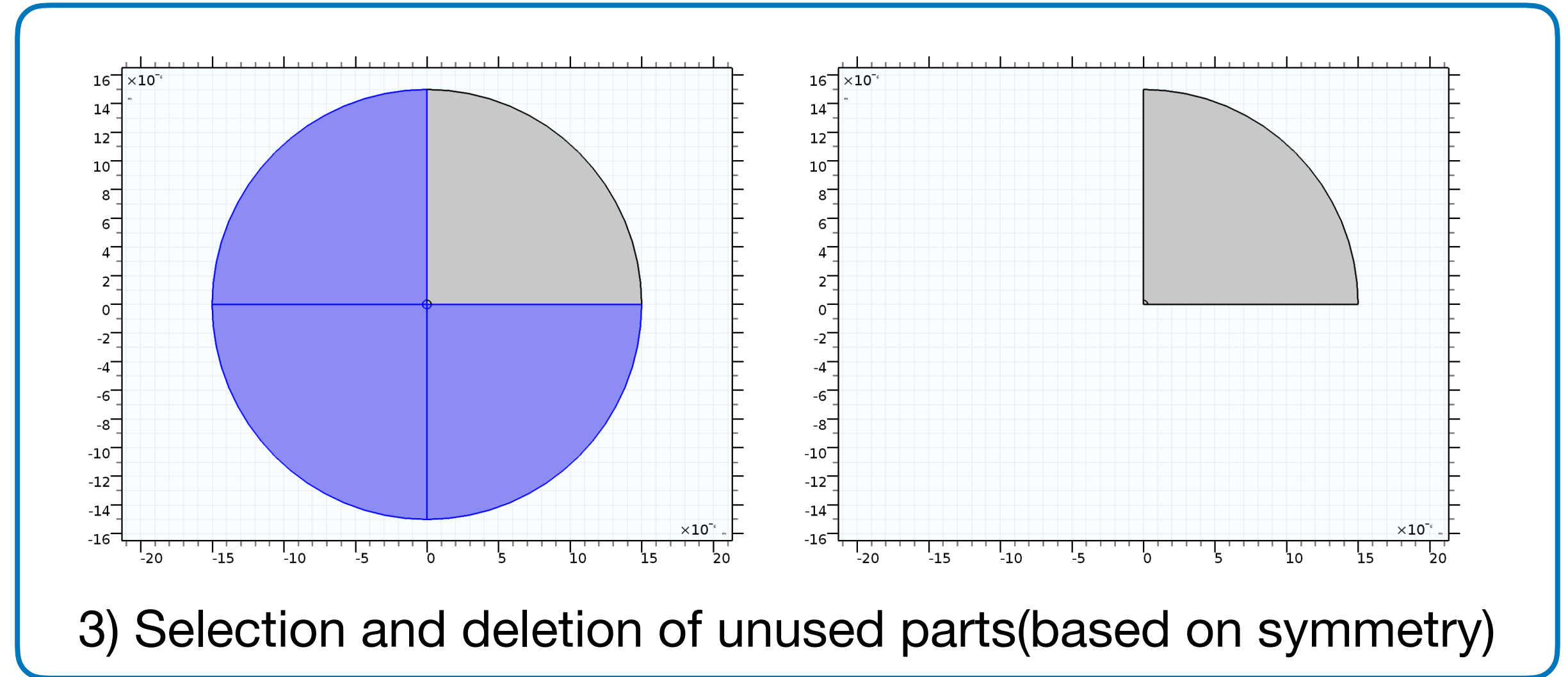
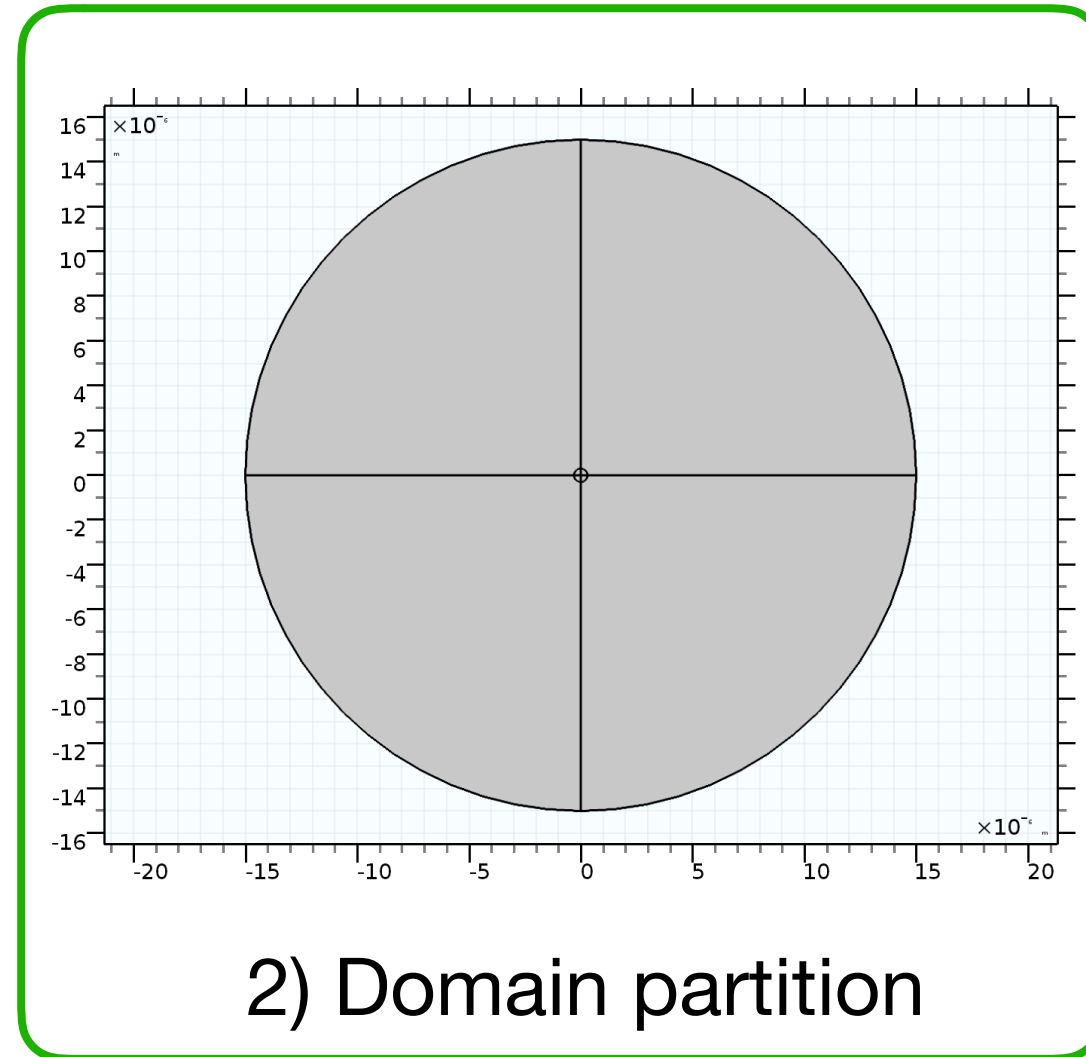
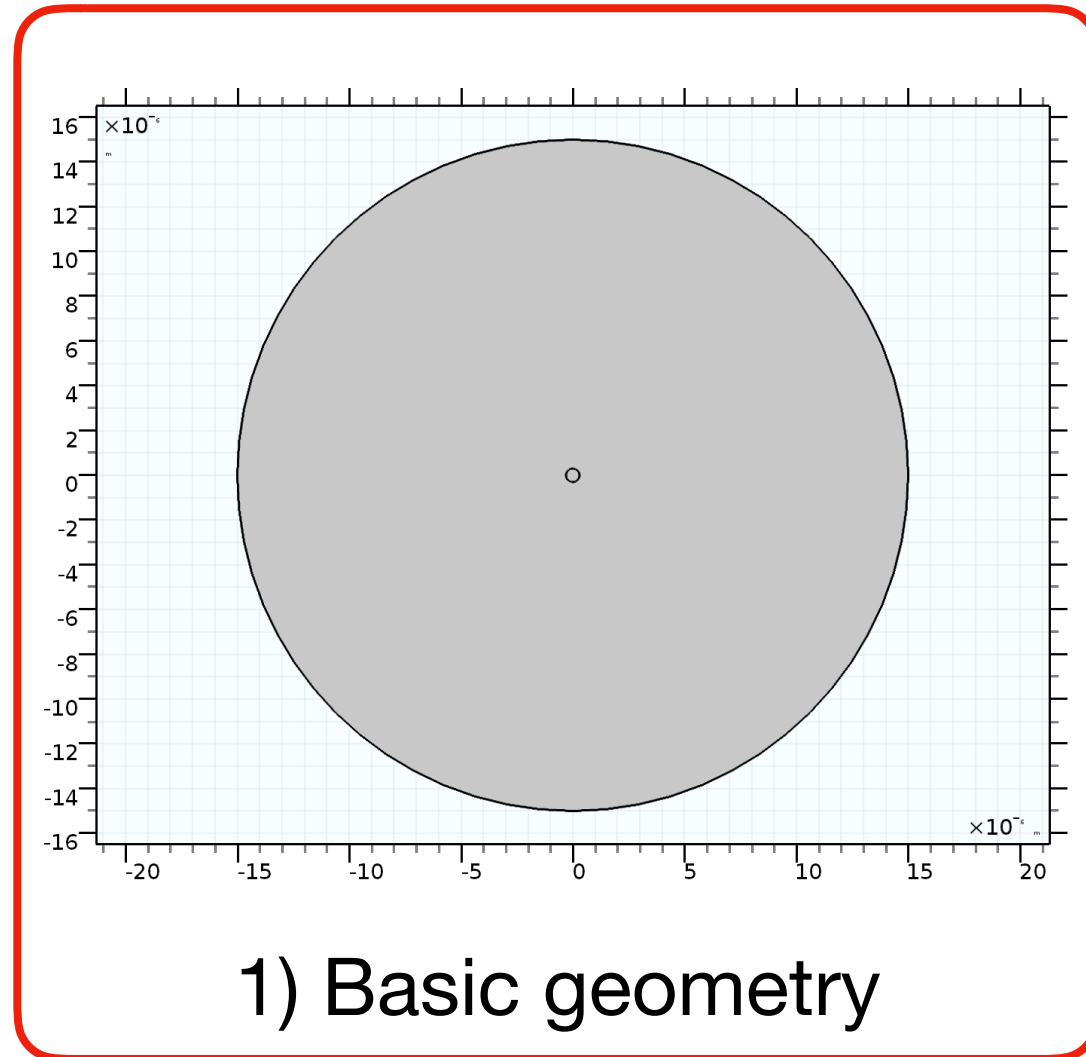


```

omega0 withsol(sol_opt_p,2*pi*emw_p.freq,setind(lambda,i_opt_p)) "Pump frequency"
beta_p withsol(sol_opt_p,abs(emw_p.beta),setind(lambda,i_opt_p)) "Pump mode wavenumber"
beta_s withsol(sol_opt_s,abs(emw_s.beta),setind(lambda,i_opt_s)) "Stokes mode wavenumber"
beta_ac (beta_p-signBS*beta_s) "Phase-matched Mechanical mode wavenumber"
  
```

Name	Expression	Unit	Description
omega0	$\text{withsol}(\text{sol_opt_p}, 2 \cdot \pi \cdot \text{emw_p.freq}, \text{setind}(\lambda, i_{\text{opt_p}}))$		Pump frequency
beta_p	$\text{withsol}(\text{sol_opt_p}, \text{abs}(\text{emw_p.beta}), \text{setind}(\lambda, i_{\text{opt_p}}))$		Pump mode wavenumber
beta_s	$\text{withsol}(\text{sol_opt_s}, \text{abs}(\text{emw_s.beta}), \text{setind}(\lambda, i_{\text{opt_s}}))$		Stokes mode wavenumber
beta_ac	$(\text{beta_p} - \text{signBS} \cdot \text{beta_s})$		Phase-matched Mechanical mode wavenumber

Geometry








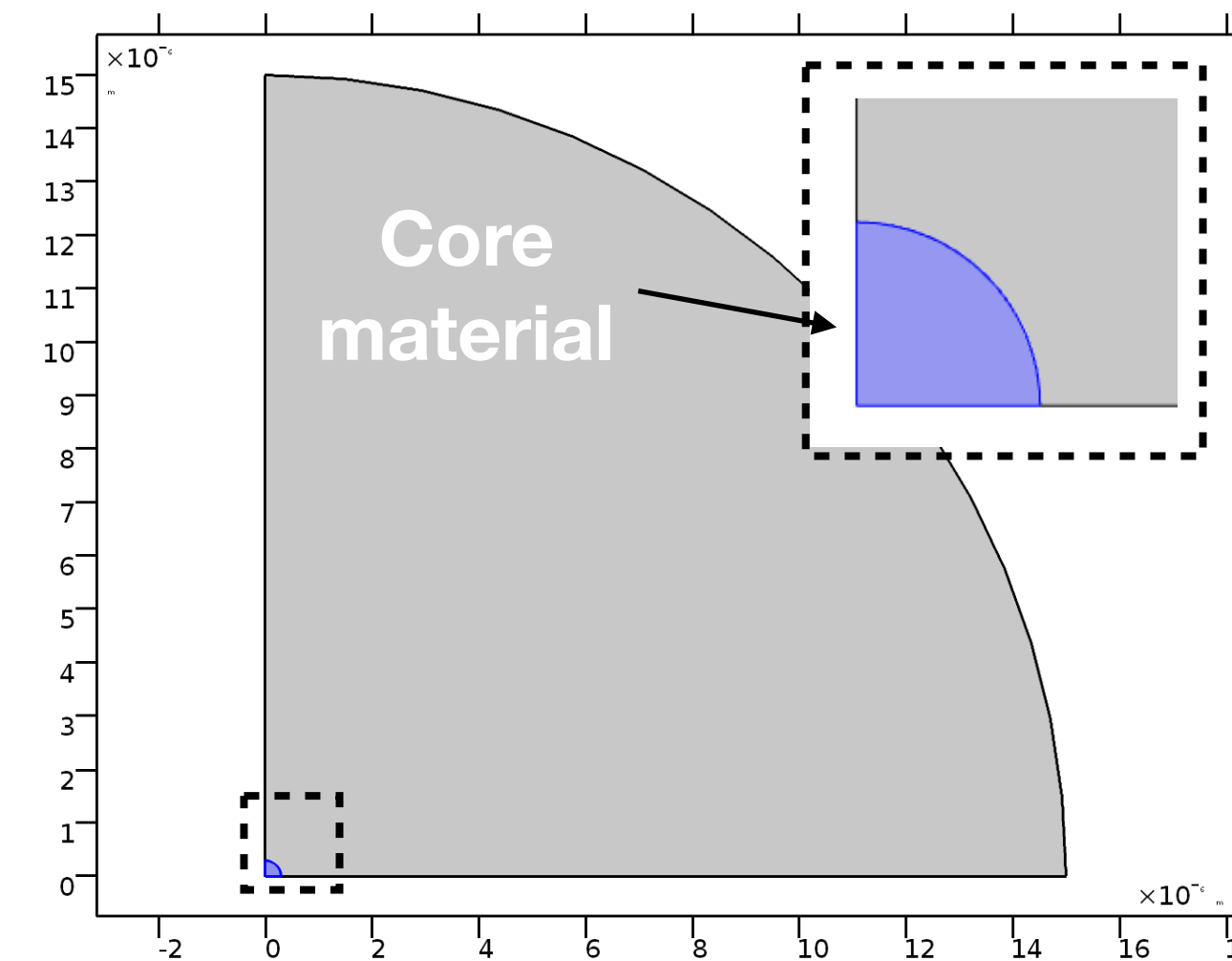
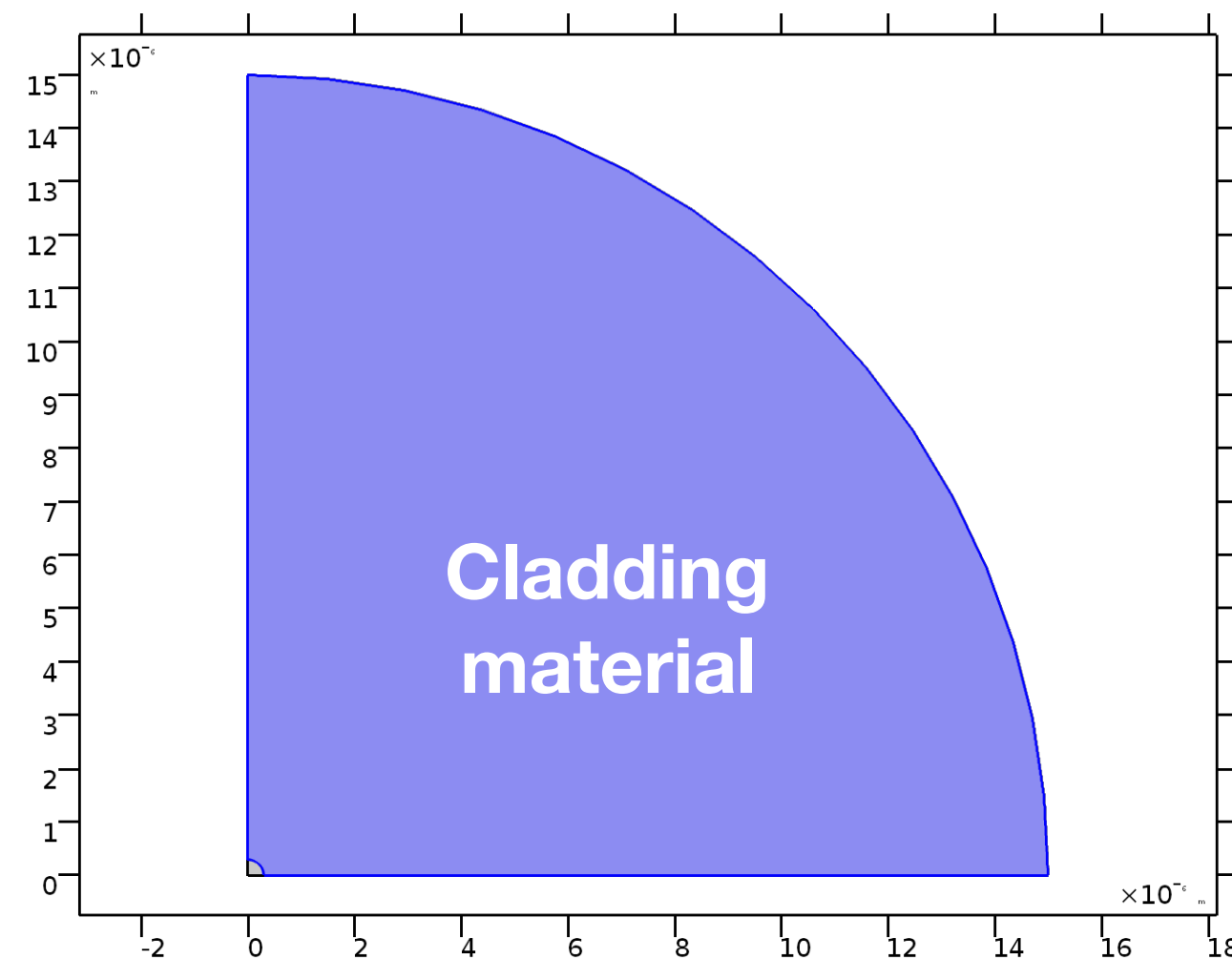
▼ Geometry 1

- Circle 1 (*c1*)
- Circle 2 (*c2*)
- ▢ Partition Domains 1 (*pard1*)
- ▢ Partition Domains 2 (*pard2*)
- 🗑 Delete Entities 1 (*del1*)
- 🔗 Form Union (*fin*)

- If symmetries are not necessary, steps 2 and 3 above can be avoided.
- Although it is not necessary to explore symmetries, it may lead not only to substantial computational time reduction but also easier selection of pump and scattered modes for intra-modal scattering.
- For instance, say one wants to investigate forward Brillouin scattering between the two polarization states (HE_{11x}, HE_{11y}) fundamental modes of a tapered fiber. Breaking up the domain as illustrated above will allow to impose boundary condition on the symmetry walls (PEC or PMC) to provide either the vertically or horizontally polarized optical modes for each of the optical solvers.
- This is specially important during parameter sweeps that would lead to random sorting of the eigenvalues order of the two modes.

Materials

- ▼  Materials
 - ▼  Air - 2019_Wiederhecker_APL_Photonics_Tutorial (mat2)
 -  Basic (def)
 - ▼  SiO2 - 2019_Wiederhecker_APL_Photonics_Tutorial (mat3)
 -  Basic (def)

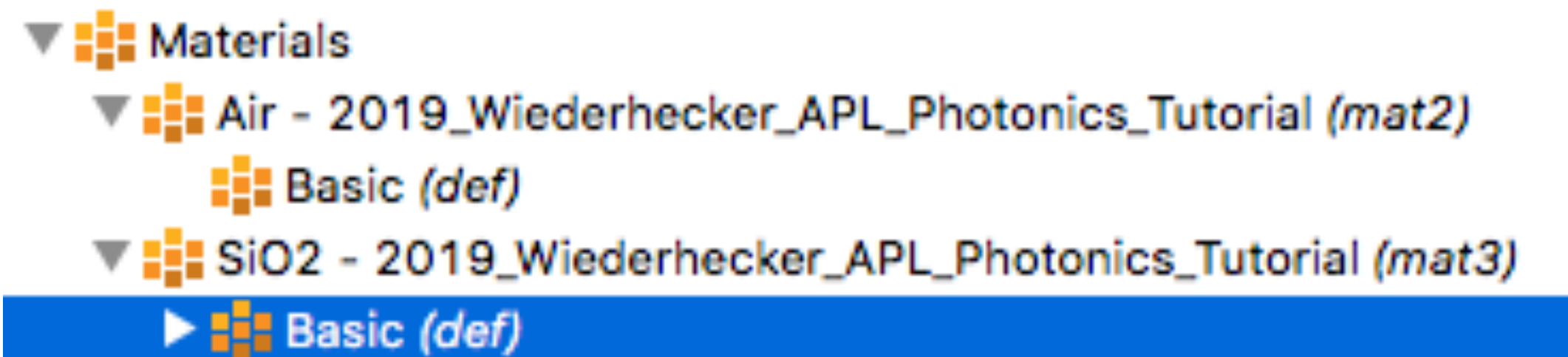


- The material properties that must be defined for the COMSOL solvers to work properly are automatically highlighted by the verified sign, as shown in the screenshot aside.
- The photo elastic constants must also be defined in order for enable Brillouin gain calculation. As shown in the next slide, these are defined as “Local Properties”.
- In this particular example, the permittivity is calculated according to Sellmeyer’s formula, as defined by the function (nref_sel) evaluated at the vacuum wavelength (nref_sel(lambda0)). This could be avoided by simply feeding a constant value for the relative permittivity.

Material Contents

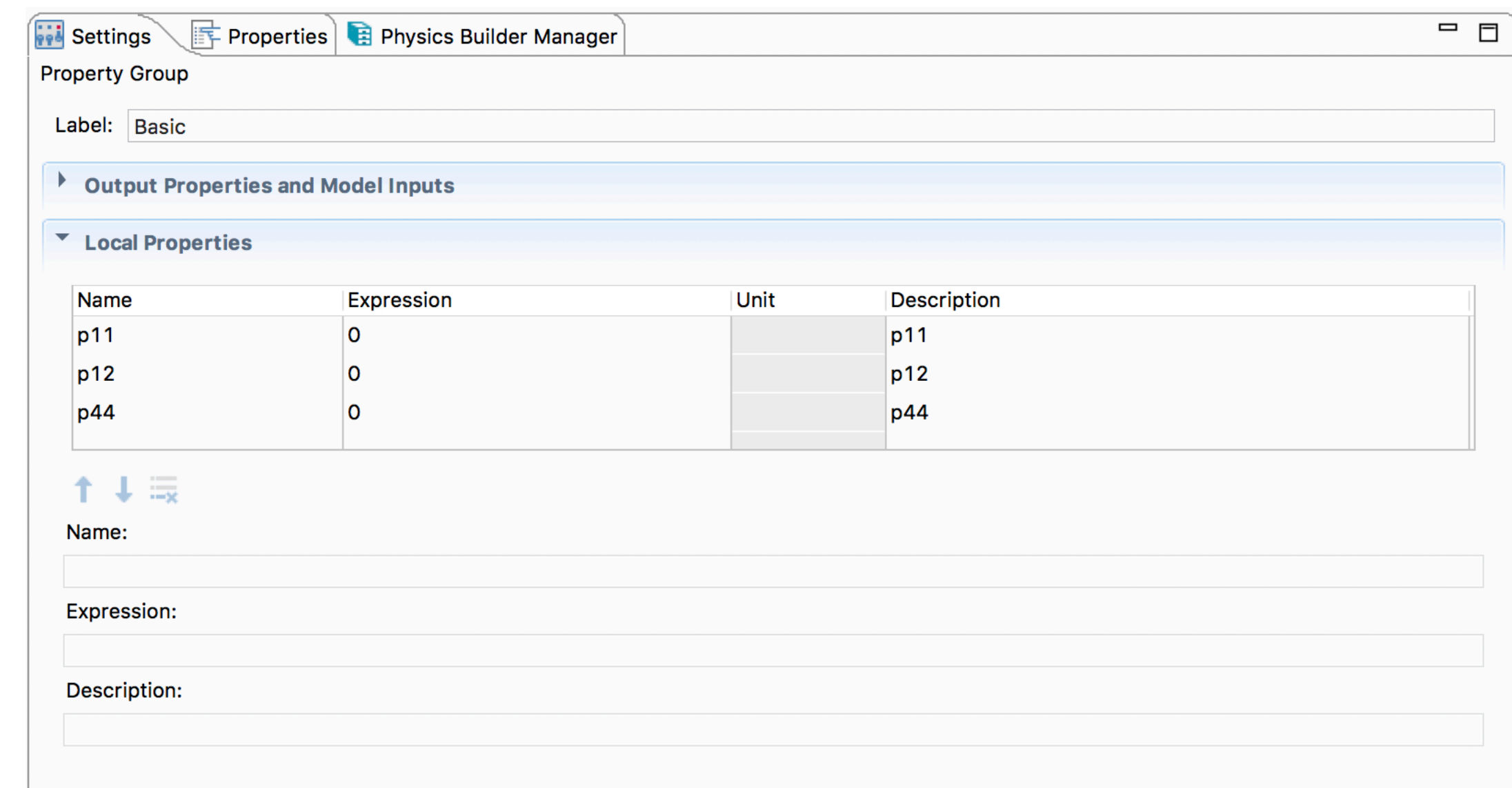
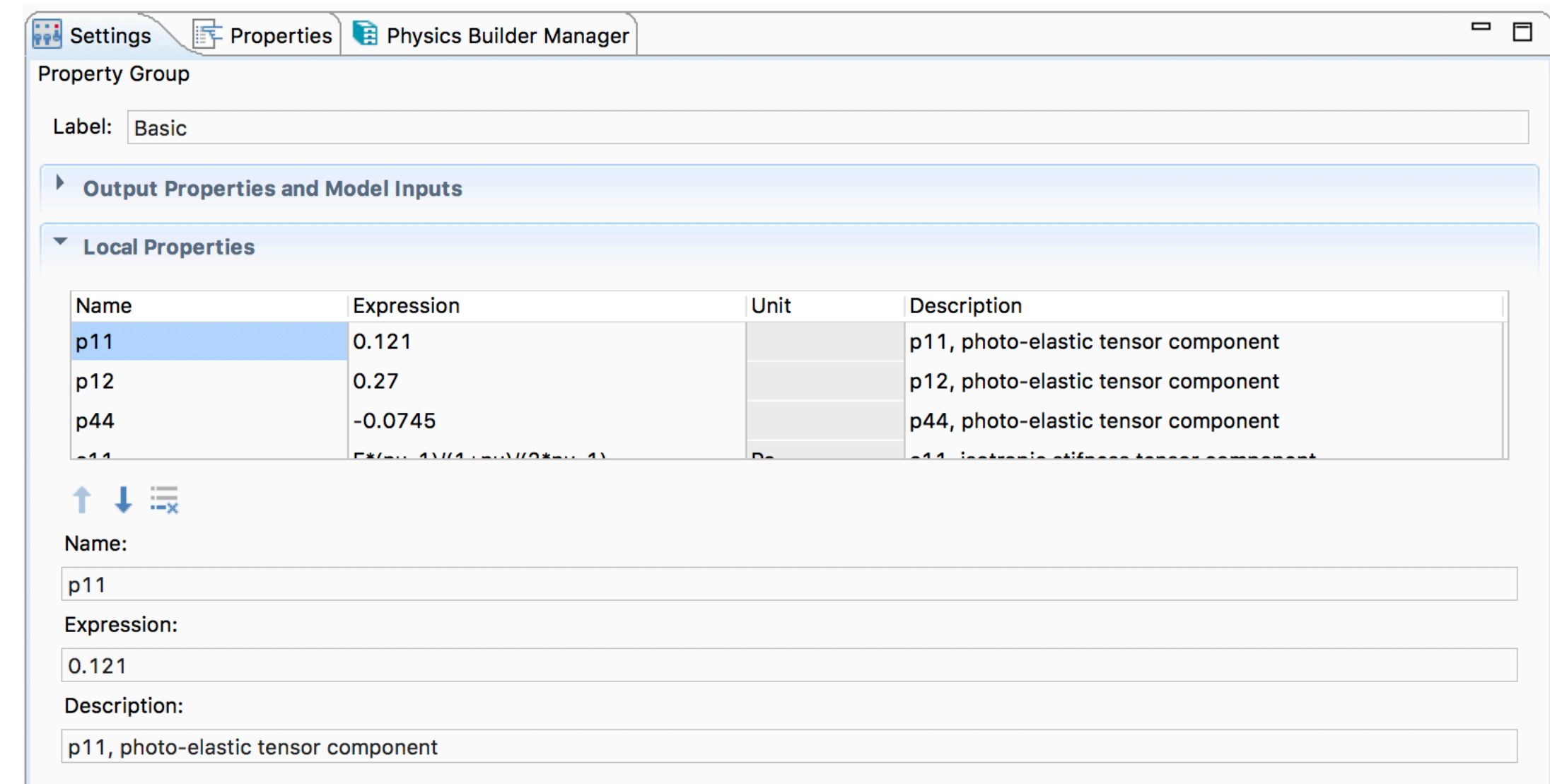
Property	Variable	Value	Unit	Property group
<input checked="" type="checkbox"/> Density	rho	2203	kg/m ³	Basic
<input checked="" type="checkbox"/> Young's modulus	E	73e9	Pa	Basic
<input checked="" type="checkbox"/> Poisson's ratio	nu	0.17	1	Basic
<input checked="" type="checkbox"/> Relative permeability	mur_iso ; m...	1	1	Basic
<input checked="" type="checkbox"/> Relative permittivity	epsilon_nr_is...	nref_sel(lambd...	1	Basic
<input checked="" type="checkbox"/> Electrical conductivity	sigma_iso ; ...	0	S/m	Basic
p11, photo-elastic tensor component	p11	0.121		Basic
p12, photo-elastic tensor component	p12	0.27		Basic
p44, photo-elastic tensor component	p44	-0.0745		Basic
Thermal conductivity	k_iso : kii =...	1.38	W/(m·K)	Basic

Materials: photo elastic properties

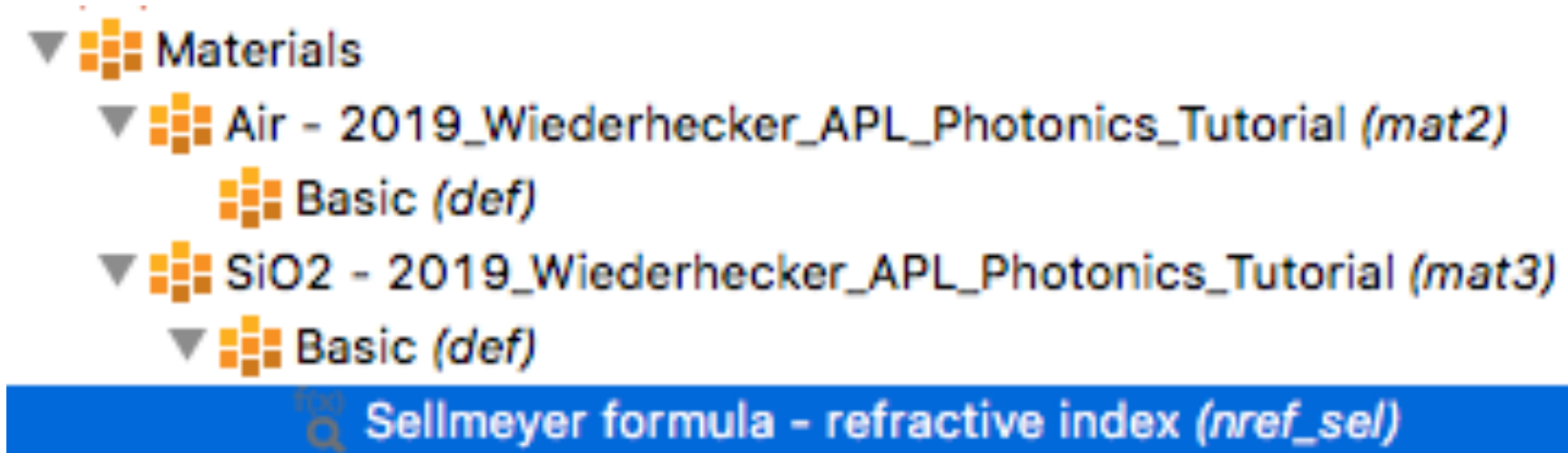


Photoelastic parameters must be defined for each material in order to calculate the Brillouin gain.

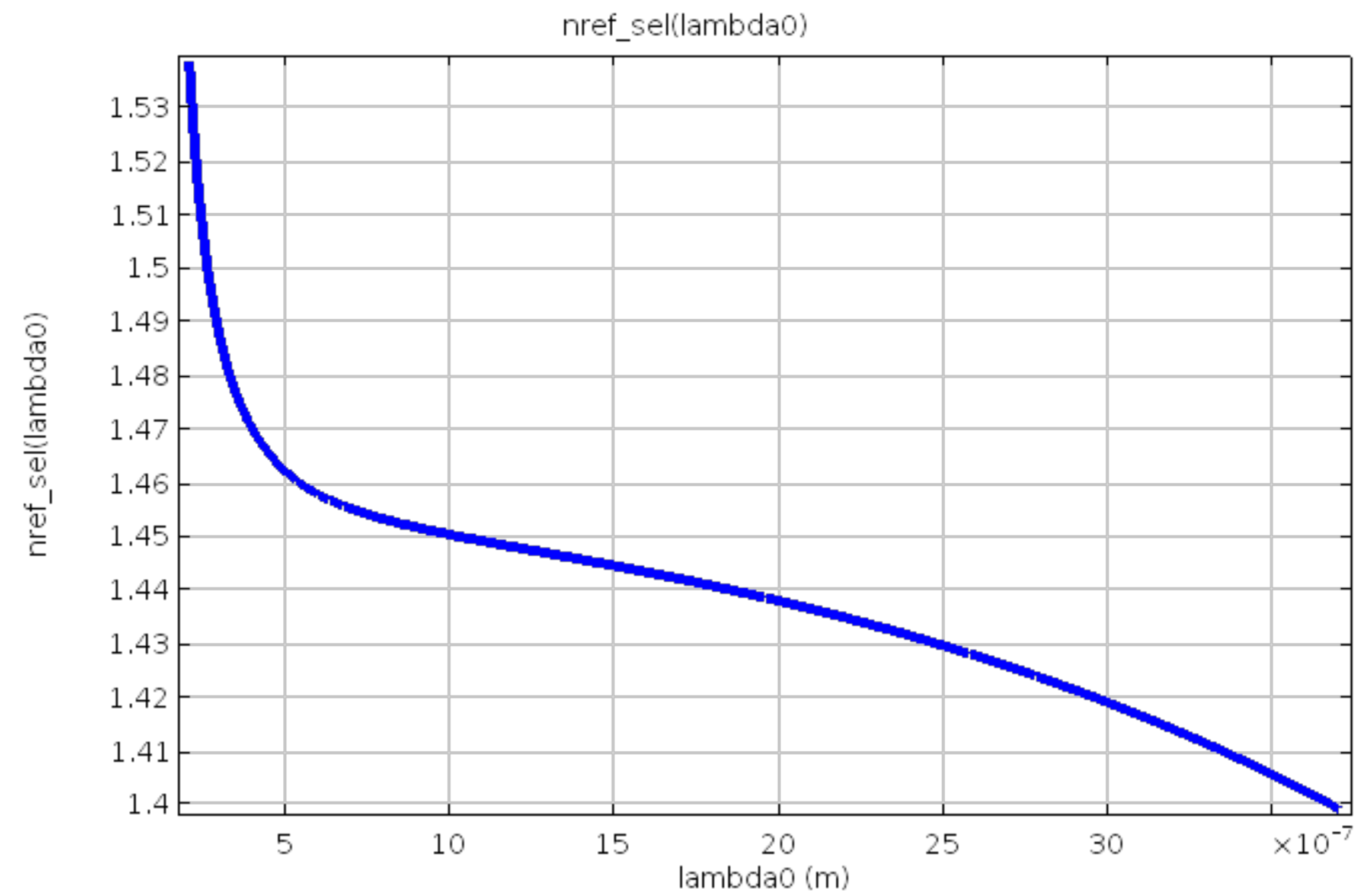
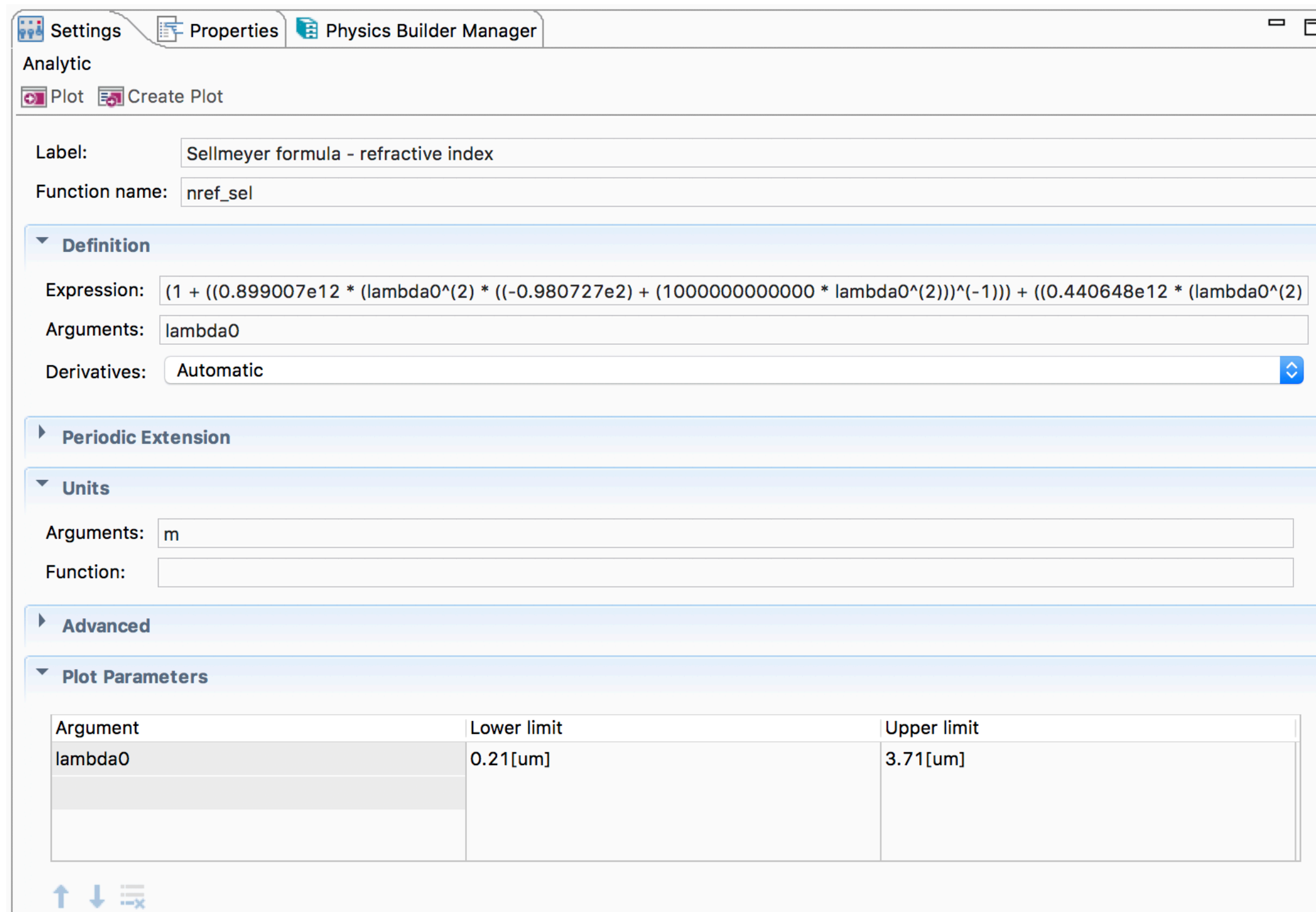
- In our simulation files, this is done by defining “Local Properties” for each material. For instance, in the “SiO2-2019-Wiederhecker_APL_Photonics_Tutorial”, the photo elastic tensor components p11, p12 and p44 are shown in the upper screenshot.
- For air, these parameters are set to a null value for convenience. Since we never solve the “air” domain in the mechanical solver, these values are never actually used in our calculations.
- Other properties, such as stiffness tensor may be defined just for convenience. In the examples shown here, the solid mechanics model uses Young modulus, Poison ratio and density for mechanical mode calculations.



Materials: Using Sellmeier formula



- It is not necessary to introduce the Sellmeier formula, however, a simple way to do it in COMSOL is by defining an “Analytic” function within the Basic (def) material properties.
- By doing so, you may also easily plot the Sellmeier expression.
- **Notice that if this approach is used**, the value of **lambda0** (within the **Global Parameters**) will be used for defining the material refractive index used for the optical mode calculations.



Optical mode physics

▼ Electromagnetic Waves, Frequency Domain (*emw_p*)

- ▶ Wave Equation, Electric 1
- ▶ Perfect Electric Conductor 1
- ▶ Initial Values 1
- ▶ Perfect Magnetic Conductor 1
- Equation View

▼ Electromagnetic Waves, Frequency Domain 2 (*emw_s*)

- ▶ Wave Equation, Electric 1
- ▶ Perfect Electric Conductor 1
- ▶ Initial Values 1
- ▶ Perfect Magnetic Conductor 1
- Equation View

- In order to be able to compute both intra and inter-modal Brillouin gain we must add twice the *Electromagnetic waves, Frequency Domain (EWFD)* physics. In order to be compatible with our custom variables definitions, **they must be labelled *emw_p*** (for the pump mode) and ***emw_s*** (for the scattered mode).

- Notice that the “Selection:” tab is chosen as “Optical domain”. This selection is the label assigned to one the selection groups in the “variables” definition.

Settings Properties Physics Builder Manager

Electromagnetic Waves, Frequency Domain

Label: Electromagnetic Waves, Frequency Domain

Name: **emw_p**

Domain Selection

Selection: Optical domain

1
2

ON Active

Equation

Equation form: Study controlled

Show equation assuming: Study 1, Mode Analysis (p)

$$\nabla \times \mu_r^{-1}(\nabla \times \mathbf{E}) - k_0^2(\epsilon_r - \frac{j\sigma}{\omega\epsilon_0})\mathbf{E} = \mathbf{0}$$

$$\lambda = -j\beta - \delta_z$$

$$\mathbf{E}(x,y,z) = \bar{\mathbf{E}}(x,y)e^{-jk_z z}$$

Settings Properties Physics Builder Manager

Electromagnetic Waves, Frequency Domain

Label: Electromagnetic Waves, Frequency Domain 2

Name: **emw_s**

Domain Selection

Selection: Optical domain

1
2

ON Active

Equation

Equation form: Study controlled

Show equation assuming: Study 1, Mode Analysis (p)

$$\nabla \times \mu_r^{-1}(\nabla \times \mathbf{E}) - k_0^2(\epsilon_r - \frac{j\sigma}{\omega\epsilon_0})\mathbf{E} = \mathbf{0}$$

$$\mathbf{E}(x,y,z) = \bar{\mathbf{E}}(x,y)e^{-jk_z z}$$

Optical mode physics

- ▼ Electromagnetic Waves, Frequency Domain (*emw_p*)
 - ▶ Wave Equation, Electric 1
 - ▶ Perfect Electric Conductor 1
 - ▶ Initial Values 1
 - ▶ Perfect Magnetic Conductor 1
 - Equation View
- ▼ Electromagnetic Waves, Frequency Domain 2 (*emw_s*)
 - ▶ Wave Equation, Electric 1
 - ▶ Perfect Electric Conductor 1
 - ▶ Initial Values 1
 - ▶ Perfect Magnetic Conductor 1
 - Equation View

• One point that might be confusing throughout the options of the EWFD physics is the “out-of-plane wave number” tab. This option may be ignored when we choose the optical wavenumber (beta) or the effective index as an eigenvalue — which is what is done here. This option is only meaningful if the optical frequency is sought from an eigenfrequency solver configuration.

▼ Equation

Equation form:
Study controlled

Show equation assuming:
Study 1, Mode Analysis (p)

$$\nabla \times \mu_r^{-1}(\nabla \times \mathbf{E}) - k_0^2(\epsilon_r - \frac{j\sigma}{\omega\epsilon_0})\mathbf{E} = \mathbf{0}$$

$$\lambda = -j\beta - \delta_z$$

$$\mathbf{E}(x,y,z) = \bar{\mathbf{E}}(x,y)e^{-ik_z z}$$

▼ Components

Electric field components solved for:
Three-component vector

▼ Settings

Formulation:
Full field

▶ Port Sweep Settings

▼ Out-of-Plane Wave Number

Out-of-plane wave number:
 k_z 0 rad/m

▼ Analysis Methodology

Methodology options:
Fast

▼ Physics-Controlled Mesh

Enable

Maximum mesh element size control parameter:
From study

Resolve wave in lossy media

▼ Out-of-Plane Wave Number

Out-of-plane wave number:
 k_z 0 rad/m

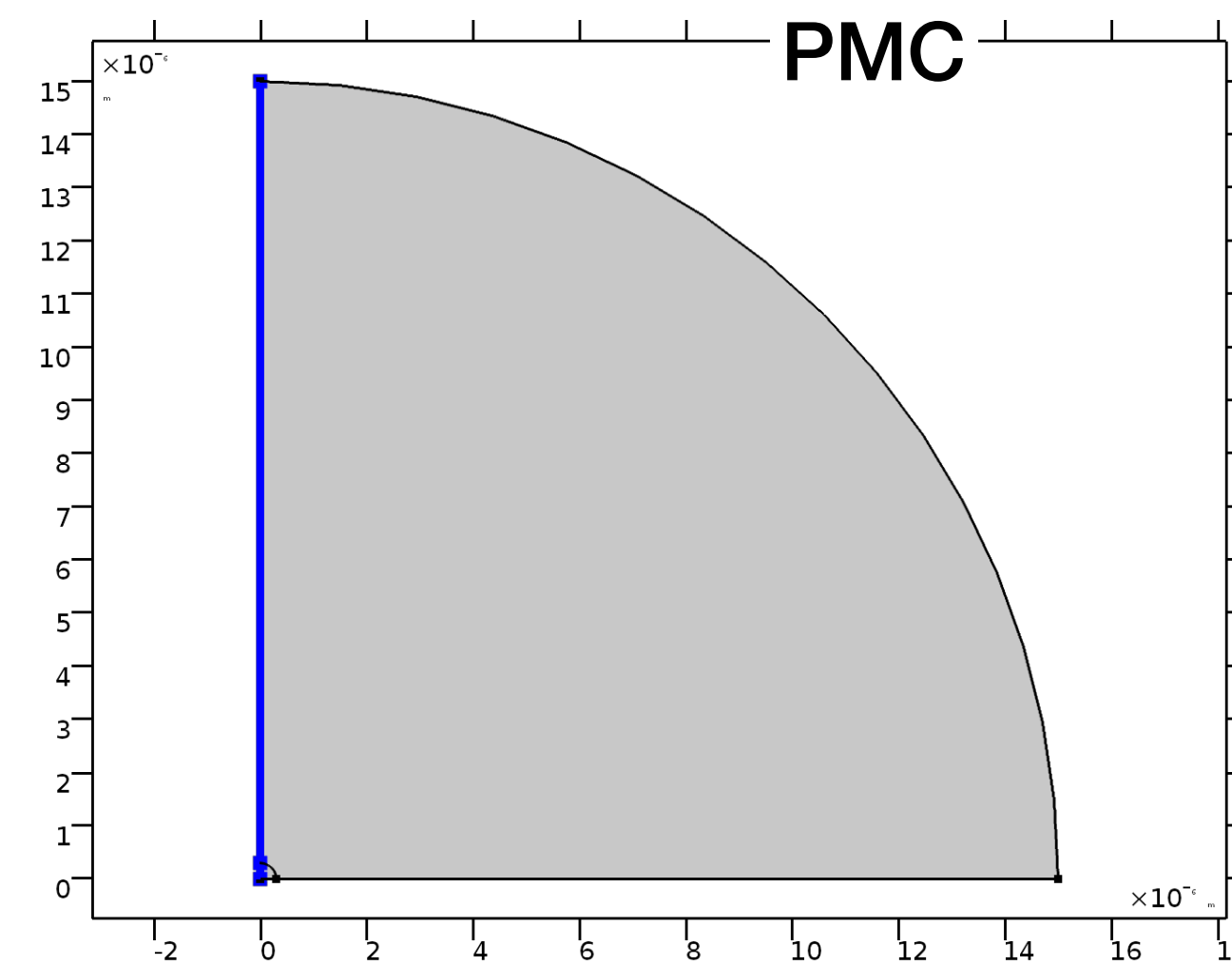
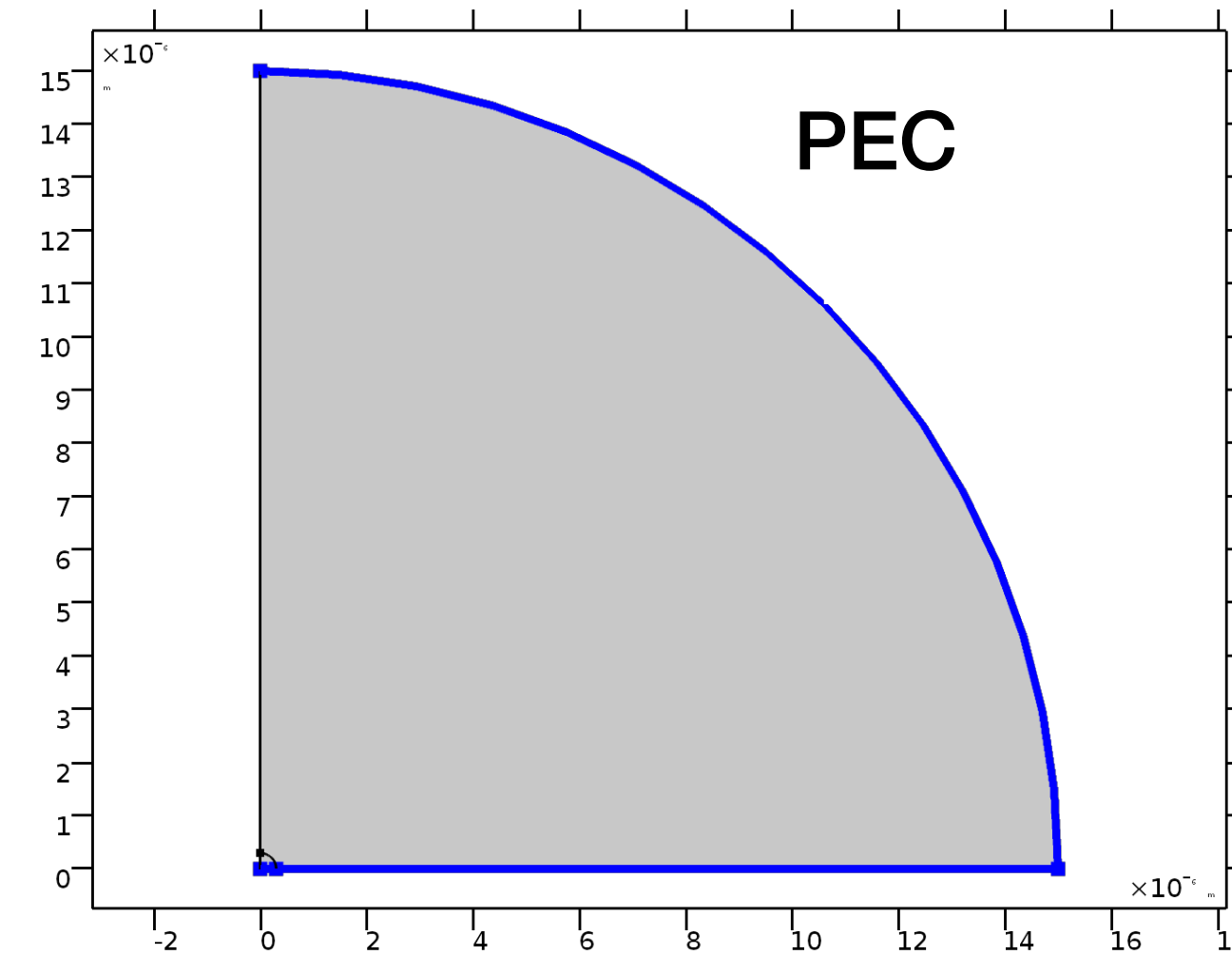
▼ Analysis Methodology

Methodology options:
Fast

Optical mode physics: boundary conditions

- ▼ Electromagnetic Waves, Frequency Domain (*emw_p*)
 - ▶ Wave Equation, Electric 1
 - ▶ Perfect Electric Conductor 1
 - ▶ Initial Values 1
 - ▶ Perfect Magnetic Conductor 1
 - Equation View

- By choosing between a PEC or a PMC along the symmetry boundaries will lead to modes with either symmetric or anti-symmetric reflection for planes.
- If the intra-modal scattering is of interest, the same boundary conditions should be applied for both the pump (*emw_p*) and scattered (*emw_s*) mode physics.



With this choice of boundary conditions, the pump mode will be vertically polarized

Mechanical mode physics

The screenshot displays the COMSOL Multiphysics interface for the Solid Mechanics physics interface. The left sidebar shows the model tree with 'Solid Mechanics (solid)' selected. The main window shows the 'Settings' tab for 'Solid Mechanics'. The 'Domain Selection' section is highlighted with a red box, showing 'Mechanical domain' selected. The '2D Approximation' section is highlighted with a blue box, showing 'Plane strain' selected, 'Out-of-plane mode extension (time-harmonic)' checked, and 'Out-of-plane wave number' set to 'abs(beta_ac) rad/m'. The 'Thickness' section shows 'd' set to '1 m'.

- Notice that the “Selection:” tab is chosen as “Mechanical domain”. This selection is the label assigned to one the selection groups in the “variables” definition.

- In the mechanical mode physics, only the solid regions are considered — air is treated as vacuum and the air-neighboring boundaries are treated as free-boundaries
- Although in the 2D approximation, the choice will be Plane-strain, once the “Out-of-plane extension (time harmonic)” option is tagged, COMSOL will use the $\exp(-i\beta_m z + i\omega t)$ ansatz.
- The thickness parameter is arbitrary and **does not affect** any of the Brillouin calculations.
- The mechanical wavenumber is β_{ac} and its value is defined within the “Basic - global” custom variables group. Within COMSOL, this variable is internally labelled as “solid.kz”.
- We use “abs(beta_ac)” here to avoid forcing the mechanical solver to use complex arithmetic. For instance, if absorbing boundary conditions are used in the optical physics, the optical wavenumber could be complex. Also, since all the mechanical waveguides treated here are bidirectional, positive or negative β_{ac} would not change the mechanical eigenfrequencies.

Mechanical material options

▼ Solid Mechanics (*solid*)

- ▶ Linear Elastic Material 1
- ▶ Free 1
- ▶ Initial Values 1
- ▶ Symmetry 1
- ▶ Equation View

Settings Properties Physics Builder Manager

Linear Elastic Material

Label: Linear Elastic Material 1

Domain Selection

Selection: All domains

ON	1		+
Active	2	(not applicable)	-

Override and Contribution

Equation

Model Input

Coordinate System Selection

Coordinate system: Global coordinate system

Linear Elastic Material

Nearly incompressible material

Solid model: Isotropic

Specify: Young's modulus and Poisson's ratio

Young's modulus: E From material

Poisson's ratio: ν From material

Density: ρ From material

Geometric Nonlinearity


Force linear strains


Additive strain decomposition


- All the isotropic materials being used in this tutorial are defined based on their Young modulus, poisson ratio, and density.
- Using other definitions, pre-defined by COMSOL, should not affect any of the results. The only mechanical property that is directly used in the Brillouin gain calculations is the density, which is necessary in any of the solid mode choices.


Study


▼ Study 1

 Step 1: Mode Analysis (p)

 Step 2: Mode Analysis (s)

 Step 3: Eigenfrequency

 Solver Configurations

 Job Configurations

- Three study steps are defined, one for each mode:
- **Step1: pump wave mode**
 - Mode Analysis : Given optical frequency, optical wavenumber (β) is sought as an eigenvalue;
- **Step2: scattered wave mode**
 - Mode Analysis : Given optical frequency, optical wavenumber (β) is sought as an eigenvalue;
- **Step3: mechanical mode**
 - Eigenfrequency: given mechanical wavenumber (solid.kz), mechanical frequency is sought as an eigenvalue.

Study options

▼ Study 1

Step 1: Mode Analysis (p)

Step 2: Mode Analysis (s)

Step 3: Eigenfrequency

Solver Configurations

Job Configurations

Settings Properties Physics Builder Manager

Mode Analysis

Compute Update Solution

Label: Mode Analysis (p)

Study Settings

Include geometric nonlinearity

Transform: Effective mode index

Mode analysis frequency: c_const/λ_0 Hz

Mode search method: Manual

Desired number of modes: 1

Unit: [dropdown]

Search for modes around: 1.45

Mode search method around shift: Closest in absolute value

Physics and Variables Selection

Modify model configuration for study step

Physics interface	Solve for	Discretization
Electromagnetic Waves, Freque...	<input checked="" type="checkbox"/>	Physics settings
Electromagnetic Waves, Freque...	<input type="checkbox"/>	Physics settings
Solid Mechanics	<input type="checkbox"/>	Physics settings

Settings Properties Physics Builder Manager

Mode Analysis

Compute Update Solution

Label: Mode Analysis (s)

Study Settings

Include geometric nonlinearity

Transform: Effective mode index

Mode analysis frequency: c_const/λ_0 Hz

Mode search method: Manual

Desired number of modes: 1

Unit: [dropdown]

Search for modes around: 1.45

Mode search method around shift: Closest in absolute value

Physics and Variables Selection

Modify model configuration for study step

Physics interface	Solve for	Discretization
Electromagnetic Waves, Freque...	<input type="checkbox"/>	Physics settings
Electromagnetic Waves, Freque...	<input checked="" type="checkbox"/>	Physics settings
Solid Mechanics	<input type="checkbox"/>	Physics settings

• It is important to restrict the variables sought for in each study step.

Study options

▼ Study 1

Step 1: Mode Analysis (p)

Step 2: Mode Analysis (s)

Step 3: Eigenfrequency

Settings Properties Physics Builder Manager

Eigenfrequency

Compute Update Solution

Label: Eigenfrequency

Study Settings

Eigenfrequency search method: Manual

Desired number of eigenfrequencies: 20

Unit: Hz

Search for eigenfrequencies around: 5.36e9 Hz

Eigenfrequency search method around shift: Closest in absolute value

Include geometric nonlinearity

Physics and Variables Selection

Modify model configuration for study step

Physics interface	Solve for	Discretization
Electromagnetic Waves, Frequen...	<input type="checkbox"/>	Physics settings
Electromagnetic Waves, Frequen...	<input type="checkbox"/>	Physics settings
Solid Mechanics	<input checked="" type="checkbox"/>	Physics settings

- For the mechanical solver, it is often necessary to include many modes to capture a full range of mechanical frequencies.
- The “search for eigenfrequencies around” option is also important

- It is important to restrict the variables sought for in each study step.

Optical post-processing I: visualization of solutions

Settings Properties Physics Builder Manager

Surface

Plot

Label: Surface

Data

Data set: From parent

Expression

Expression: $emw_p.normE/maxop_opt(emw_p.normE)$

Unit: 1

Settings Properties Physics Builder Manager

2D Plot Group

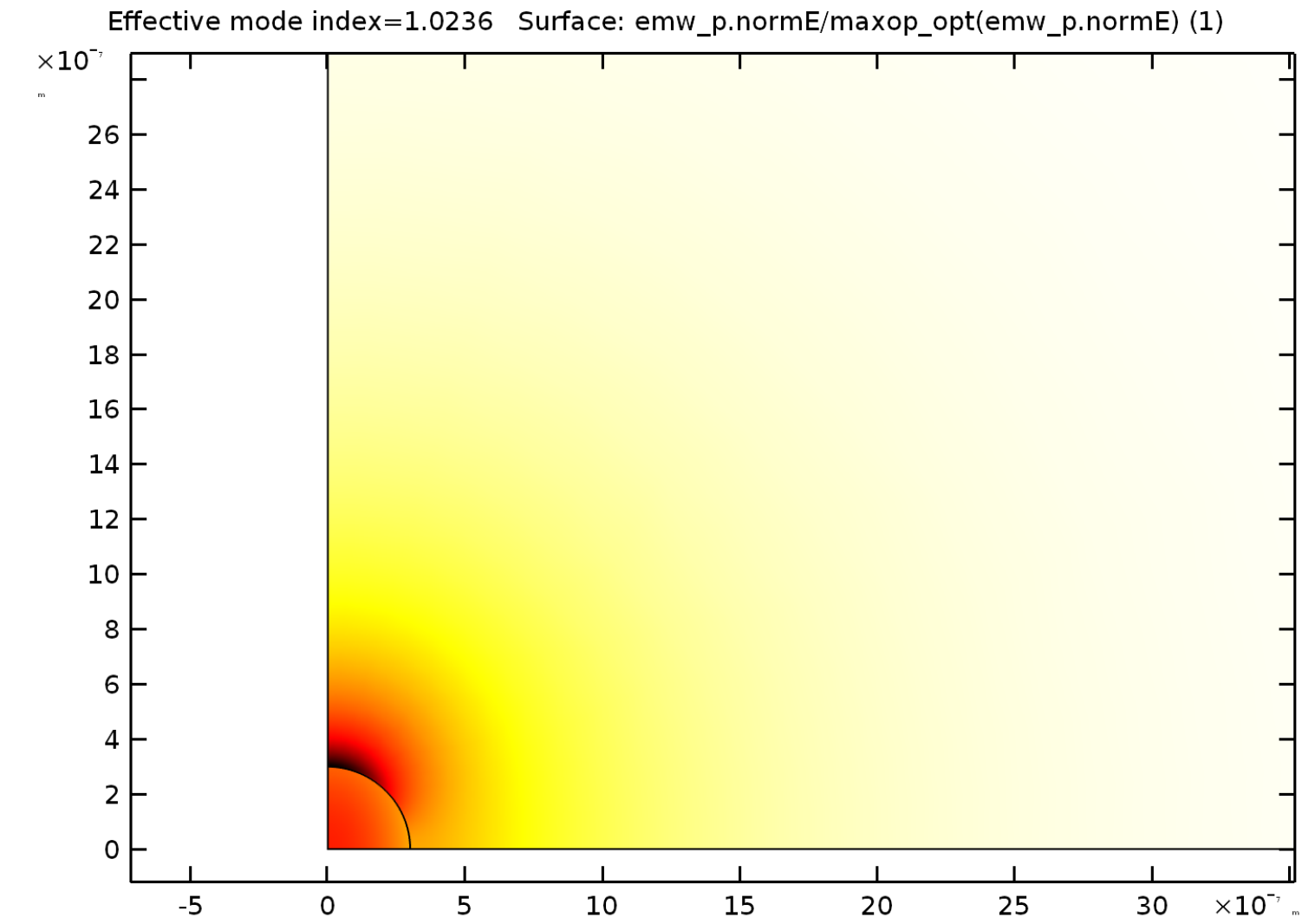
Plot

Label: Electric Field (emw_p)

Data

Data set: Study 1/Solution Store 1 (sol2)

Effective mode index: 1.0236



Settings Properties Physics Builder Manager

Surface

Plot

Label: Surface

Data

Data set: From parent

Expression

Expression: $emw_s.normE/maxop_opt(emw_s.normE)$

Unit: 1

Settings Properties Physics Builder Manager

2D Plot Group

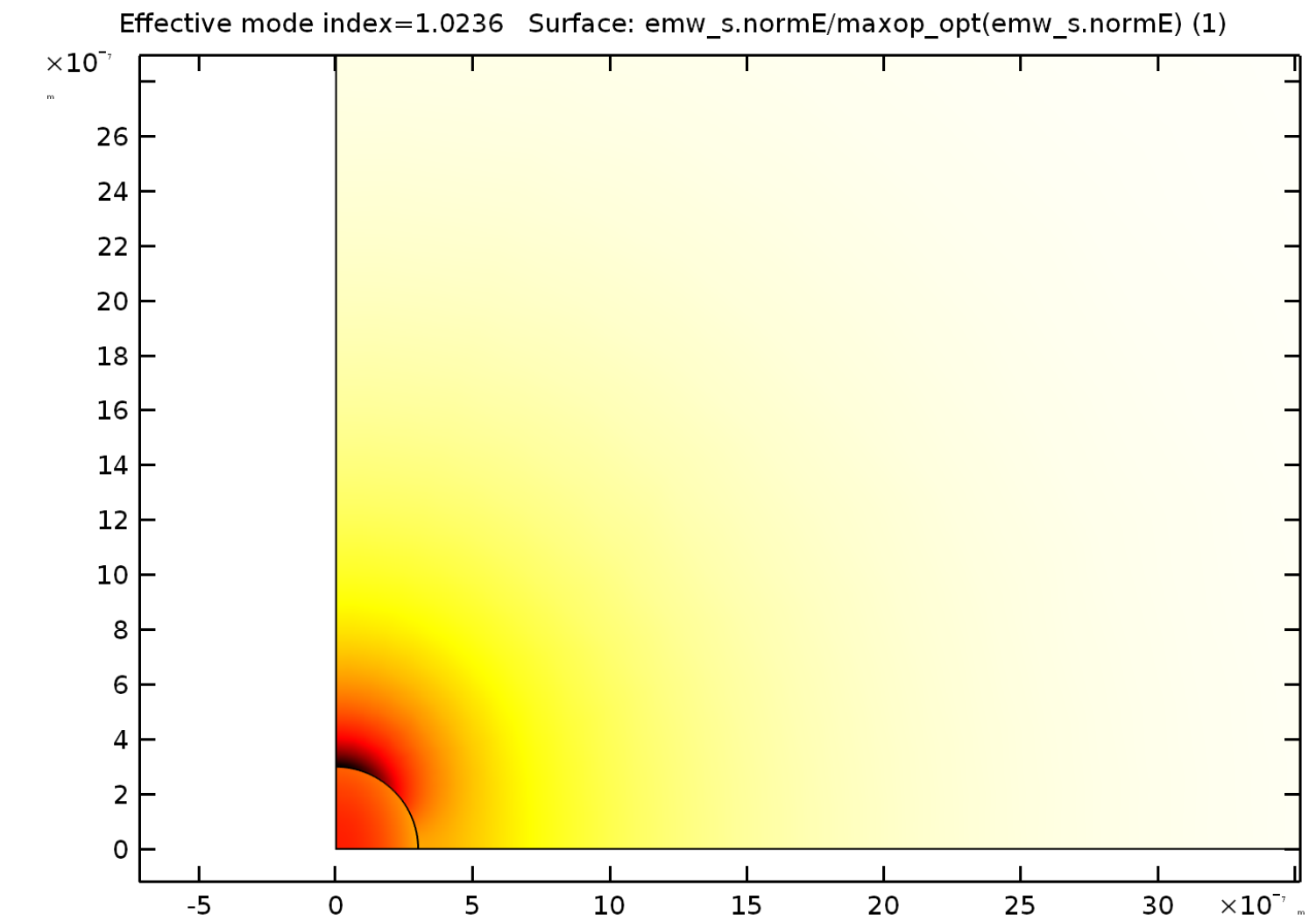
Plot

Label: Electric Field (emw_s)

Data

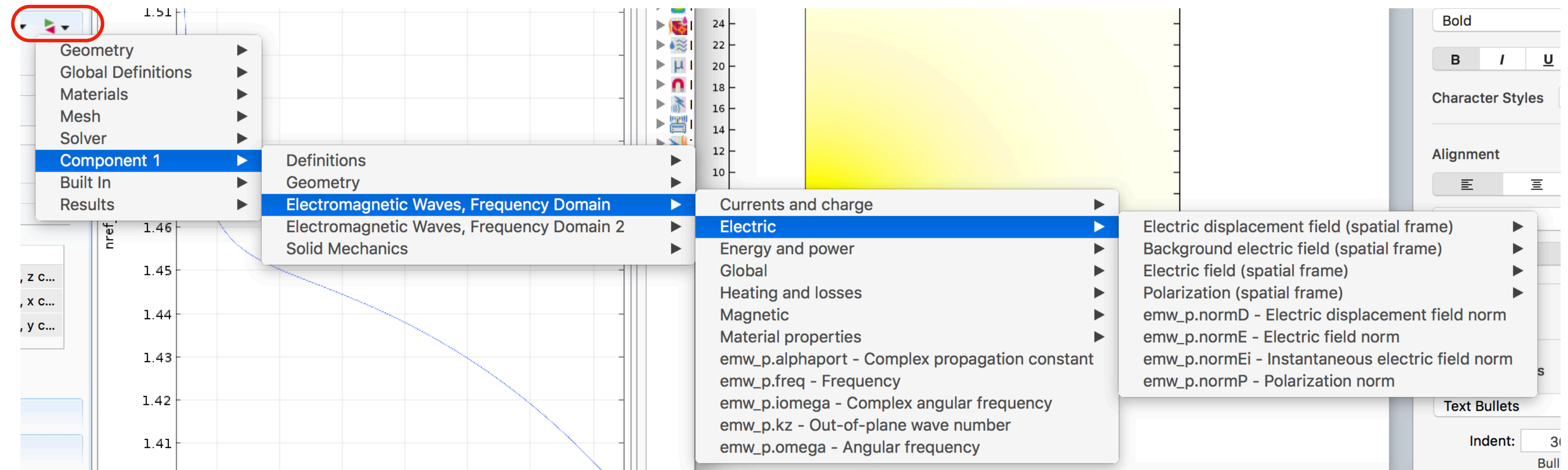
Data set: Study 1/Solution Store 2 (sol3)

Effective mode index: 1.0236



Optical post-processing I: visualization of solutions

- All the electromagnetic variables that are defined within COMSOL's module are directly accessible using `emw_p.var_name`, where `var_name` is the COMSOL variable name. A full list of those is at reach.



Mechanical post-processing I: visualization of solutions

Settings Properties Physics Builder Manager

2D Plot Group

Plot

Label: Mode Shape (solid)

Data

Data set: Study 1/Solution 1 (sol1)

Eigenfrequency (Hz): 5.652E9

Settings Properties Physics Builder Manager

Surface

Plot

Label: Surface 1

Data

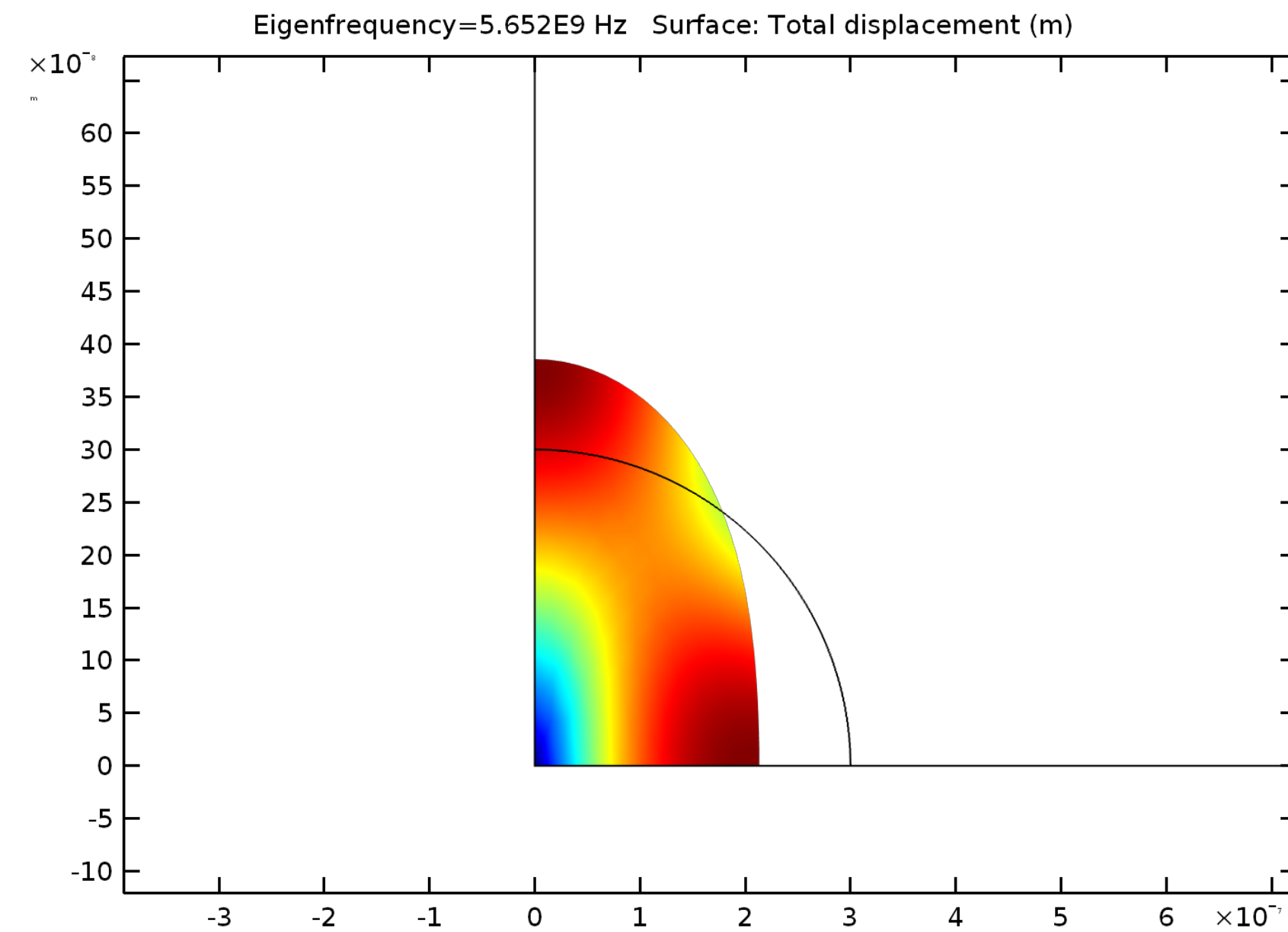
Data set: From parent

Expression

Expression: solid.disp

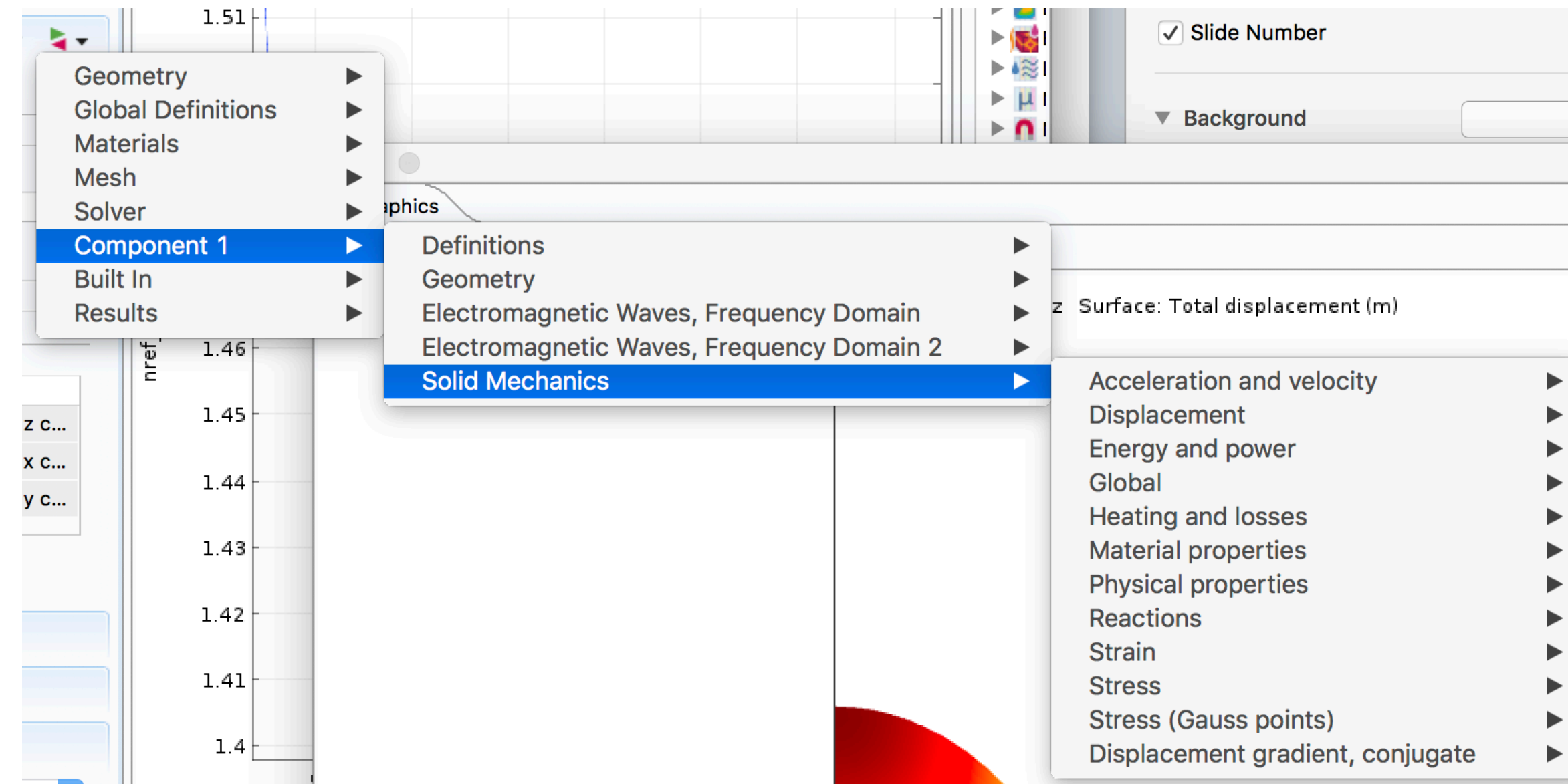
Unit: m

Description:



Mechanical post-processing I: visualization of solutions

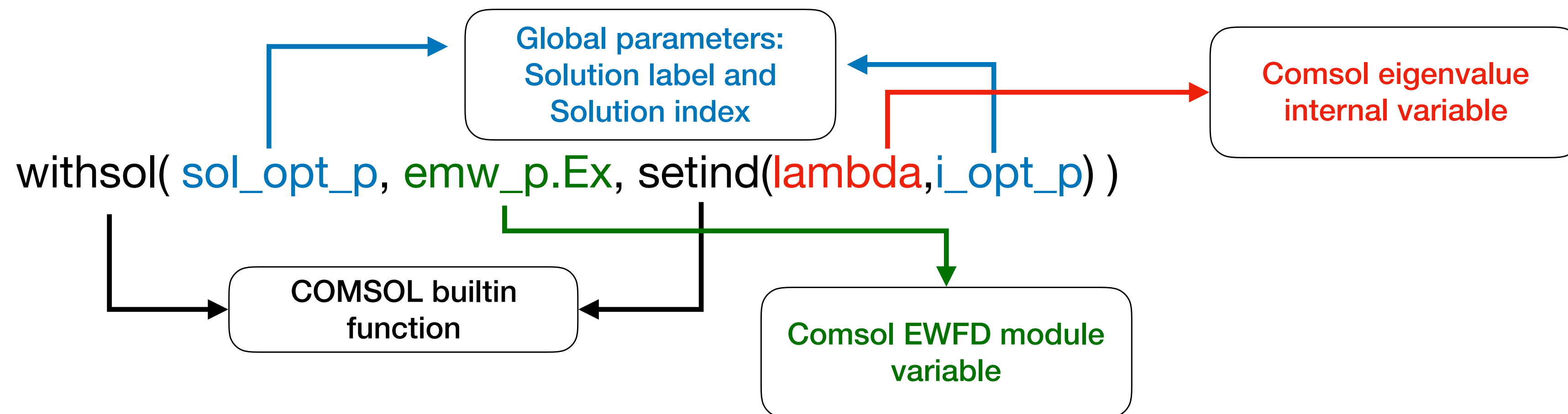
- All the solid mechanics variables that are defined within COMSOL's module are directly accessible using `solid.var_name`, where `var_name` is the COMSOL variable name. A full list of those is at reach.



Optical post-processing II: visualization of solutions - Ep and Es variables

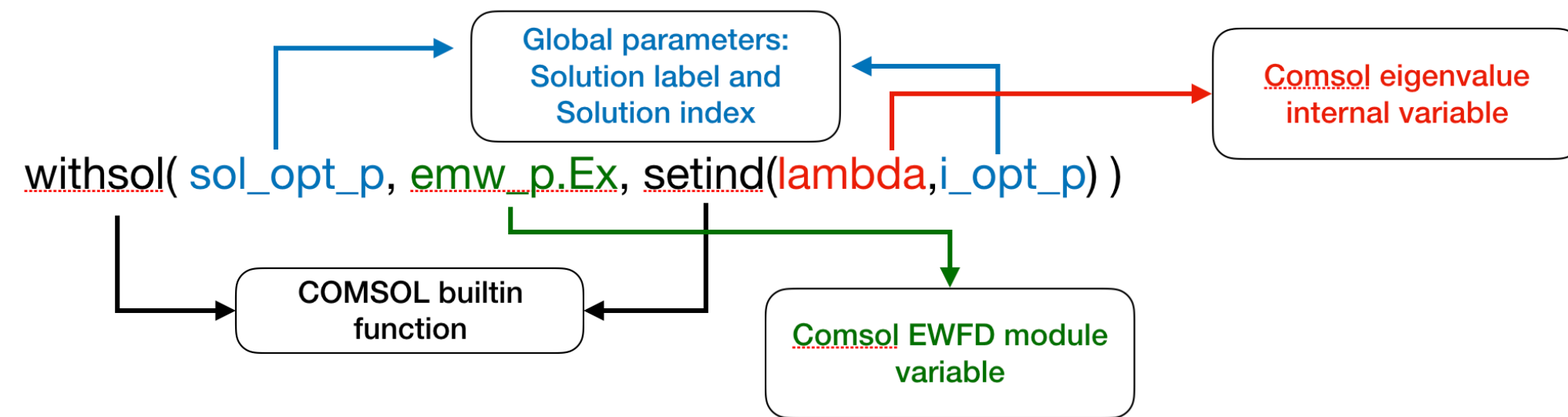
- These COMSOL variables are not directly used in the Brillouin gain calculations. Instead, within the “custom variables” defined earlier, the optical fields for the pump and scattered field used in the gain calculation are defined as:
- Ep:
 - Ep1: Ex-component for the pump mode
 - Ep2: Ey-component for the pump mode
 - Ep3: Ez-component for the pump mode
- Es:
 - Es1: Ex-component for the scattered mode
 - Es2: Ey-component for the scattered mode
 - Es3: Ez-component for the scattered mode

To define these variables we explore the built-in COMSOL function `withsol()`. For instance, the Ep1 variable is given by:



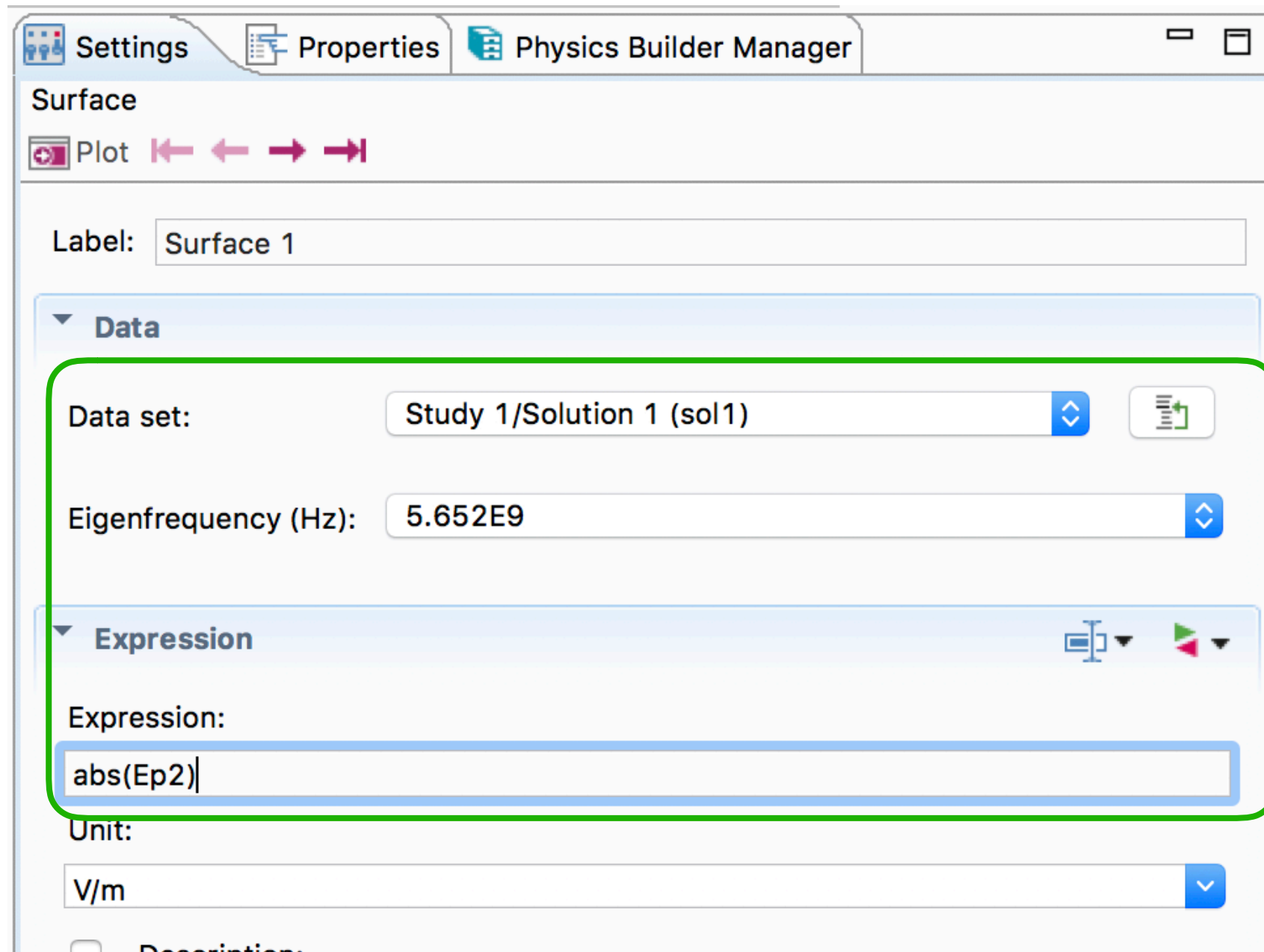
The output of the `withsol()` function is always the `emw_p.Ex` field stored in ‘`i_opt_p`’ index of solution ‘`sol_opt_p`’. **This function will override, any selection of dataset during a plot or evaluation of this function.**

Optical post-processing II: visualization of solutions - Ep and Es variables

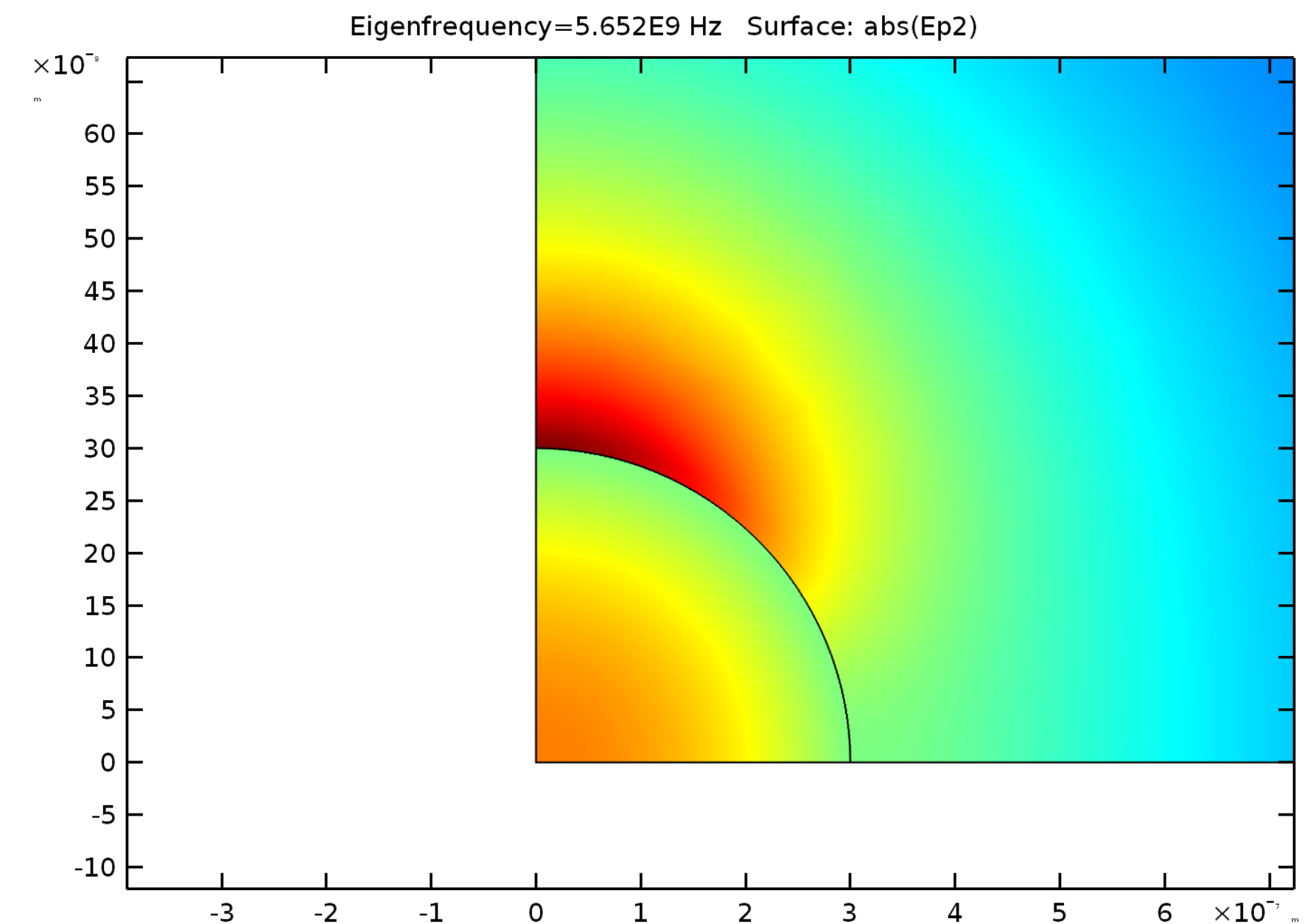


This function will override, any selection of dataset during a plot or evaluation of this function.

For example, during a plot — even if one selects the mechanical mode dataset — the resulting plot will be the one defined by the output of `withsol()`:



Even though "sol 1" is chosen (which is the mechanical solver solution) and a mechanical eigenfrequency is selected, the expression `abs(Ep2)` will ignore these settings and use its definitions based on the argument of **`withsol()`**



Optomechanical post-processing I: visualization of mixed variables

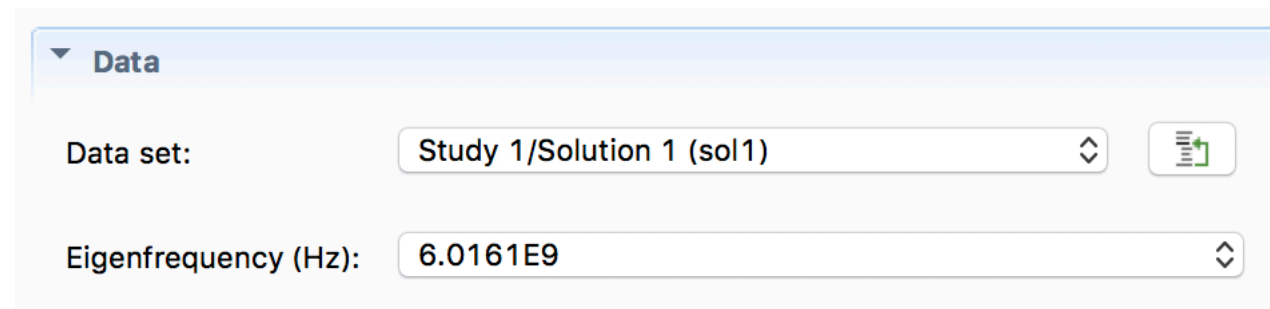
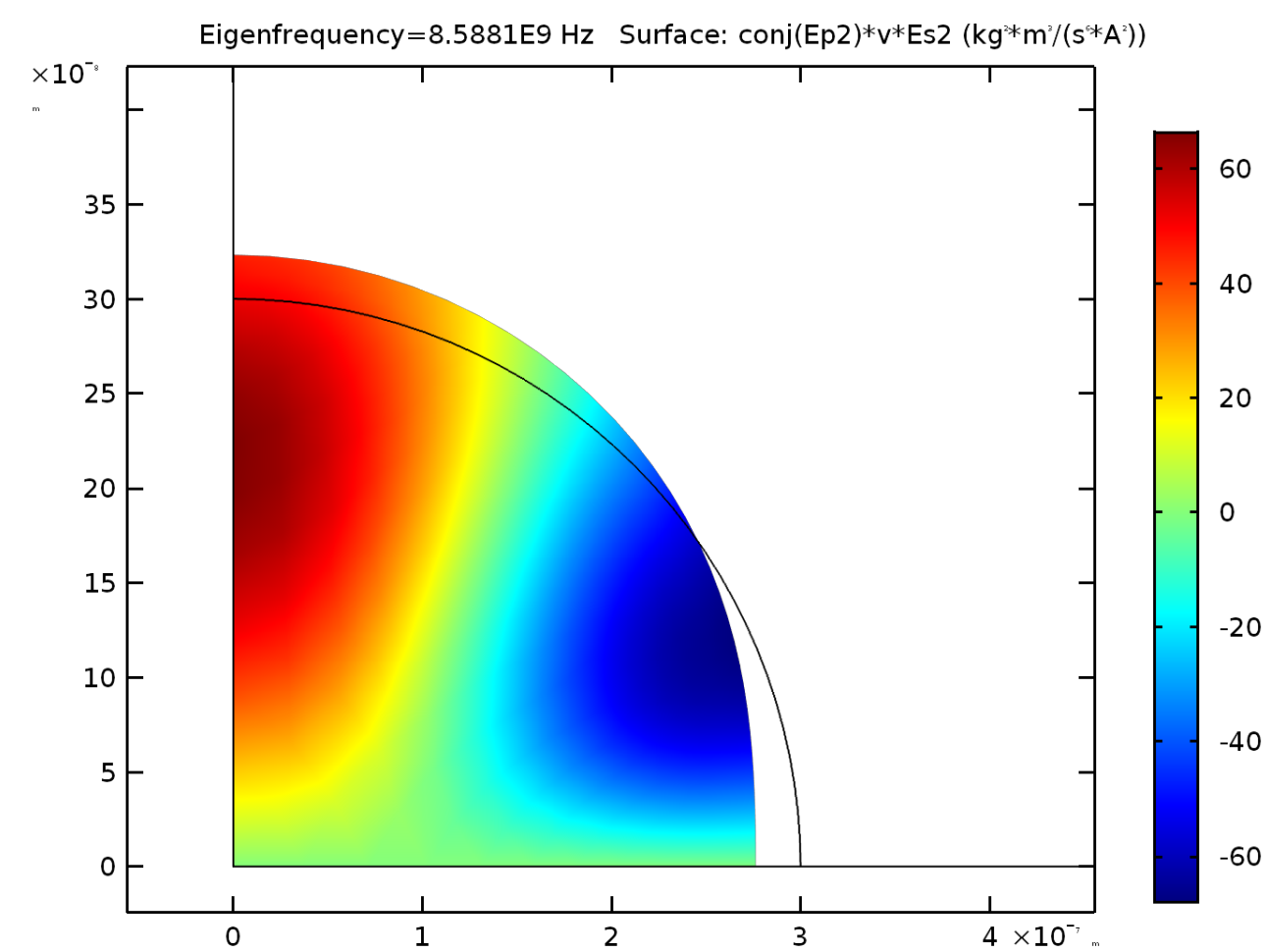
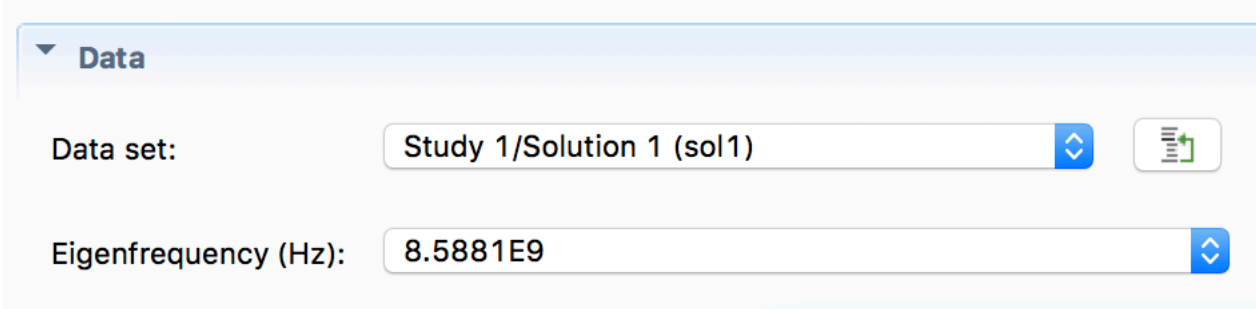
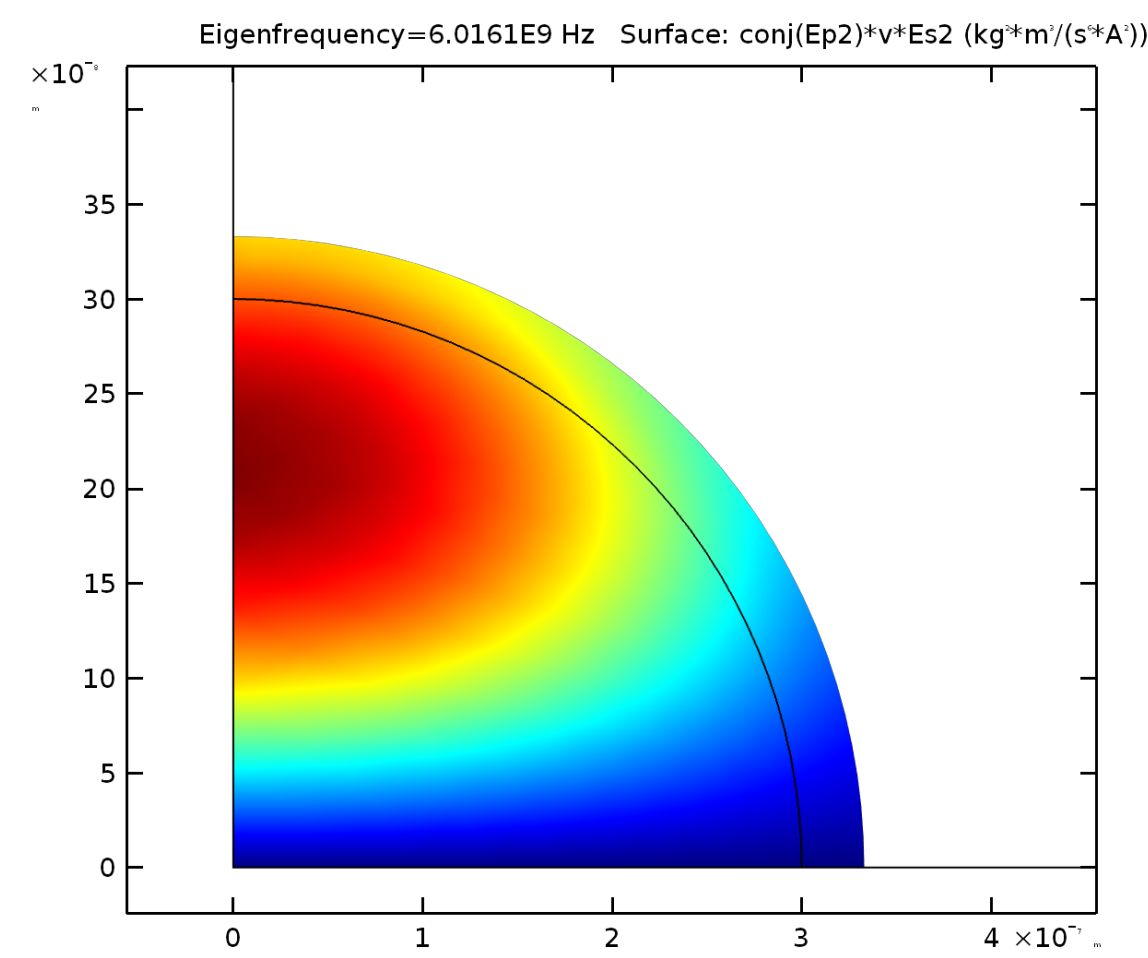
The great advantage of defining the pump and scattered wave mode variables (E_p, E_s) arises when one wants to visualize a given optomechanical product. Let's take the example of the quantity:

$$E_{py}^* u_y E_{sy} : \text{conj}(E_{p2}) * v * E_{s2}$$

u_y : mechanical displacement y-component: v in comsol

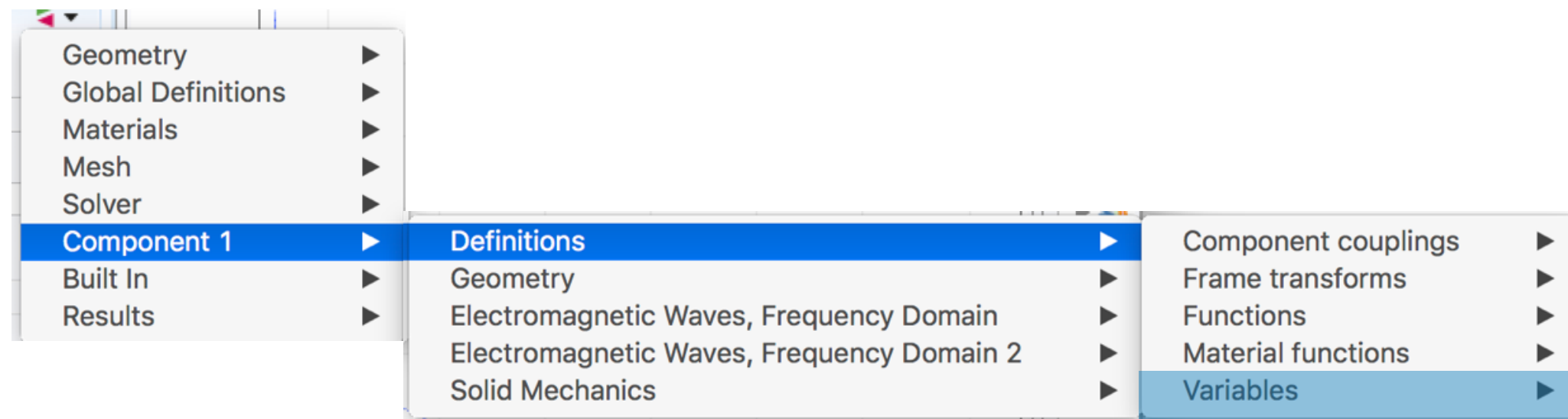
E_{py} : pump electric field y-component: Ep2 in comsol

E_{sy} : scattered electric field y-component: Es2 in comsol



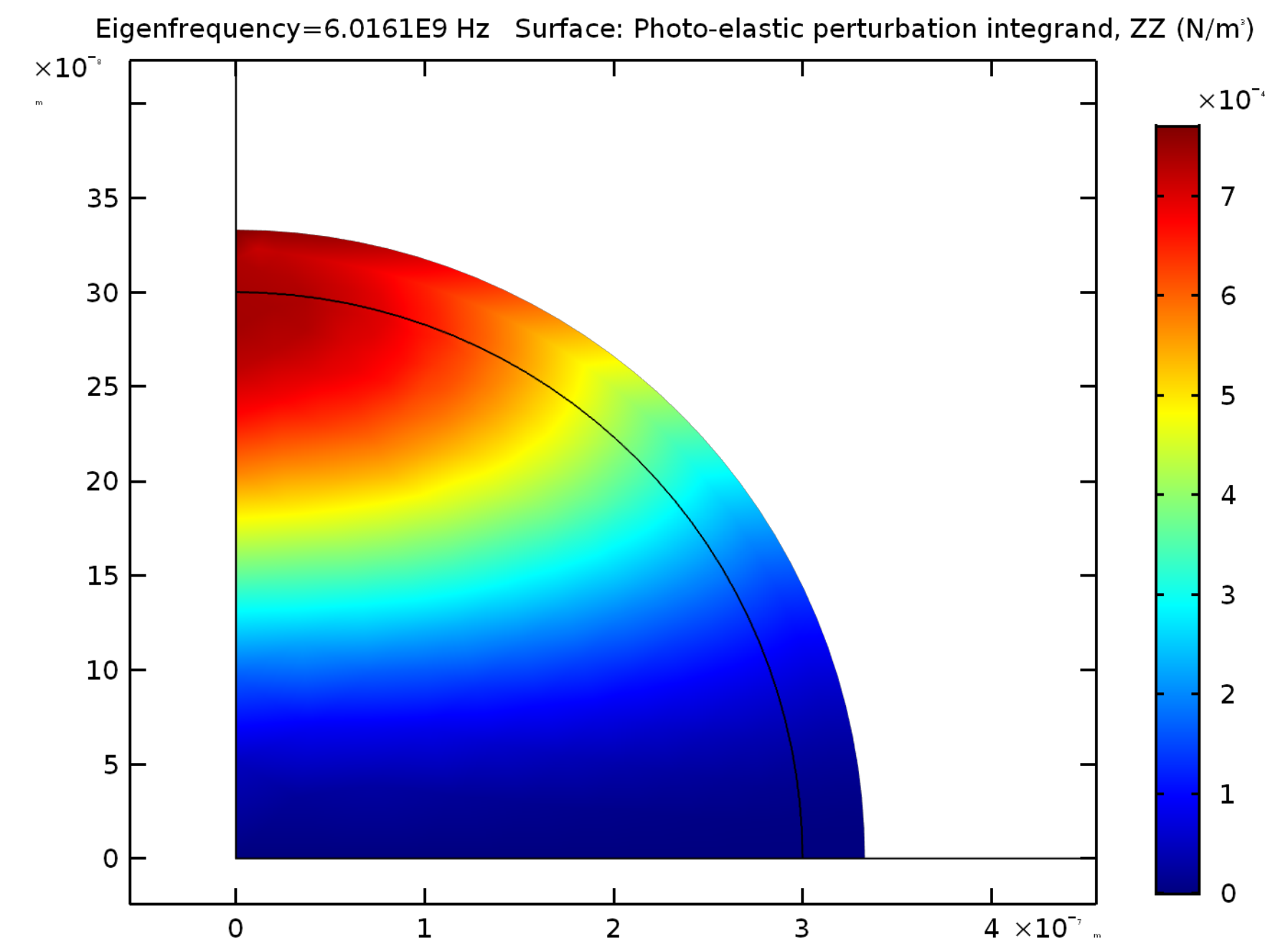
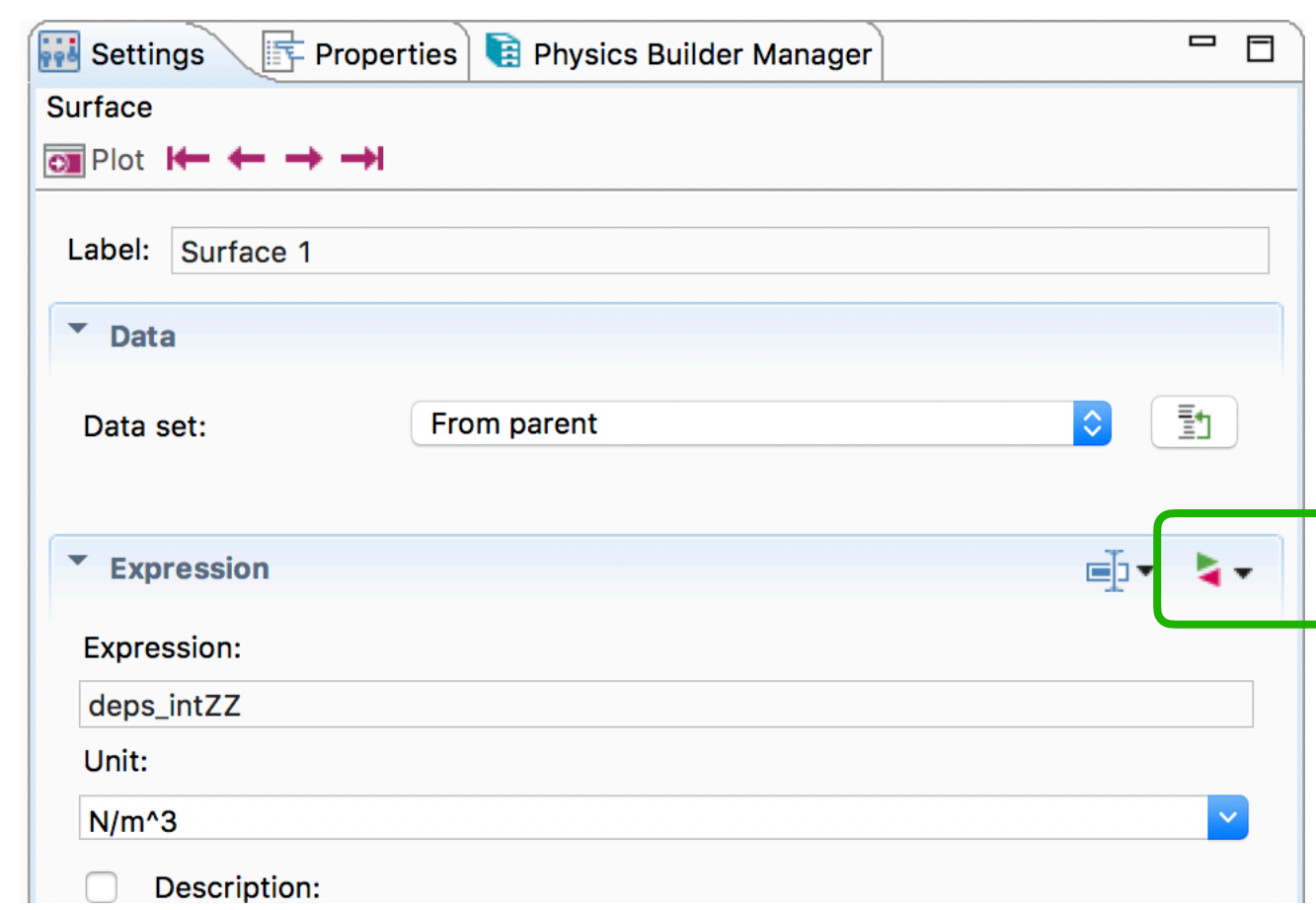
To be able to quickly skip through the several sought mechanical modes, the `withsol()` operator is **not used** in the mechanical solutions. This allows one to quickly choose the desired mechanical dataset (sol1) and the desired eigenfrequency to visualize the quantity of interest.

Optomechanical post-processing II: visualization of mixed, pre-defined, variables - **photoelastic**



- Dp1 - Displacement field (p) - comp. 1
- Dp2 - Displacement field (p) - comp. 2
- Dp3 - Displacement field (p) - comp. 3
- Ds1 - Displacement field (s) - comp. 1
- Ds2 - Displacement field (s) - comp. 2
- Ds3 - Displacement field (s) - comp. 3
- Ep1 - Electric field (p) - comp. 1
- Ep2 - Electric field (p) - comp. 2
- Ep3 - Electric field (p) - comp. 3
- Es1 - Electric field (s) - comp. 1
- Es2 - Electric field (s) - comp. 2
- Es3 - Electric field (s) - comp. 3
- Hp1 - Magnetic field (p) - comp. 1
- Hp2 - Magnetic field (p) - comp. 2
- Hp3 - Magnetic field (p) - comp. 3
- Hs1 - Magnetic field (s) - comp. 1
- Hs2 - Magnetic field (s) - comp. 2
- Hs3 - Magnetic field (s) - comp. 3
- Popt_p - Optical power integrated (p)
- Popt_s - Optical power integrated (s)
- S1 - Strain per unit length tensor 1, Voigt notation
- S2 - Strain per unit length tensor 2, Voigt notation
- S3 - Strain per unit length tensor 3, Voigt notation
- S4 - Strain per unit length tensor 4, Voigt notation
- S5 - Strain per unit length tensor 5, Voigt notation
- S6 - Strain per unit length tensor 6, Voigt notation
- Sz_p - time-average Poynting vector (p) - z-component
- Sz_s - time-average Poynting vector (s) - z-component
- We_p - Electrical energy (p)
- We_s - Electrical energy (s)
- beta_ac - Phase-matched Mechanical mode wavenumber
- beta_p - Pump mode wavenumber
- beta_s - Stokes mode wavenumber
- depsXX - Photo-elastic perturbation, XX**
- depsXY - Photo-elastic perturbation, XY
- depsXZ - Photo-elastic perturbation, XZ
- depsYY - Photo-elastic perturbation, YY
- depsYZ - Photo-elastic perturbation, YZ
- depsZZ - Photo-elastic perturbation, ZZ
- deps_int - Photo-elastic perturbation integrand, total
- deps_intXX - Photo-elastic perturbation integrand, XX
- deps_intXY - Photo-elastic perturbation integrand, XY
- deps_intXZ - Photo-elastic perturbation integrand, XZ
- deps_intYY - Photo-elastic perturbation integrand, YY
- deps_intYZ - Photo-elastic perturbation integrand, YZ
- deps_intZZ - Photo-elastic perturbation integrand, ZZ
- dispmax - max. displacement
- disptot - total displacement
- disptot2 - total displacement square
- fPE - PE force (N/W/m) per unit length
- fPE_density - PE force density (N/W/m^3)
- gOPE - cavity-equivalent PE coupling rate [Hz]
- gOTotal - Total g0[rad/s]
- gainPE - PE gain[1/W/m]
- gainTotal - Total gain[W^-1*m^-1]
- int_norm_energy_p - Electric energy normalization integrand (s)
- int_norm_energy_s - Electric energy normalization integrand (p)
- keff - effective stiffness per unit length
- meff - effective mass per unit length

All the variables that are predefined in our “custom variable” list are directly accessible in COMSOL’s plotting expression:



Anisotropic Silicon

There are three main changes needed to enable the use of anisotropic materials

▼ Definitions

- a= Basic - optical
- a= Basic - mechanical
- a= MB - optical
- a= MB - mechanical
- a= MB - mixed
- a= PE - mechanical
- a= PE - mixed

Coupling coefficients under “PE - mixed” definitions

▼ Solid Mechanics (solid)

- ▶ Linear Elastic Material 1
- ▶ Free 1
- ▶ Initial Values 1
- ▶ Linear Elastic Material 2
- ▶ Fixed Constraint 1
- $\frac{\partial U}{\partial t}$ Equation View

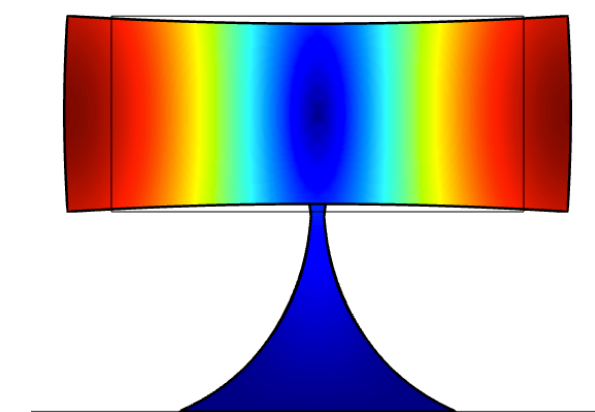
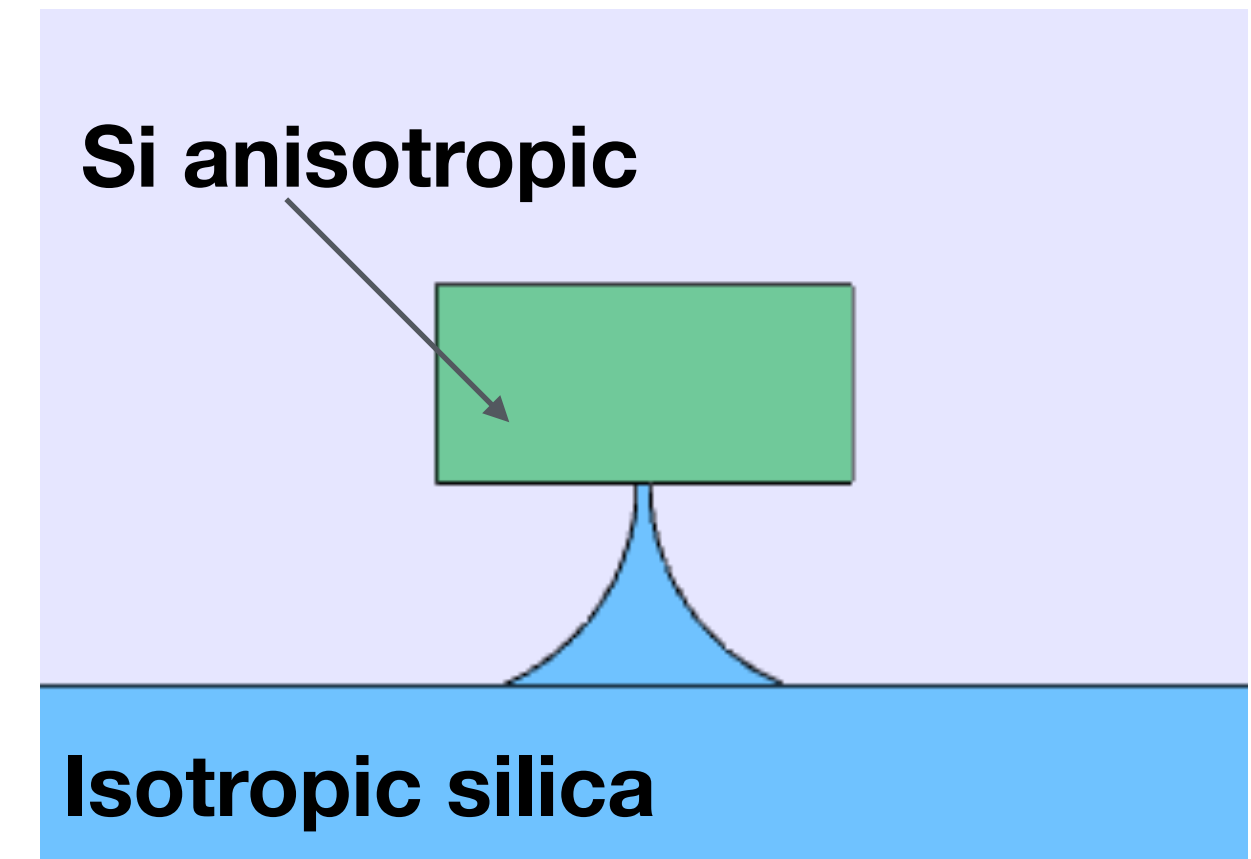
Linear Elastic physics definition

▼ Materials

- ▶ Silica - 2019_Wiederhecker_APL_Photonics_Tutorial (mat1)
- ▶ Air - 2019_Wiederhecker_APL_Photonics_Tutorial (mat2)
- ▼ Si Anisotropic - 2019_Wiederhecker_APL_Photonics_Tutorial (mat4)
 - ▶ Basic (def)
 - ▶ Anisotropic, Voigt notation (AnisotropicVoGrp)

Material properties

Silicon undercut waveguide were chose as an example of the use of Anisotropic materials



Mechanical mode calculated using anisotropic Si

Anisotropic Silicon: Coupling coefficients

Coupling coefficients under “PE - mixed” definitions

▼ Definitions

- a= Basic - optical
- a= Basic - mechanical
- a= MB - optical
- a= MB - mechanical
- a= MB - mixed
- a= PE - mechanical
- a= PE - mixed

Photo-elastic perturbation has now to include all terms

Name	Expression	Unit	Description
depsXX	$-\epsilon_{0_const} * \text{material.def.epsilonr_iso}^2 * (...)$	F/m ²	Photo-elastic perturbatio...
depsYY	$-\epsilon_{0_const} * \text{material.def.epsilonr_iso}^2 * (...)$	F/m ²	Photo-elastic perturbatio...
depsZZ	$-\epsilon_{0_const} * \text{material.def.epsilonr_iso}^2 * (...)$	F/m ²	Photo-elastic perturbatio...
depsXY	$-\epsilon_{0_const} * \text{material.def.epsilonr_iso}^2 * (...)$	F/m ²	Photo-elastic perturbatio...
depsXZ	$-\epsilon_{0_const} * \text{material.def.epsilonr_iso}^2 * (...)$	F/m ²	Photo-elastic perturbatio...
depsYZ	$-\epsilon_{0_const} * \text{material.def.epsilonr_iso}^2 * (...)$	F/m ²	Photo-elastic perturbatio...

Notice that we chose to use symmetric materials in the Voigt notation which is also the reason for the factor of two in the mixed strain components (S_4 , S_5 e S_6).

$$\begin{bmatrix} \delta\epsilon_{xx} \\ \delta\epsilon_{yy} \\ \delta\epsilon_{zz} \\ \delta\epsilon_{xy} \\ \delta\epsilon_{xz} \\ \delta\epsilon_{yz} \end{bmatrix} = -\epsilon_0 n^4 \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\ p_{12} & p_{22} & p_{23} & p_{24} & p_{25} & p_{26} \\ p_{13} & p_{23} & p_{33} & p_{34} & p_{35} & p_{36} \\ p_{14} & p_{24} & p_{34} & p_{44} & p_{45} & p_{46} \\ p_{15} & p_{25} & p_{35} & p_{45} & p_{55} & p_{56} \\ p_{16} & p_{26} & p_{36} & p_{46} & p_{56} & p_{66} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ 2S_4 \\ 2S_5 \\ 2S_6 \end{bmatrix}$$

Anisotropic Silicon: Solid Mechanics Physics

Linear Elastic physics definition

- ▼ Solid Mechanics (*solid*)
 - ▶ Linear Elastic Material 1
 - ▶ Free 1
 - ▶ Initial Values 1
 - ▶ Linear Elastic Material 2
 - ▶ Fixed Constraint 1
 - Equation View

Linear Elastic Material 1 was set only for the waveguide (silicon) and assigned as Anisotropic material

Linear Elastic Material

Nearly incompressible material

Solid model:

Anisotropic

Material data ordering:

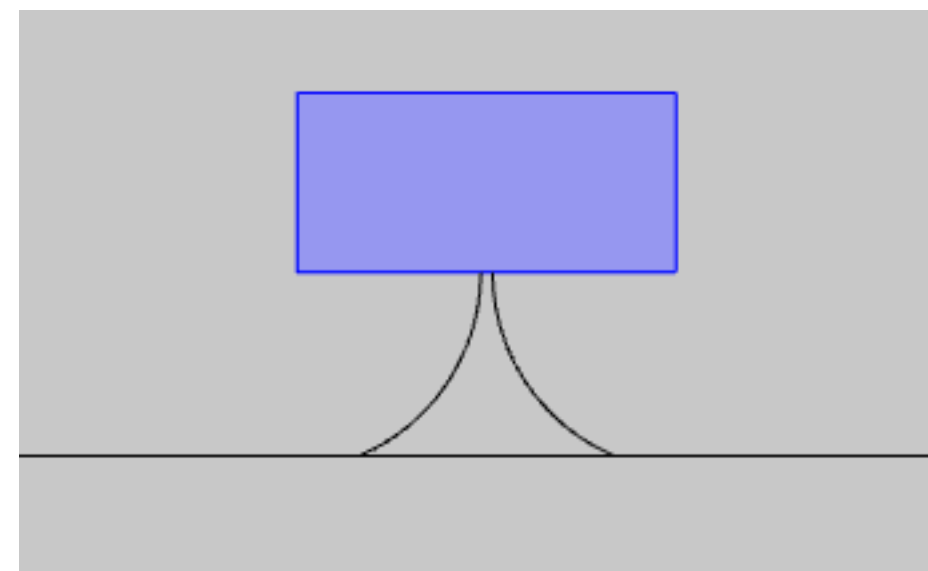
Voigt (XX, YY, ZZ, YZ, XZ, XY)

Elasticity matrix, Voigt notation:

D From material

Density:

ρ From material



Very important: Since Voigt notation was used for defining the coupling coefficients, it must also be used in the linear elastic settings

Linear Elastic Material 2 was set only for the pedestal (silica) and remained as an Isotropic Material with elastic properties given by the Young's modulus and Poisson's ratio

Linear Elastic Material

Nearly incompressible material

Solid model:

Isotropic

Specify:

Young's modulus and Poisson's ratio

Young's modulus:

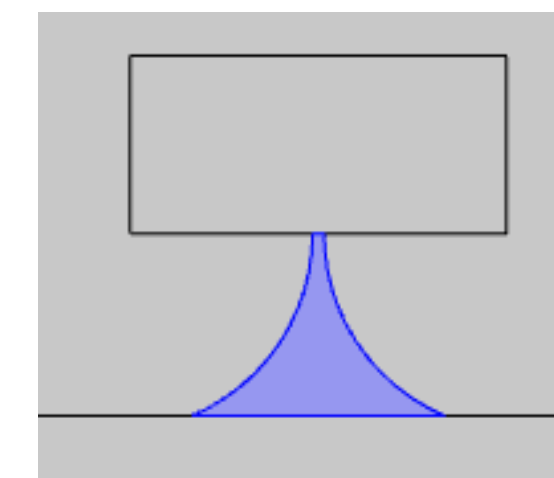
E From material

Poisson's ratio:

ν From material

Density:

ρ From material



Anisotropic Silicon: Material Properties

Material properties: Elasticity Matrix

Materials

- Silica - 2019_Wiederhecker_APL_Photonics_Tutorial (mat1)
- Air - 2019_Wiederhecker_APL_Photonics_Tutorial (mat2)
- Si Anisotropic - 2019_Wiederhecker_APL_Photonics_Tutorial (mat4)
 - Basic (def)
 - Anisotropic, Voigt notation (AnisotropicVoGrp)

Material Contents

Property	Variable	Value	Unit	Prop
Electrical conductivity	sigma...	0	S/m	Basi
Relative permeability	mur_is...	1	1	Basi
Relative permittivity	epsilo...	3.45^2	1	Basi
Density	rho	2329(kg...	kg/m³	Basi
Elasticity matrix, Voigt notation	{C11, C...		Pa	Anis
Young's modulus	E	165.26e9	Pa	Basi
Poisson's ratio	nu	0.046	1	Basi
p11 - photoelastic tensor com...	p11	-0.094		Basi
p12 - photoelastic tensor com...	p12	0.017		Basi
p44 - photoelastic tensor com...	p44	-0.051		Basi

Edit

Elasticity matrix, Voigt notation

Symmetric

C11	C12	C13	C14	C15	C16
C11	C12	C13	C14	C15	C16
C12	C22	C23	C24	C25	C26
C13	C23	C33	C34	C35	C36
C14	C24	C34	C44	C45	C46
C15	C25	C35	C45	C55	C56
C16	C26	C36	C46	C56	C66

Cancel OK

Elasticity matrix can be edit using right-click over it which pops-up a window for inserting each of the elasticity matrix components.

- Symmetric elasticity matrix were used;
- Each component component (Cij - capital "C") is defined under "Local Properties" of material's "Anisotropic" tab.

Si Anisotropic - 2019_Wiederhecker_APL_Photonics_Tutorial (mat4)

- Basic (def)
- Anisotropic, Voigt notation (AnisotropicVoGrp)

Local Properties

Name	Expression	Unit	Description
C11	c11-((c11-c12)...	Pa	c11 - elasticity matrix rota...
C12	c12+((c11-c12)...	Pa	c12 - elasticity matrix rota...
C13	c12	Pa	c13 - elasticity matrix rota...
C14	0[Pa]	Pa	c14 - elasticity matrix rota...

- To allow for crystalline rotations of the material properties along the z-axis, we have written every elasticity matrix components (Cij) as a function of its principal axis components (cij) and the rotation angle (theta).
- The principal axis components (c11, c12 and c44) are also defined under "Local Properties" of material's "Anisotropic" tab.

Si Anisotropic - 2019_Wiederhecker_APL_Photonics_Tutorial (mat4)

- Basic (def)
- Anisotropic, Voigt notation (AnisotropicVoGrp)

Local Properties

Name	Expression	Unit	Description
c11	166e9[Pa]	Pa	xx-elasticity matrix for silicon - Voigth notation
c12	64e9[Pa]	Pa	yy-elasticity matrix for silicon - Voigth notation
c44	79e9[Pa]	Pa	xy-elasticity matrix for silicon - Voigth notation

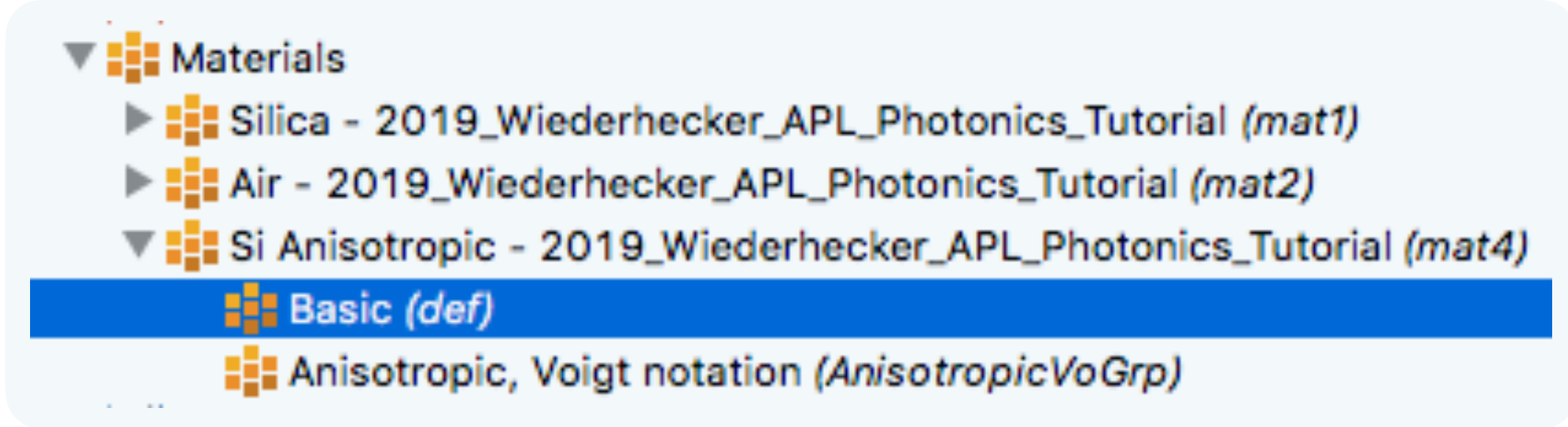
- The rotation angle (theta) is defined as a "Parameters" under the "Global Definitions" tab

Parameters

Name	Expression	Value	Description
theta	45 [degree]	0.7854 rad	rotation angle over z-axis...

Anisotropic Silicon: Material Properties

Material properties: Photoelasticity Matrix



As in the isotropic case, each photoelastic tensor components (P_{ij} - capital "P") are defined under the "Local Properties" tab into material's "Basic (def)"

Name	Expression	Unit	Description
P11	$p_{11} - ((p_{11} - p_{12}) / 2 - p_{12})$		p11 - photoelastic component r...
P12	$p_{12} + ((p_{11} - p_{12}) / 2 - p_{12})$		p12 - photoelastic component r...
P13	p12		p13 - photoelastic component r...
P14	0		p14 - photoelastic component r...

- To allow for crystalline rotations of the photo elastic tensor along the z-axis, we have written every photoelastic tensor components (P_{ij}) as a function of its principal axis components (p_{ij} - lowercase "p") and the rotation angle (θ).
- The principal axis components (p_{11} , p_{12} and p_{44}) are also defined under "Local Properties" tab into material's "Basic (def)"

Name	Expression	Unit	Description
p11	-0.094		p11 - photoelastic tensor compo...
p12	0.017		p12 - photoelastic tensor compo...
p44	-0.051		p44 - photoelastic tensor compo...

- The rotation angle (θ) is defined as a "Parameters" under the "Global Definitions" tab

Name	Expression	Value	Description
theta	45 [degree]	0.7854 rad	rotation angle over z-axis...

Anisotropic Silicon: z-axis Tensor Rotation

For both the elasticity matrix (**C**) and the photoelastic tensor (**P**) we have allowed for rotation of the crystalline axis with respect to the geometry. Our choice was to explicitly write each of the rotated tensor components (C_{ij} or P_{ij}) following Auld's book^[1]. Below we show the example for the elasticity matrix in the Voigt notation, but the same approach can be used for the photoelastic tensor since it holds the same symmetry of the elasticity matrix:

$$\mathbf{C} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{12} & C_{11} & C_{13} & 0 & 0 & -C_{16} \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ C_{16} & -C_{16} & 0 & 0 & 0 & C_{66} \end{bmatrix}$$

Notice that for materials with cubic symmetry and without any anisotropy:

$$c_{44} = \left(\frac{c_{11} - c_{12}}{2} \right)$$

and the **C** matrix is reduced back to the isotropic matrix

$$C_{11} = c_{11} - \left(\frac{c_{11} - c_{12}}{2} - c_{44} \right) \sin^2(2\theta)$$

$$C_{12} = c_{12} + \left(\frac{c_{11} - c_{12}}{2} - c_{44} \right) \sin^2(2\theta)$$

$$C_{13} = c_{12}$$

$$C_{16} = - \left(\frac{c_{11} - c_{12}}{2} - c_{44} \right) \sin(2\theta)\cos(2\theta)$$

$$C_{33} = c_{11}$$

$$C_{44} = c_{44}$$

$$C_{66} = c_{44} + \left(\frac{c_{11} - c_{12}}{2} - c_{44} \right) \sin^2(2\theta)$$

[1] B. A. Auld, Acoustic fields and waves in solids, 2nd ed., no. v. 2. Wiley, 1992.

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