



LIX. Some Researches upon the connexion between the rotation of the Earth and the geological changes of its surface

Henry Hennessy Esq.

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The bleaching action which the substance contained in electrolytical oxygen produces when dry on litmus paper, is a fact which of itself indicates that it must be a peroxide. It is well known that chlorine does not possess that property, but only such combinations of oxygen as give up this element with great ease, as for instance hypochlorous acid.

I cannot conclude this notice without expressing the obligation I am under to Professors Liebig and Buff for their kind direction and assistance during these experiments.

LIX. *Some Researches upon the Connexion between the Rotation of the Earth and the Geological Changes of its Surface.*
By HENRY HENNESSY, Esq.*

IT is now generally acknowledged that the agency of modern causes in producing geological changes on the surface of the earth is of no inconsiderable importance. Although the amount of matter which in the course of centuries may change its position on the surface of the earth, by the action of elevating and degrading forces, is thus locally important to the earth's surface, it must appear insignificant when compared with the mass of the entire globe. From this circumstance, it may at first appear futile to examine how the rotation of the earth could be affected by the mechanical action of the changes in position of such comparatively minute portions of its mass. It may be said that observation has not yet disclosed any irregularity in the period of the earth's rotation, but this could occur when the causes producing such an irregularity would be counteracted by others having a contrary tendency.

It is here proposed to examine how certain changes of position, with respect to the earth's centre of masses on its surface, would affect its rotation. Let ω represent the earth's angular velocity of rotation, M its mass, v the sum of the velocities impressed on it, by which both its rotation on its axis and its translation through space are produced, h a perpendicular from its centre to a plane passing through the centre of the impressed forces, and I the moment of inertia of the whole mass. Then

$$\omega = \frac{M v h}{I} (1.)$$

From this well-known formula, it is evident that any change in the value of I must produce a corresponding change in the value of ω . If θ represent the arc through which a point on the surface of the earth is carried during a certain time t , and if t' represent t when ω becomes ω' by a change of I into I' , then

* Communicated by the Author.

$$\omega = \frac{\theta}{t}, \quad \omega' = \frac{\theta}{t'}, \quad \omega' = \omega \frac{I}{I'}.$$

Hence

$$t' = t \frac{I'}{I}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

It is thus apparent that when the moment of inertia of the earth changes by a known quantity, the change in sidereal time can be calculated.

This fact furnishes a method for ascertaining whether, during certain defined intervals of time, an equilibrium has been maintained between the forces which tend to elevate or to depress the external crust of the earth. If any unknown *residual phenomena* exist on either side, their existence will be made known by finding the difference between the total mechanical actions exerted by the known elevatory and degrading forces.

The earth may be considered as a spheroid surrounded by a thin shell, whose external surface is covered with protuberances and depressions. A change in the moment of inertia of this shell produces a change in the moment of inertia of the whole earth, which is compounded of that moment, and of the moment of inertia of the internal spheroid. In finding both of those moments of inertia, the following general method is employed.

It is well known that if a solid of revolution be supposed to consist of an infinite number of plates perpendicular to the axis, and each plate of an infinite number of concentric rings, the moment of inertia of any zone will in general be expressed by

$$\int_l^l \int_0^y 2 \pi \rho r^3 dr dx; \quad . \quad . \quad . \quad . \quad (a.)$$

π being the ratio of the diameter of a circle to its circumference, r the radius of one of the rings, ρ its density, x the abscissa, and y the ordinate of the generating curve. The distance of the extremities of the zone from the origin of the co-ordinates, are expressed respectively by l and l' . When the body is heterogeneous, it will be found more convenient to transform the rectangular into polar co-ordinates. Let R represent the radius vector of a point in the ring, θ the angle comprised between the plane of R, y and the plane of x, y , and Ψ the angle comprised between R and the plane of y, z . As R is the same for all points in the ring, the plane of R, y and the plane of x, y coincide, and $\theta = 0$. If the origin of the co-ordinates be at the extremity of the axis of x , and if the distance from the point where the radius vector of the ring

touches the axis be called A' , then $x = A' (1 - \sin \Psi)$, $R = \sqrt{r^2 + a^2 \sin^2 \Psi}$. Thus, if ρ be given as a function of a or R , it can be expressed as a function of r and Ψ , or $\phi(r, \Psi)$. y , a function of x , can also be expressed as a function of Ψ or $F(\Psi)$, and (a) then becomes

$$-\int_{\Psi_2}^{\Psi_1} \int_0^{F\Psi} 2\pi A' \phi(r, \Psi) r^3 \cos \Psi dr d\Psi. \quad (3.)$$

In the case of a spheroid, where A' is the semi-polar axis, and the plane of y, z coincides with the plane of the equator, Ψ is evidently the latitude of the ring, and $y = a(1 + e) \cos \Psi$, e representing the ellipticity of the generating curve. If the earth be supposed to consist of an infinite number of spheroidal shells, it has been shown that the density of any shell which will agree best with the known ellipticity of the earth and its mean density, is represented by the formula*

$$\rho = \frac{A \sin qa}{a},$$

where ρ represents the density of the shell, a its semi-polar axis, and A and q constants. The value assigned to q is expressed by $q = \frac{5\pi}{6p}$, p representing the semi-polar axis of the entire spheroid. If I_1 represent the moment of inertia of the internal spheroid, then

$$a = \frac{r}{(1 + e) \cos \Psi}, \quad \dots \dots (4.)$$

and

$$I_1 = - \int_{\Psi_2}^{\Psi_1} \int_0^{b \cos \Psi} 2\pi A p r^2 \cos^2 \Psi \cdot (1 + e) \sin \left(\frac{5\pi r}{6p(1 + e) \cos \Psi} \right) dr d\Psi,$$

b representing the semi-equatorial axis of the entire spheroid. But†

$$e = \frac{5m}{2} \cdot \left\{ \frac{\left(1 - \frac{qa}{\tan qa}\right)^2}{2 - q^2 a^2 - \frac{qa}{\tan qa} - \frac{q^2 a^2}{\tan^2 qa}} \right\} \left\{ \frac{1 - \frac{3}{q^2 a^2} + \frac{3}{qa \tan qa}}{1 - \frac{qa}{\tan qa}} \right\}.$$

To completely eliminate e and a between this equation and (4.), is a step which must be at present considered impossible.

An approximate solution could be obtained, but the resulting expression would be so long and complicated as to be entirely useless. It is evident, however, that when e is small, its equality in every shell can be assumed without material error. Let e therefore be equal to the ellipticity of the external shell, so that

* Airy on the Figure of the Earth.

† Ibid.

$$a = \frac{r p}{b \cos \Psi}; \quad \dots \quad (5.)$$

then

$$\begin{aligned} I_1 &= - \int_{\Psi_2}^{\Psi_1} \int_0^{b \cos \Psi} 2\pi A b r^2 \sin\left(\frac{5\pi r}{6b \cos \Psi}\right) \cos^2 \Psi \, dr d\Psi; \quad (6.) \\ &= -2\pi A \left\{ \frac{5}{3} \pi \sin \frac{5}{6} \pi - 2 - \left(\frac{25}{36} \pi^2 - 2 \right) \cos \frac{5}{6} \pi \right\} \frac{216}{125 \pi^3} \int_{\Psi_2}^{\Psi_1} b^4 \cos^5 \Psi \, d\Psi. \end{aligned}$$

When $\Psi_1 = \frac{\pi}{2}$, and $\Psi_2 = -\frac{\pi}{2}$, or when the above is integrated throughout the whole mass of the spheroid, it becomes

$$I_1 = \frac{6912A}{1875\pi^3} \left\{ \left(\frac{25}{36} \pi^2 - 2 \right) \cos \frac{5}{6} \pi + 2 - \frac{5}{3} \pi \sin \frac{5}{6} \pi \right\} b^4 \pi. \quad (7.)$$

When a becomes p and ρ becomes ρ' , the density at the surface of the spheroid

$$\rho' = \frac{A}{p} \sin \frac{5}{6} \pi, \quad A = \frac{\rho' p}{\sin \frac{5}{6} \pi}.$$

Substituting this value of A in (7.), and remembering that $M_1 = \frac{8}{3} \pi \rho' b^2 p$, M_1 being the mass of the spheroid, we shall obtain

$$I_1 = \frac{864}{625\pi^3} \left\{ \left(\frac{25}{36} \pi^2 - 2 \right) \cot \frac{5}{6} \pi + \frac{2}{\sin \frac{5}{6} \pi} - \frac{5}{3} \pi \right\} M b^2. \quad (8.)$$

If the spheroid were homogeneous, we should have, after making the necessary substitutions in (3.) or (6.), and integrating throughout the whole mass of the body,

$$I_2 = - \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \int_0^{b \cos \Psi} 2\pi \rho r^3 \cos \Psi \, dr d\Psi = \frac{8}{15} \pi \rho a b^4 = \frac{2}{5} M_2 b^2, \quad (9.)$$

a result easily obtained by the ordinary methods.

In finding the moment of inertia of the external shell so as to be able to appreciate the changes which it may undergo, it should be remembered that as geological changes have not the same magnitude or importance at every latitude, a change in the moment of inertia of the whole shell can be found only by considering the changes of the moments of inertia of its different parts.

Let the shell be supposed therefore to consist of a series of zones, on each of which geological changes occur in a comparatively uniform manner, whatever may be the nature or extent of the changes on any of the others. Let $\xi_1, \xi_2, \xi_3, \&c.$ represent the moments of inertia of the zones in the northern

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hemisphere, and $\zeta_1, \zeta_2, \zeta_3$, &c. the moments of inertia of the zones of the southern hemisphere, ξ_1 and ζ_1 being those of the zones in the immediate vicinity of the equator. Then J the moment of inertia of the entire shell will be obtained by the equation

$$J = \xi_1 + \xi_2 + \xi_3 + \&c. + \zeta_1 + \zeta_2 + \zeta_3 + \&c. - I_2; \quad (10.)$$

the external shell being so thin that the variation in its density may be neglected, and its moment of inertia found as if it were homogeneous. Let the zone ξ_1 be included between the latitude ψ_1 and the equator, the zone ξ_2 between ψ_1 and ψ_2 , and so on to the zone ξ_n , included between ψ_{n-1} and ψ_n , or $\frac{\pi}{2}$. Similarly, let ζ_1, ζ_2 and ζ_n , be included respectively between the equator and θ_1, θ and θ_2, θ_{n-1} and θ_n .

The values of the moments of inertia of the zones will be then found by the equations which follow, where A_1, B_1, A_2, B_2 , &c. represent the semi-polar and semi-equatorial axes of the imaginary spheroidal shells which correspond to each zone of the northern hemisphere; and A'_1, B'_1, A'_2, B'_2 have a similar meaning with respect to the zones of the southern hemisphere.

$$\left. \begin{aligned} \xi_1 &= \int_0^{\psi_1} \frac{\pi}{4} \rho A_1 B_1^4 \cos^5 \Psi d\Psi \\ \xi_2 &= \int_{\psi_1}^{\psi_2} \frac{\pi}{4} \rho A_2 B_2^4 \cos^5 \Psi d\Psi \\ &\dots \dots \dots \\ \xi_n &= \int_{\psi_{n-1}}^{\frac{\pi}{2}} \frac{\pi}{4} \rho A_n B_n^4 \cos^5 \Psi d\Psi. \end{aligned} \right\} (11.) \quad \left. \begin{aligned} \zeta_1 &= \int_0^{\theta_1} \frac{\pi}{4} \rho A'_1 B'^4_1 \cos^5 \Psi d\Psi \\ \zeta_2 &= \int_{\psi_1}^{\psi_2} \frac{\pi}{4} \rho A'_2 B'^4_2 \cos^5 \Psi d\Psi \\ &\dots \dots \dots \\ \zeta_m &= \int_{\psi_{n-1}}^{\frac{\pi}{2}} \frac{\pi}{4} \rho A'_n B'^4_n \cos^5 \Psi d\Psi \end{aligned} \right\} (12.)$$

The values of A_1, B_1 , &c.; A'_1, B'_1 , &c., must be found for each zone by the following method.

Let T represent the thickness in feet or miles of a zone of the external shell, Θ the mean latitude of the zone, α the polar semi-axis of the earth, β its equatorial semi-axis, D the depth of the lower surface of the shell below the surface of the sea, R_θ the radius of the earth at Θ , and R the radius of the internal spheroid at the same latitude. Then

$$R = R_\theta - (D - T), \quad R_\theta = \sqrt{(\alpha^2 \sin^2 \Theta + \beta^2 \cos^2 \Theta)}.$$

Let γ correspond to α , η to β , as $D - T$, or G does to R_θ .

$$\text{Then } \alpha \pm \gamma = \frac{\alpha (R_\theta \pm G)}{R_\theta}, \quad \beta \pm \eta = \beta \frac{(R_\theta \pm G)}{R_\theta};$$

$$\text{or, } \left. \begin{aligned} A &= \frac{\alpha (\sqrt{(\alpha^2 \sin^2 \Theta + \beta^2 \cos^2 \Theta)} \pm G)}{\sqrt{(\alpha^2 \sin^2 \Theta + \beta^2 \cos^2 \Theta)}} \\ B &= \frac{\beta (\sqrt{(\alpha^2 \sin^2 \Theta + \beta^2 \cos^2 \Theta)} \pm G)}{\sqrt{(\alpha^2 \sin^2 \Theta + \beta^2 \cos^2 \Theta)}} \end{aligned} \right\} . \quad (13.)$$

A and B representing the general values of the polar and equatorial semi-axes of the shell corresponding to the zone of the shell whose thickness is T.

When investigating the moment of inertia of the shell for any zone, it will be found most convenient to consider the centres of gravity of the parts of the shell, or the surface passing through the centres of gravity of all its parts, as equidistant from its interior surface.

No matter how irregular any zone of the shell may be, its thickness can be considered without much chance of error, as the distance between two imaginary surfaces each of which is equidistant from the surface passing through the centres of gravity of all the parts of the shell. Let the internal surface of the shell be at the mean depth of the sea, the position of its external surface being determined by the distances of the centres of gravity of its parts. Let the entire shell be supposed to consist of an infinite number of pyramidal frusta, which if prolonged would form pyramids meeting at the earth's centre. The height and dimensions of the base of each frustum being infinitely small compared to the height of its entire pyramid, we can without sensible error consider it as a parallelopiped. Let H represent the mean height of the land above the level of the sea, and D the mean depth of the ocean. The height of a parallelopiped whose external surface is on dry land is then H + D, and the distance of its centre of gravity from each of the surfaces of the shell $\frac{1}{2} (H + D)$. Similarly, $\frac{1}{2} D$

represents the distance of the centre of gravity of a parallelopiped whose upper surface forms a part of the surface of the ocean, from the interior surface of the shell. Let Δs represent an indefinitely small portion of the surface of the land, $\Delta s'$ a similar portion of the surface of the sea, δ_1 the density of the land, and δ_2 the density of the ocean. The equation for the value of C_1 , the thickness of the shell, will be—

$$\begin{aligned} C_1 &= \frac{H \Sigma (H + D) \Delta s \delta_1}{\{(H + D) \delta_1 \Sigma \Delta s + \delta_2 D \Sigma \Delta s'\}} + \frac{D (\delta_2 \Sigma \Delta s' + \delta_1 \Sigma \Delta s)}{2 (\Sigma \Delta s + \Sigma \Delta s')} \\ &= \frac{H s (D + H) \delta_1}{\{\delta_1 (H + D) s + \delta_2 D s'\}} + \frac{D (\delta_2 s' + \delta_1 s)}{2 (s' + s)} \quad (14.) \end{aligned}$$

In an examination of the equilibrium of the degrading and elevatory forces on the surface of the earth, it is indifferent

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whether we examine the action of one or both as affecting the earth's rotation, we can therefore examine the change in C_1 either when it is diminished or increased: it will be found most convenient to examine its change in the former case. Of the two principal causes which operate in diminishing C_1 or degrading the level of the land, a knowledge of the intensity of one will suffice for our purposes. We shall therefore proceed to find an equation expressing merely the diminution of C_1 from the transportation of solid matter from the high levels of the parallelopipeds to their lower levels. This course is adopted from the conviction, that however little is our present knowledge of the degradation of land, an accurate estimate of its annual amount can be obtained with more certainty than a knowledge of the subsidences of portions of the earth's crust produced by causes comparatively hidden.

Let h_1, h_2, h_3 , &c. be the heights of any places above the level of the internal surface of the shell, from which are transported in the same time the masses p_1, p_2, p_3 , &c., then by the theory of moments, the result will be in effect the same as if the entire mass $p_1 + p_2 + p_3 + \dots$, or P were carried from the distance

$$U_1 = \frac{h_1 p_1 + h_2 p_2 + h_3 p_3 + \dots}{P} \dots \dots \dots (15.)$$

Similarly, if i_1, i_2, i_3 , &c. represent the heights of any places above the same level to which any masses q_1, q_2, q_3 , &c. are transported, the resulting mechanical action will be in effect the same as if the whole mass $q_1 + q_2 + q_3 + \dots$, or Q were removed to the distance from the internal surface of the shell expressed by the formula

$$U_2 = \frac{i_1 q_1 + i_2 q_2 + i_3 q_3 + \dots}{Q} \dots \dots \dots (16.)$$

After the removal of P from U_1 to U_2 , let C_1 become C_2 , then C_2 will evidently be expressed by the formula

$$C_2 = C_1 - \frac{P}{\mu} (U_1 - U_2); \dots \dots \dots (17.)$$

where $\mu = \delta_1 (H + D) s + \delta_2 D s'$, the mass of the zone of the shell. By means of the equation (14.), we can calculate the thickness of the shell when P is at U_1 ; and its thickness when P is at U_2 , can be obtained by (17.). We can then find G , which being used in the equations (13.), will serve to point out any particular values of A and B . The same process being performed for every zone, the value of J in (10.) will be obtained, and by the final substitution in (2.) of $I_1 + J$ for I , and $I_1 + J'$ for I' (J' being the value of J when C_1 changes to

C_2), we can ascertain the actual change in sidereal time produced by the removal of masses on the surface of the earth from high to low levels. It is evident, that when calculating the value of J' by means of C_2 , the masses of the zones of the shell must not be changed from what they were when obtained from C_1 .

To show that the general equations which have been obtained in the preceding part of this memoir can be applied to the determination of geological questions, the following numerical application of them has been made. This problem would be solved if it could be shown that an appreciable change in the length of the day would be produced by an amount of degradation of the earth's surface, such as would be within the limits of what geological considerations render probable for the interval of time between the periods when the data may have been obtained for determining the change in time.

Let τ represent any appreciable change in the length of the day. Then in the equation $t' = t \frac{I'}{I}$, t' will become $t \pm \tau$, the upper sign being taken when $I' > I$, the lower when $I' < I$, and the former notation being used. Hence $\tau = t \left(1 - \frac{I'}{I}\right)$ in the case now under consideration. But

$$I' = I_1 + I'_2 - I_3, \quad I = I_1 + I_2 - I_3,$$

I_1 representing the moment of inertia of the internal heterogeneous spheroid, $I'_2 - I_3$ the moment of inertia of the external shell with the mean thickness C_2 , and $I_2 - I_3$ the moment of inertia of the shell with the thickness C_1 . The value of τ will therefore be represented by the formula

$$\tau = t \left(\frac{I_2 - I'_2}{I_1 + I_2 - I_3} \right) \dots \dots \dots (18.)$$

If C_1 , C_2 , and D the mean depth of the sea, be supposed the same at every latitude, and if we substitute the values of I_1 , I_2 , I'_2 and I_3 , found upon these suppositions in the above equations, we shall have, when we solve it with respect to C_2 ,

$$C_2 = D - \beta + \sqrt{\frac{1}{k_1} \left\{ k(\alpha - D + C_1)(\beta - D + C_1)^4 - \frac{\tau}{t} \left[K(\alpha - D)(\beta - D)^4 + k((\alpha - D + C_1)(\beta - D + C_1)^4 - (\alpha - D)(\beta - D)^4) \right] \right\}}.$$

But

$$C_2 = C_1 - \frac{P V}{\mu},$$

V representing $U_1 - U_2$, or the mean vertical space through which the mass P is transported. Eliminating C_2 between the last two equations, and we shall obtain

$$P = \frac{SC_1}{V} \left\{ \beta - D + C_1 - \sqrt{(\beta - D + C_1)^2 - \frac{\tau}{t} \left[\frac{K}{k_1} (\alpha - D) \right.} \right. \\ \left. \left. (\beta - D)^4 + (\beta - D + C_1)^2 - \frac{(\alpha - D)(\beta - D)^4}{(\alpha - D + C_1)(\beta - D + C_1)^2} \right] \right\}, \quad (19.)$$

substituting for μ its value SC_1 , S being the surface of the land.

In the above formula, and the first of the two equations expressing the value of C_2 , we have used for brevity the letters k , k_1 and K , for the following expressions:—

$$k = \frac{8}{15} \pi, \quad k_1 = \frac{8}{15} \pi (\alpha - D + C_1) (\beta - D + C_1)^2, \\ K = \frac{6912}{1875 \pi^2} \left\{ \left(\frac{25}{36} \pi^2 - 2 \right) \cot \frac{5}{6} \pi + \frac{2}{\sin \frac{5}{6} \pi} - \frac{5}{3} \pi \right\}.$$

If we assume that the mean height of the land above the level of the sea is equal to the mean depth of the ocean, and for the value of the latter two miles, the surface of the sea twice that of the dry land, and the density of the water half that of the land, the value of C_1 will evidently be two miles. A value of V not very inconsistent with observation would be half a mile; those of α and β are obtained from Mr. Airy's treatise on the Figure of the Earth. We have made

$\tau = \frac{1}{1000}$ of a second, as it has been shown that if the length

of the day had varied since the time of Hipparchus by the one-three-thousandth part of a second, the value of the secular equation of the moon would be changed by more than $4''$.

The sidereal day being represented by t , it follows that $\frac{\tau}{t}$

$= \frac{1}{86400000}$. The value of P calculated from these data will be less than five feet multiplied into the area of all the dry land.

The amount of denudation of the earth's surface represented by the above quantity appears to be within the limits which can be assigned by geological observations.

Dublin, July 9, 1845.

LX. On a Peculiar Method of obtaining the Sesqui-ferrocyanide of Potassium. By Prof. SCHÖNBEIN, of Basle*.

MY experiments on ozone have shown that this substance, like chlorine, is capable of transforming the yellow ferrocyanide of potassium into the red one. The other day

* Communicated by the Author.