ON FERMAT'S NUMBERS.

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[Communicated May 14th, 1903.]

Abstract.

FERMAT'S numbers are of the form $2^{2^n}+1 = F_n$, say. The following results have been obtained in regard to the factorisation of these numbers :—

n	Factors of F_n.	Discoverer.	Year.
0–4	$F_0 \ldots F_4$, all prime.		
5	$2^7 . 5 + 1 = 641$ $2^7 . 62347 + 1$	L. Euler...	1732
6	$2^8 . 9 . 7 . 17 + 1$ ... ... $2^8 . 5 . 52562829149 + 1$...	Landry ... ... Landry and Le Lasseur ...	1880 1880
9	$2^{16} . 37 + 1$	A. E. Western	1903
11	$2^{13} . 3 . 13 + 1$ $2^{13} . 7 . 17 + 1$	A. Cunningham	1899
12	$2^{14} . 7 + 1$ ... ... $2^{16} . 397 + 1$ $2^{16} . 7 . 139 + 1$	E. Lucas and P. Pervouchine A. E. Western	1878 1903
18	$2^{20} . 13 + 1$	A. E. Western	1903
23	$2^{25} . 5 + 1$	P. Pervouchine	1878
36	$2^{39} . 5 + 1$	Seelhoff	1886
38	$2^{41} . 3 + 1$	J. Cullen, A. Cunningham, A. E. and F. J. Western	1903

Abstract of Negative Results.

No more factors (f) of F_n, as follows :—

1. None $< 10^6$; (certified by each of the authors).
2. No $f = 2^a \pm 2^x + 1 < 10^8$.
3. No $f = 2^r . Q + 1 < 9 . 10^6$; $[Q = 1$ to $1220]$.
4. No $f < 1\frac{1}{4}$, $2\frac{1}{2}$, 5, 9 million of F_8, F_9, F_{10}, F_n $(n > 10)$.
5. No $f = 2^{16} . Q + 1 < 10^8$.
6. No $f < 10^8$ of F_n, $(n \not< 14)$.