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LXVI. *On the Dimensions of a Magnetic Pole in the Electrostatic System of Units.* By C. K. WEAD*.

IN the May number of this Magazine, p. 376, Dr. Everett presents Clausius's deduction of the dimensions of a magnetic pole in the electrostatic system, finding them to be $P_e = M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2}$, instead of $M^{\frac{1}{2}} L^{\frac{1}{2}}$, as Maxwell gave it without explicit statement of the method of derivation. In Maxwell's treatise this unit is obtained, along with all the rest, from fifteen simultaneous equations involving twelve unknown quantities; the substitution of the final results in the original equations shows that the suggestion of a misprint may be set aside at once.

In the smaller books that use the clearer method of deriving the dimensions step by step, the development does not usually include magnetic quantities under the electrostatic system: they are not found in the British-Association Reports on Electrical Standards, Reprint, p. 80, nor in Kohlrausch's 'Physical Measurements,' nor in Everett's 'Units and Physical Constants.' Only in Herwig's *Physikalische Begriffe*, p. 78, do I find any values for intensity of a magnetic field, moment of a magnet (strength of pole is involved in this, but not given separately), and magnetic potential in this system. An examination of his method, which leads to Maxwell's results, shows at once that the whole question turns on the step, or steps, between current-strength and magnetic moment. Using so far as possible Everett's notation, but with the subscript letter e or m to distinguish the two systems when necessary, we shall have to consider Strength of current C , Intensity of a magnetic field I , Moment of a magnet μ or PL , Strength of magnetic pole P , Quantity of electricity Q .

Herwig gives $I^e = \frac{2\pi r^2 C_e}{L^3}$; and since $C_e = M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2}$ (agreeing with Everett), $I_e = \frac{C_e}{L} = M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-2}$.

Then, since a magnet whose moment μ in the field I is subject to a couple, $I_e \mu_e =$ a couple,

$$\mu_e, \text{ or } P_e L = \frac{ML^2 T^{-2}}{M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-2}} = M^{\frac{1}{2}} L^{\frac{3}{2}};$$

$$P_e = M^{\frac{1}{2}} L^{\frac{1}{2}};$$

$$V_e = \frac{\text{work}}{P_e} = \frac{ML^{\frac{1}{2}} T^{-2}}{M^{\frac{1}{2}} L^{\frac{1}{2}}} = M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1} = \text{magnetic potential.}$$

* Communicated by the Author.

Clausius gives " $C \times L^2 = P \times L$ in any consistent system ;" so

$$P_e = \frac{C_e L^2}{L} = \frac{L^2 \times M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-2}}{L} = M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2}.$$

The equation $C \times L^2 = P \times L$ leads to $\frac{P}{L^2} = \frac{C}{L}$; which would in the electromagnetic system be consistent, since each member = I. Again, $PL = \mu$; that is, PL must represent a magnet; consequently $C \times L^2$ is put equal to a magnet. But the passage of the current ordinarily produces effects, such as the movement of a galvanometer-needle, which we explain more naturally by saying that the circular current produces a magnetic field at its centre, than by saying that the current is, or makes, or even is equivalent to, a magnet. Herwig's derivation, therefore, in which a magnet placed in a field experiences a couple, conforms to the ordinary way of thinking better than the way of Clausius, and is the way used in the derivation of the electromagnetic system. Again, if

$$P_e = M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2},$$

then

$$I_e P_e L = M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2} \cdot M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2} \cdot L = M L^4 T^{-4} = \text{a couple} \times L^2 T^{-2}.$$

How can this result be explained, consistently with the known effect of a circular current on a magnet? It may be noted that, in all discussions except that of Clausius, the magnet pole P is introduced into a field. Clausius produces a field by the pole P. Also that the ratio of the two values of P is the square of a velocity.

Herwig says, with regard to the step from I to μ in the electrostatic system:—"It may be remarked that for this purpose we cannot use the formula of § 64, $\mu = \pi r^2 C$, which expresses the relation between the magnetic moment μ and the current-strength C; for the validity of this formula is dependent (*geknüpft*) on the use of the magnetic system" (p. 78).

A comparison of the derivation of the same units in the two systems will strengthen our belief in the view, that Clausius has proposed a new way, for which, rather than for the older one, a justification is needed. Taking the least number of steps that will lead from P to Q, or *vice versa*, we have these equations in each system:—

Magnetic,

$$\frac{PP}{L^2} = F (= \text{a force}). \quad PL = \mu; \quad I\mu = \text{a couple}; \quad \frac{CL}{L^2} = I; \\ Q = CT. \quad . \quad . \quad (1)$$

Electrostatic,

$$\frac{QQ}{L^2} = F, \quad \frac{Q}{T} = C; \quad \frac{CL}{L^2} = I; \quad I\mu = \text{a couple}; \quad \mu = PL. \quad (2)$$

Electrostatic (Clausius),

$$\frac{QQ}{L^2} = F, \quad \frac{Q}{T} = C; \quad CL^2 = \mu = PL. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

It is obvious at a glance that, after the fundamental operation has been performed of finding P_m or Q_e , the remaining operations take exactly the opposite order in the two systems; the corresponding equations in the two are the same; while this is not true of series (1) and (3). In (1) we may find the dimensions of Q in terms of P , and from (2) and (3) of P in terms of Q . The results are:—

$$Q_m = ML^{\frac{3}{2}}T^{-1} \div P_m, \quad . \quad . \quad . \quad . \quad . \quad (1')$$

$$P_e = ML^2T^{-1} \div Q_e, \quad . \quad . \quad . \quad . \quad . \quad (2')$$

$$P_e = LT^{-1} \div Q_e. \quad . \quad . \quad . \quad . \quad . \quad (3')$$

Since in the two systems P and Q play similar parts, are we not justified in expecting such a symmetry as is shown by (1') and (2'), rather than the want of it shown in (1') and (3')? If the values of P_m and Q_e , $M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$ are substituted in the above, we may group the results thus:—

$$P_m = M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}, \quad Q_e = M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1};$$

$$Q_m = M^{\frac{1}{2}}L^{\frac{1}{2}}, \quad P_e = M^{\frac{1}{2}}L^{\frac{1}{2}} \quad (2'), \text{ or } M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-2} \quad (3').$$

Of course these are not presented as three independent lines of argument in favour of Maxwell's view, as they are all involved in the series of operations (1), (2), and (3).

It will now be shown that to write " $C \times L^2 = PL$ in any consistent system" is simply begging the whole question.

No physical formulæ are better established than those used in finding with the magnetometer M and H , the moment of a magnet and the horizontal component of the earth's magnetism:—

$$MH = \frac{\pi K}{t^2(1 + \theta)}; \quad \frac{M}{H} = \frac{1}{2} \frac{\gamma^5 \tan \phi - \gamma'^5 \tan \phi'}{\gamma^2 - \gamma'^2};$$

in which K = a moment of inertia = ML^2 , t = a time = T , γ and γ' are lengths = L , and the other quantities in the second members are numerical. In the usual notation, therefore, $\mu I = ML^2T^{-2}$, $\mu \div I = L^3$; whence we find the dimensions of $\mu = M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$, and of $I = M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$, the same as in the magnetic system. But one has the same right to declare that "in any consistent system" both the above formulæ must be

true as that $C \times L^2 = P \times L$. In the magnetic system both are true; in the electrostatic system with Maxwell's value the first is true, the second is not; with Clausius's values the second, and not the first. Maxwell (or at least Herwig) openly uses the first; Clausius impliedly uses the second; for if $CL^2 = PL = \mu$, and $C \div L = I$,

$$(C \div L) \times L^3 = \mu, \therefore \mu \div I = L^3.$$

In deducing the dimensions of physical quantities, there is much that is as arbitrary as the order in which several numbers shall be multiplied together*. Thus, the familiar equation

$$I = k \frac{2\pi r^2 C}{(l^2 + d^2)^{\frac{3}{2}}}$$

is true in any conceivable system of units, where r is the radius of the current-circle, and d the distance from the centre on a normal to the plane. In the electrostatic and electromagnetic systems, the dimensions of either C or I being given to find the other, r , l , and d being lengths, k is arbitrarily made equal to 1, and then I or C is found: if I and C had both been given, ordinarily its value would not be 1, nor its dimensions $= L^0 M^0 T^0$. To pass from I to P or to $PL = \mu$, either of two equations may be used:—

$$I = kP \div l^2; \dots \dots \dots (1)$$

$$\mu I = k' \times \text{a couple.} \dots \dots \dots (2)$$

Clausius arbitrarily makes $k=1$ in (1), letting k' assume whatever concrete value will satisfy (2). Herwig, to obtain Maxwell's result, as arbitrarily makes $k'=1$, paying no attention to k . If arbitrarily we make $k=k'=1$, μ and I must come out as in the magnetic system.

In making $k'=1$ rather than k , there is the advantage of introducing a mechanical unit; and we use the equation (2) that is both more familiar in experimental work, and the one used in the derivation of the magnetic system. Further, if P be changed, three other quantities of the twelve that Maxwell discusses must have their dimensions changed, and confusion would be introduced into his system, that is based on fifteen equations, in each of which the second member is some simple mechanical quantity, as work, time, &c. Until it has been clearly shown how this system will be affected by the proposed change, and why the new expression is to be preferred to the older one, that has been "unimpeached" for some twenty years, is it not clearly better to write $P_e = M^{\frac{1}{2}} L^{\frac{1}{2}}$? It is not a question merely of correctness, but of consistency, simplicity, and usefulness; and on all these grounds Maxwell's expression seems to the writer to deserve the preference.

University of Michigan,
Ann Arbor, May 29, 1882.

* On this point, compare Everett's 'Deschanel,' p. 783.