occurring in the smaller islands. Meyer's story of the whimbrels nesting on trees (probably *Numenius uropygialis*, Gould, by the way—not *N. phæopus*) is quoted, but without comment, and it is worthy of remark that no naturalist has as yet confirmed it. Dr. Hickson is not quite accurate in his statement that there are only two Celebean birds which are likewise English. He must often have noticed, in his rambles along shore, not only the common sandpiper, but also the wide-ranging *Strepsilas interpres* and one or more of the genus *Totanus*, which are not unfamiliar to us at home.

Perhaps one of the best passages in the book is that describing a mangrove-swamp, where the extraordinary conditions of life obtaining within its limits, and the interdependence of that tree and the coral reef, are well illustrated. The scenery of Talisse Island is not particularly beautiful, although the author does not tell us so; but that of the district of Minahassa on the mainland is strikingly lovely, and he describes the view of the Tondano Lake as one without an equal. It was unspoilt to him even by the thought of the "*heerendienst*"—that system of compulsory service which has acted as a red rag to so many Englishmen. Dr. Hickson is not so prejudiced, and is wise enough to recognize—as did Wallace—the enormous advantage which it has conferred upon the people.

"I cannot help thinking," he says (p. 208), "that everyone who is really acquainted with the circumstances of these colonies and the character and condition of the people must admit that it is a service both necessary and just. The Dutch Government has brought to the people of Minahassa not only the blessings of peace and security, but also the possibilities of a very considerable civilization and commercial prosperity. . . . In return for all this, it is only just that every able-bodied man should be compelled to lend a hand in maintaining this happy condition of affairs. In a land where the necessities of life are so easily obtained, . . . it would be impossible for the Government to obtain a sufficient number of them to labour on the roads at a reasonable wage."

The consequence is that they would be neglected. The *heerendienst*, then, as Dr. Hickson shows, is the only system possible, without overburdening the Exchequer, or increasing the taxation beyond the endurance of the people.

We have not space to dwell upon the description of the Sangir Islands, or on the mythology and customs of the natives of Minahassa, which Dr. Hickson has done well to put within the grasp of those who are unacquainted with the Dutch language. Among the folk-lore it is interesting to notice (p. 241) the story of Lumimuüt's impregnation by the west wind—a story which, if we mistake not, is almost identical with one of Egyptian source. The "swan-maiden" tale—which, perhaps, has as wide a distribution over the surface of the globe as any other—again occurs in Celebes. Enough has been said to show that "a naturalist in North Celebes" had a varied interest in his surroundings, which he has contrived to communicate to his readers with success. A little more care, perhaps, would have purged the volume of several misprints, and one or two instances of involved diction.

The woodcuts with which the book is furnished are well enough. We wish that anything could be said in favour of the "process" illustrations. That at p. 33 is bad, and another at p. 137 still worse. But anything more muddy and meaningless than that facing p. 45 we confess never to have seen.

F. H. H. GUILLEMARD.

SAINT-VENANT'S ELASTICAL RESEARCHES.

The Elastical Researches of Barré de Saint-Venant. (Extract from Vol. II. of Todhunter's "History of the Theory of Elasticity.") Edited, for the Syndics of the University Press, by Karl Pearson, M.A., Professor of Applied Mathematics, University College, London. (Cambridge: At the University Press. London: C. J. Clay and Sons. 1889.)

OUR fears lest this "History of the Theory of Elasticity" should, like Thomson and Tait's "Natural Philosophy," remain a magnificent mathematical torso have been agreeably falsified by the early appearance of this instalment of the second volume. It is devoted entirely to the work of Saint-Venant, the distinguished French mathematical engineer.

Saint-Venant is one of the rare examples of a writer who is equally popular with the mere mathematician and with the practical engineer. To quote from the author's preface to this part of the "History of Elasticity," "we live in an age when the physicist awaits with not unreasonable excitement for greater revelations than even those of the past two years about the ether and its atomic offspring; but we live also in an age when the engineer is making huge practical experiments in elasticity, and when true theory is becoming an absolute necessity for him, if his experiments are to be of practical as well as of theoretical value." This is the opinion of the theorist; but the engineer points to his work as magnificent experiments on a gigantic scale, to which he invites the theorist to an inspection, for him to deduce his theoretical laws.

So far as pure theory is concerned, the engineer trusts only to Hooke's law, and Euler's theory of the beam, which neglects the warping of the cross-sections. But Hooke's law is shown by the testing-machine to be only a working hypothesis within very narrow limits of extension and compression, after which the baffling phenomena of plasticity make their appearance, and destroy all the simple mathematical harmony; while as to Euler's theory of the flexure of the beam, the editor, Prof. Pearson, is at present engaged on the mathematical discussion of the permissible limits of the application of the ordinary theory, and, so far, the result of his investigations (in the *Quarterly Journal of Mathematics*) is such as to strike dismay in the heart of the practical man who would be willing to apply his conclusions.

The purely mathematical theory of Elasticity is, at the present moment, in a very curious condition, for a subject in the exact science *par excellence*. Not only are elasticians divided into opposite camps of *multi-constancy* and *rari-constancy*, but we find a war of opinion raging among the most recent investigators—Lord Rayleigh, Chree, Love, Basset, and others. All are compelled to violate apparently the most fundamental rule of mathematical approximation; and, in considering the elasticity of a

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curved plate, to begin by neglecting the terms depending on the stretching of the material, which involve the first power of the thickness of the plate, in comparison with the terms depending on the bending, involving the cube of the thickness; thus apparently neglecting the first power compared with the third power of small quantities. But, if we take a thin sheet of brass or iron in our hands, we shall find it quite easy to bend, but apparently impossible to stretch or shear in its own plane, showing that the stretching stresses may be considered as non-existent, by reason of requiring such large forces to produce them.

Before pure mathematical treatment can make much progress in Elasticity, much more experimental demonstration is required of the behaviour of pieces of metal of mathematical form under given applied forces; and such experiments can be carried out in testing-machines, now forming an indispensable part of a physical laboratory.

Saint-Venant's memoir on torsion, analysed in Section I., is familiar to us through its incorporation by Thomson and Tait, and shows that Saint-Venant carried out, with the comparatively crude methods at his disposal, valuable experiments, from which much theoretical deduction has been made; the analogues of the mathematical analysis in the problem of the torsion of the cylindrical beam of given cross section, and of the flow of viscous liquid through a pipe of the same section, or of the rotational motion of a frictionless liquid filling the cylinder being very striking. Prof. Pearson introduces great elegance and interest into the series which arise by a free use of the notation of hyperbolic functions, and we think there is still some interesting work for pure mathematicians in the identification of those series which are expressible by elliptic functions. But it certainly looks curious to find in § [287] the old familiar polar co-ordinates treated as mere conjugate functions, without reference to their geometrical interpretation.

Section II. is occupied with the analysis of Saint-Venant's memoirs of 1854 to 1864, in which he attacks such questions in practical elasticity as the longitudinal impact of bars, illustrated by very ingenious graphic diagrams, and also the conditions of stress of a cylindrical shell, in equilibrium under given applied internal and external pressures. This is the problem required in the scientific design of modern built-up artillery; and it is noticeable that Saint-Venant's solution differs materially from Lamé's, subsequently popularized by Rankine, the theory employed, as far as it will go, by scientific gun-designers all over the world.

The researches in technical Elasticity of Section III. arose in the annotations of Navier's "Leçons sur la Résistance des Corps solides"; the mantle of Navier descended on the shoulders of Saint-Venant, and ultimately the notes of Saint-Venant overwhelmed the original text of his master Navier; and, according to Section IV., Saint-Venant has practically done the same thing with Clebsch's "Elasticität."

Being the mathematical referee for all the difficult theoretical problems arising with the extensive use of the new materials iron and steel in architecture and engineering, Saint-Venant was provided with a number of useful problems on which to exercise his ingenuity; such as the impact of bars, the flexure of beams due to a falling weight or a travelling load, the critically dangerous speeds of fly-wheels and piston-rods, and so on; all problems hitherto solved by practical rule of thumb, the practical constructor encountering and opposing the difficulties without knowing why and how they arose.

Saint-Venant's investigations urgently need extension and application to the critically dangerous conditions which can arise in the stresses in artillery, when the dynamical phenomena are analysed, due to the sudden and periodic application of the powder pressure, and to the wave-like propagation and reflection of the stresses in the material. At present, we can only investigate the theoretical strain set up in the material of the gun by a steady hydrostatic pressure equal to the maximum pressure of the powder, employing Lamé's formulas, and then employ an arbitrary factor of safety, say 10, in the design of the gun, to provide against the contingencies of the dynamical phenomena we have not yet learnt how to discuss.

In the old times, before the Cambridge Mathematical Tripos was reduced to its present meagre curriculum, the examiner would have found the present volume very useful in suggesting good ideas, capable of testing reasonably the mathematical power of the candidates; at present, the chief class to profit by the present work are the practical constructors, who will learn where to look for the useful information on the narrow technical point which concerns them.

Prof. Pearson has brought his onerous task one step nearer to completion in this interesting volume, a monument of painstaking energy and enthusiasm.

A. G. GREENHILL.

GLOBES.

Hues's Treatise on the Globes (1592). Edited by Clements R. Markham, C.B., F.R.S. (London: Reprinted by the Hakluyt Society, 1889.)

THE Hakluyt Society has for its object the reprinting of rare or unpublished voyages and travels, and few are worthier of this honour than the "Tractatus de Globis" of Robert Hues. The author of this work was an intimate friend of Sir Walter Raleigh, and combined book-learning with practical knowledge gained by joining in some of the voyages to the New World with navigators whose names have made the sixteenth century famous. He strongly urged that his countrymen would have still further surpassed their Spanish and Portuguese rivals if they had "but taken along with them a very reasonable competency and skill in geometry and astronomy." In those days logarithms were unknown, and the solution of the problems of nautical astronomy required advanced mathematical knowledge. It was hoped that this difficulty would be overcome by the extended use of globes, which at once reduces these complex questions to approximate solution by inspection. After the construction of the Molyneux globes, Hues's treatise came into very general use, and no doubt played an important part in the explorations of the succeeding century.

It seems strange in these days, when a globe can be purchased for a few shillings, to read that only three centuries ago the construction of globes entailed such great expense that the liberal patronage of a merchant

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