

IV.—THE PRINCIPLE OF INDUCTION.

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IN MIND No. 58 I discussed the value of the Experimental Methods, and came to the conclusion that the Method of Difference supplies us with a type of valid inference from the particular to the general. But this conclusion raises a more general question. "On what principle," it may be asked, "can you ever generalise from a true particular?" Such a process must always take you beyond your premisses, whereas true reasoning must only elicit what is already implied in the premisses. And it may be said that the objection so raised is admitted rather than answered by Mill. It is true that he proposes the Law of Causation as the ultimate principle of Inductive argument; but, when we come farther to inquire into the grounds of this Law itself, we find that it rests on a form of generalisation which undoubtedly does involve a "leap" from known to unknown, and the very nature of which, on Mill's own showing, is to proceed on no fixed or definite principle at all.

On the view which I propose in this and in a following article, all inference involves a generalisation from observed particular cases. But definite principles may be laid down on which such generalisation proceeds. The nature of reasoning, then, consisting in the assertion or application of general truths upon these principles, such generalisation involves not a "leap" unwarranted by the premisses but a regular process from the known to the unknown. Such "advance" I take to be an essential part of the reasoning process.

In the present article I shall try to find the axiom on which inductive inferences proceed. In the following article I hope to discuss the principles of deductive inference and the relation between the two forms of reasoning.

The ultimate major premiss of Induction according to Mill is, we have seen, the Law of Causation. But this Law, as he treats it, is not so much a principle tacitly or openly implied wherever we draw inductive inferences, as a wide generalisation true of sequences just as other generalisations are true of the facts of space. Hence, further, it is itself an induction like other inductions. What we want on the

other hand is an axiom expressing in general terms what we do when we make a particular statement universal, which makes explicit the truth implied by the making of any generalisation whatever, and which thus, so to say, generalises generalisation. The Law of Causation will, I think, be found to be a particular application of this wider axiom, and the axiom itself must be sought from the analysis of ordinary simple generalisations.

Now, when I connect truths together, or reason, what do I do that I leave undone in judgment? I *support* my inferred judgment by some other assertion. If I say A will —B,¹ and am asked why I say so, I answer because A was —B. If I say the clear sunset this evening will be followed by a fine day to-morrow, I give some proof of my assertion when I adduce the clear sunset of yesterday and the fineness of to-day. Now, I may be answered by a doubter upon two lines. He may say A_1 and A_2 are not really alike. Yesterday's sunset was clear in sense of cloudless, to-day's in the sense that the air is transparent. Or we may say: Yes, A_1 and A_2 are alike, but there is something beyond these which makes the difference. With yesterday's sunset went (say) a certain electrical condition of the air, and it was that which really determined the fine day. That condition is not present now. This gives us roughly the two conditions of inference, which we have now to define further.

1. A_1 and A_2 must be alike. I use the notation $A_1 A_2$ to express that they are different facts, observed, that is, at different times or places, but that in character they are precisely similar. I say precisely, because it is only so far as they are similar that I have any basis for inference. It may be that I never get precise similarity, but I do find *points* of precise similarity, and it is from these that I argue. The terms I use in describing a fact always allow a certain latitude. I call many different shades of colour red. But the more latitude is allowed, the more difficult it is to argue with precision. If I can argue at all from one red to another, it must be because just in point of redness there is no difference between them; they are both equally red; in that point they are precisely similar. Argument, then, is precise in proportion as similarity is precise.

We may, if we like, use the word "same" to express this precise similarity. But if we do, we must observe that we intend by it something quite different from the sameness of

¹ I use the symbol — to express any sort of relation between two terms.

an individual with itself. I am the same man that I was five years ago, in one sense of the word ; that is, there is a *continuity* in my existence. I am not the same man as I was then in the sense of being precisely similar, for propositions true of me then are no longer true of me now. I *cannot* infer at once from a past attribute of myself to a present one. On the other hand, the blue which I see in the picture is the same tint that I see in the sky. They are, in point of mere colour, precisely similar. They are not continuous, and one might disappear and leave the other. But what is true of the one colour as such is true of the other.

It may be asked how I can reason from one thing to another when they are not the same? That is precisely what I wish to show. I take this "advance" to be essential to real inference, and my present aim is to prove that it is made upon a single definite principle.

2. But though A_1 and A_2 are precisely similar, there may be some change in the concomitants of A , outside A . This change, again, may or may not affect B . When I infer A_2-B_2 then, I assume either that there is no such change, or that no change outside A makes any difference to B . We will consider presently what we mean by "making a difference".

3. Observe now the implication of inference. If I do argue from A_1-B_1 to A_2-B_2 I imply that $A-B$ holds always ; that given an A we shall always have a B in the same relation to it. This, of course, is the point always brought out by cross-examination:—"You think *laissez-faire* best in this case—do you think it *always* the best thing?" The implication is that, if not, you must be prepared to adduce that circumstance in the case which makes *laissez-faire* the true policy here ; and this circumstance must be one which always makes it the best policy, unless, again, to take a further complication, there are special circumstances which always make in the opposite direction. Without pursuing such complication further, we see that in arguing from A_1-B_1 to A_2-B_2 we commit ourselves to the assertion A always $-B$, or Any $A-B$.

And we can, in fact, always argue from A_1-B_1 to A_2-B_2 unless there is some change in the concomitant parts which makes a difference. This formula holds of any sort of inference, from the barest analogy upwards ; only, in the case of a mere analogy we have really no sort of ground for supposing that there will not be some change which "makes a difference". If I argue:—"X sat down thirteenth at table

and died within the year; you have done the same; therefore you will die,"—I pay no attention whatever to the concomitant facts. X may have been in a consumption. The consumption then is the fact that makes the difference. It was the consumption which produced X's death, and having assigned that as the cause, and discovered that it is not present here, I have no ground for the conclusion. But it remains that there must be some such fact discoverable; or otherwise the inference from A to B will hold universally. The fact in question may be something of which A is really a part, or it may be something quite separable from A, or it may be the absence of counteracting causes, or, to phrase it differently, the presence of conditions which are neutral to the effect. We will go further into these cases presently. Meanwhile we must observe that the phrase "makes a difference" requires further analysis. Such an expression involves some activity or causative power. This is really a specific conception, and we want one that is general. What we really mean when we say there must be some change which makes the difference, is that there must have been some fact which is always connected with the consequent, and which is not present now; in other words, if A is not always in the relation which we observed between it and B, then there was along with A some third fact C, which does always go with B but not always with A. As I said above, this third fact may bear any sort of relation to A: it need not be entirely separate from A, but involves more than A pure and simple. We now see that, whether we accept or reject an inference, we make the same assumption of the universality of relations and no other. In the one case we assume A—B universal, in the other some C—B.

There are then two conditions of inference: (1) that A and A₁ should be alike, and (2) that there should be no third C, other than A and the universal correlates of A, which is always in a definite relation to B. And there is a single implication in inference, namely, that the relation we are now asserting holds always. We thus see that the conditions of inference and the implication of inference rest all on one principle. *B must have some fact with which it is always in relation.* That fact may be A, and if so we can infer from A to B; but it may be C, in which case we cannot infer from A to B. It may be that A which struck us in connexion with B is the fact always related to B. If not, it is some other fact. There always is some such fact to be found. Thus in inferring to A₂—B₂ I imply A always —B; and that again

implies that there is no C always —B which is not itself always related to A.

That we should be able to reason at all, then, involves that any fact, as B, should have some other fact, as C, to which it is always related. By this is meant that any fact precisely resembling this B, whatever its other attributes and concomitants may be, will be found in a precisely similar relation to a precisely similar C. It does not involve that any A to which B happens to be related here should be always related to B. And hence the proposition which is to hold good of any two facts whatever, that are observed in any relation, must present us with an alternative. Either the relation observed holds always, or there is some other fact present in the observed case always related with one of our two facts and not with the other. Now, when we draw an inference, it is implied that the relation asserted holds always, and we see that this implies the absence of any such other fact. Hence we may put the axiom of Reasoning thus :—

If a fact A_1 is observed in any relation to a fact B_1 , then any A will be in that relation to B, unless among the facts in relation to the B observed there was some fact other than A alone which is always in that relation to B in which it stood in the observed case but does not always stand in the relation to A in which it then stood.

I have here put the axiom as if A were the fact presented to us in some second case. But obviously the order makes no difference. If it were B that were presented to us we could say just the same of A.

My object in putting the axiom thus is to phrase it so that it may hold of any sort of fact, and by "fact" I mean anything that strikes our attention, and that we speak of as a fact, whether we bring it into a unity naturally or artificially. However much or however little of the "work of the mind" there may be in it, whether it be a substance, or a well-defined attribute, or the first rough apprehension of an attribute, or a statement involving a complicated system of parts, I understand this judgment to hold true of it. The word "relation" also needs a little explanation. As I use it here, I mean to assert it not only of a fact that is before or after another, or near or far from another, or like or unlike another, but also of a fact which is an attribute of another which is its substance, or which is conjoined with another as a second attribute of the same substance, or as a second aspect of a complex mass of facts. I hold a relation to exist between two facts whenever the mind can at once

distinguish the facts as two, and at the same time attend to them together and assert something of them considered together.

To illustrate my meaning, let the observed relation be exposure to cold followed by inflammation on the lungs. Here A is exposure to cold. Now I am quite aware that such a fact as this cannot exist in isolation. It was of course a particular concrete case of the exposure to cold of a particular person. Quite so; but *all we may know of it* may be quite adequately represented by the bare words "exposure of a man to cold". Of course the more I know of it the better for my powers of drawing inferences, but as soon as I begin to know such a bare fact as those words express I begin to have some basis for reasoning. The same remarks apply to the term B. Hence without knowing anything more of A and B than is expressed by such words as are used above, and the fact that B did follow A, I can say that in this case again, or in any case, B will follow unless in the first case B was related to some third, C. Now this third, C, might be something quite apart from A: it might be, for instance, the continual inhaling of iron dust; or again, it might involve A and something more, *e.g.*, it might be exposure to cold following great heat and in an exhausted condition on the part of a man with weak lungs. To get at the whole fact which would really and strictly be *always* followed by inflammation of the lungs, we should doubtless have to go through something very complex. But in the broad sense I have given to the word "fact," with the object of abbreviating the formula, it would hold that *some* fact could be found always related to the fact inquired about.

Let us take another case: a pistol-shot, A, caused death, B. Now a pistol-shot might not cause death. What does cause death? Let us say a projectile aimed in one or other of certain definite directions and with not less than a certain energy. If I shoot a man and aim straight at his brain or his heart and am near enough for the ball to penetrate, I shall kill him. Thus I can find a fact standing in universal relation to my B. But it is not something out of all relation to A. A, the pistol-shot, is a vague phrase expressing one aspect of the whole fact—the aspect which would first strike a bystander. The C which is really connected with B involves A and something more. The whole fact can be analysed of course into any number of "latent processes," and again has any number of aspects. Now A is just that

aspect which happens to have struck us. C is here some fuller account of the whole fact.

Again, "This oxygen has an atomic weight of nearly 16. Any oxygen will have the same." The assumption here is that there is no further fact to be taken into account. It is merely as oxygen that the substance has the assigned atomic weight. There is nothing that can make a difference, and the relation must always hold. And this illustrates the way in which I wish the term "relation" to be understood in the formula above given. I speak of a *relation* existing between oxygen and its atomic weight. Of course I do not mean that they are in any way separate existences, but they are different aspects of the same existing thing, and may come before us at different times. We may thus speak of a relation of coherence between them, knowing at the same time that such a coherence constitutes the whole thing a unity.

In a purely frivolous or false inference, the C which is really in relation to B is something quite foreign to A. If, to take Grote's instance, I assert that on the day of the battle of Salamis rain fell on the site of New York, and if I were to go further and say that therefore if another battle were fought at Salamis rain would again fall on that site, the obvious answer is that the rain depended on meteorological conditions quite foreign to the political causes which led up to Salamis, *i.e.*, there is a third C totally disconnected with A. If, again, because the battle of Himera was fought on the same day as Salamis or at least about the same time, I were to expect a similar conjunction to repeat itself, I should be very far out, though not quite so far as before. For the conjunction of Himera and Salamis may have been remotely due to some correspondence between Persia and Carthage. The cause of Himera, then, was remote from that of Salamis, but not, so to say, infinitely remote. I should have to insert a great number of links connecting my A and my C before I could infer universally. I should have to observe the repetition of a great number of military and political conditions.

If we now develop our axiom for a while, we shall see that certain broad cases of its application may be distinguished, and we may with advantage restate it in view of them. The first and most important distinction is between cases where the third fact C, the fact that is always accompanied by B, is a fact which includes A or is closely related to A, and those in which it is not. The first case will give us good ground for inference as a general rule,

though it will not give absolute certainty. The second allows us no ground for inference at all.

The first case would be represented in the instance above given by the man who caught inflammation from exposure under special circumstances; the second, by the man who was indeed exposed to cold but in fact caught his illness from inhaling iron dust. A still better instance of the first case is afforded by mechanical pressure. Let a force P act on a body at A in the direction AB ; we may infer that the body will begin to move in the direction AB unless there are some other forces acting on it in the contrary direction. In other words, the case of inference which we are now considering is that in which counteracting causes are possible; in which we have one or more main determinants of the fact B , but the whole complex of conditions is not given us. Restating the axiom from this point of view, we get a double distinction.

If A_1-B_1 , then

Either any $A-B$, as *e.g.*, A_2-B_2 ,

Or with A was some third C which always $-B$,

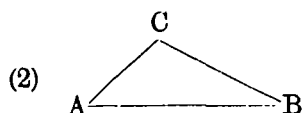
Or with A_2 will be some third D which always —
absence of B .

Before showing how the axiom in this form can be brought into relation with the Inductive Methods, I want to meet in advance a criticism which might be passed upon it as here given. It may be said my symbol ($-$) is so vague that it would comprehend any relation in the universe; or, if for it I substitute "coexists with or is followed by," that I am not much better off unless I specify the limits of space or time within which the coexistence or sequence must take place if it is to affect our inference. In other words, such an expression as "in the case observed there was a third fact C " is either so vague as to be of no service, or else implies that I have already isolated a certain group of facts from the surrounding universe and know that I need not consider the rest. To escape from this dilemma, it is, I think, only necessary to state the axiom with greater accuracy, if also more clumsily:—

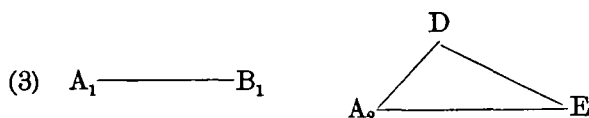
If a fact A_1 is observed in any relation to another fact B_1 , then either any other A_2 will be in the same relation to B_2 ; or with A_1 and B_1 was a third fact C which always stands in the relation to a B in which it there stood; or in relation to A_2 there is a third fact D which in such a relation to A is always so related to a fact E , unlike B , that E will occupy a relation to A_2 similar to that occupied by B_1 to A_1 in the case observed.

To illustrate this complicated statement, let us use a graphic method representing a relation now by a straight line of definite length and direction. We have :—

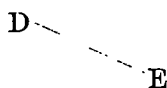
$$(1) \quad A_1 \text{ ————— } B_1, \quad A_2 \text{ ————— } B_2$$



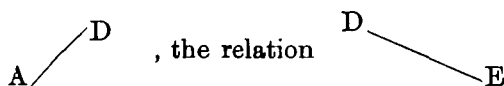
Here the relation C—B holds always, but we know nothing of A—C. Therefore, given A again, we have no reason to infer B.



Here we should have A ————— B if it were not for D, but



is a relation which *always* holds, hence, if we have the relation



will bring E into the precise place previously occupied by B. For instance, supposing A is a force acting on a body in a given direction, and B is the resulting movement when A acts alone, then D would be an equal and opposite force. Now if D does not act on the same body at the same time it will not, of course, counteract A ; but if it does so act the result will be equilibrium, E, instead of the movement B. Then to secure true counteraction of B, D must be in a definite relation to A : it must act at the same moment on the same body.

I have, I think, said enough to show that the three alternatives afforded by the Axiom as thus stated correspond to the three cases in which A is the "sum of the conditions of B," or in any way a universal correlate of B ; in which it is the cause of B in the popular sense of the term ; and in which its connexion with B is merely "casual". In other

words, the Law of Causation is the Axiom of Reasoning as applied to the sequences of phenomena.

I have not space to exhibit the connexion of the Axiom with many forms of reasoning, but I wish to show its application to one form in particular—the Method of Difference.

How does the Axiom help us in dealing with the Method of Difference? In a very simple way; the method proves that there is no third C universally related to B. We have, under a rigid application of the method,

Acdecde.

Bfghfgh.

Now this proves that none of the concomitants of A (*viz.*, *cde*) are universally related to B. In other words, we have a case in which A is followed by B without the presence of any third C, which is always related to B. Hence A will always be followed by B unless we have counteracting causes. The Method of Difference, then, is simply a particular application of our Axiom.

But we have to remember Mr. Bradley's exposition of the defects of the Method. It might be that, though *cde* would not produce B without A, A in its turn would not produce B without them, or some of them. Then it would be true that there would be no third C altogether apart from A which would produce B, and yet not true that A would always produce B. To save our Axiom, we should have to regard the absence of *cde* as a counteracting cause. But this would generally be very far-fetched, and an unfruitful way of looking at it. Let us then modify the second alternative of our axiom, thus:—

Either A will always be in that relation to B, or there is a third C which either is always in the relation which it here holds to B, or whenever it is in the relation which it here holds to A is in the relation which it here holds to B.

This statement gives us the distinction popularly drawn—and, like most popular distinctions, resting on a fundamental truth—between cause and condition. The first subdivision of my second alternative makes C the cause of B, and leaves A nothing to do with it. It is this which is eliminated by a single application of the Method of Difference. The second subdivision leaves it open for C to be a joint condition of B, together with A. If C is not really such, that may be proved by repeated applications of the Method. Thus, summing up, one good instance under the Method proves that A is followed by B without being accompanied by any third fact

which is always followed by B, whether or not A is present. Repeated instances may prove that A is followed by B, whether or not any other conditions are present.

I can argue then from any fact of observation, provided that fact gives me in some way or other the means of sifting it. But I can do this, because any and every fact observed stands in universal relation to some other fact. I start from particulars, and I reason about them, but the reasoning itself is the assertion of a universal. It is the judgment that certain facts are always in such and such a relation. This judgment is implied in the rudimentary inference which states only the particular fact observed and the particular fact now expected. It is explicit in the reason that is conscious of its own grounds and methods, and takes there the form of the universal judgment, or major premiss.

I may be asked, "How can we ever outside mathematics attain to propositions so rigidly universal? There is always a chance that we may be deceived." There is. And to go into the whole question of Chance, and the relation of its assumptions to the assumption of reason, is out of my power here. At present, I can only repeat what I have said before of chance—that, though, in fact, chance does interfere with our reasoning, still reasoning can and does go on, and I am concerned here with the implications of reasoning alone. The "simple enumerations" on which most of our "judgments of allness" depend rest, of course, on the Law of Chances. They are to be accepted so far as the instances taken have been diverse, numerous, and "random" enough to eliminate chance. In fact the statement of allness according to our Axiom would mean primarily that we had reason to believe that there was no third factor in the instances observed that was responsible for the relation found in them. The assumption of a Simple Enumeration is that we have observed A and B together so many times that the chances are against the combination being casual—that is, due to some third C. The repetition of the instance guarding us more or less (according to the nature of the instances) against this, we are enabled to draw an inference.

Leaving, for the present, the question of chance, I must say a word in conclusion on the question of the proof of our Axiom. Mill held that the Law of Causation could be proved by a simple enumeration. Now, his law of causation is the axiom of reasoning as applied to sequence. Let us then

apply his proof to our axiom. The axiom holds good in cases *a, b, c, d . . .*, therefore it holds good in all cases. Therefore? Why? Because that which holds good in many cases will hold good in all. But that is precisely what our axiom asserts. Our axiom is, that what holds once will hold always, unless something makes a difference. We have then used our axiom as major and conclusion in the same syllogism.

If there is any axiom involved in reasoning it can never be proved by reasoning. The only kind of test of which it is susceptible is the test of self-consistency in all its applications. If we could find a case in which it could be proved not to hold, it is true that we should be using the axiom itself to prove its own nullity, but still we should be bringing ourselves by that very fact to an inconsistency, and should be certain that somehow we had got wrong. On the other hand, so far as we find it applying and "coming out right," we get a certain test of its truth. And this should be carefully noticed, that it is one thing to speak of proving the Axiom of Induction, and another to show that a particular formula is that axiom. There we are liable to mistake, and therefore we not only can but must attempt to prove. We cannot set up dogmas of our own and say these are the axioms of reasoning, and therefore above proof. We must prove that they are in fact the statement of that which is implied in reasoning. I have tried to do this by the analysis of some simple inferences, and my main conclusion is that a definite principle can be laid down on which we form our generalisations from the particular cases that we observe.