SOME MEASUREMENTS OF THE TEMPERATURE VARIATION IN THE ELECTRICAL RESISTANCE OF A SAMPLE OF COPPER.[1]

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I.

PRECISION in the determination of the temperature variation of resistance in copper is important not only to electrical science, but also to its applications. Our estimate of the temperature of remote or inaccessible positions, as, for example, the ocean bed on which a submarine cable lies stretched, or the interior layers of a dynamo armature winding, are often directly dependent for their accuracy upon the completeness of our knowledge of this temperature coefficient.

Electrical text-books, in stating the temperature coefficients of copper, usually quote the results of Dr. Matthiessen, or of Dr. Siemens, or both. The results of these two authorities are discordant.

Within the range of chamber temperatures, say from 0° C. to 35° C., the difference between these results is practically of little importance. Taking, however, the resistivity of copper at zero centigrade as unity, its resistivity at 100° C. is 1.422 by Matthiessen's observations, and 1.388 by those of Siemens, a variation of nearly $2\frac{1}{2}$ per cent, while above 100° C., or below 0° C., this discrepancy increases rapidly.

First in order of date are the elaborate researches of Matthiessen (and of his collaborator Von Bose), appearing in the *Philosophical Magazine* for February, 1857, and February, 1861, also in the *Philosophical Transactions* for 1858, 1860, 1862, and 1864, the most important series from our present standpoint being those for 1862. The wires tested were varnished with shellac,

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and immersed in a bath of oil whose temperature was raised by the application of Bunsen burners, and read off by an immersed thermometer. Readings were taken both in ascending and descending series of temperature, and observations are adduced in support of the statements that the application of varnish did not affect the results, and that the observations on a wire heated in a bath of oil were sensibly the same as when the heating took place in air. Matthiessen took six copper wires, all from the same electrolytic source, three annealed and three hard-drawn. Six observations are given of the resistances of each between 0° and 100° C. Having ascertained that all 36 observations accorded very fairly with a parabolic relation between conductivity and temperature, the parabola of closest conformity computed by the method of least squares was $\lambda = 1 - at + bt^2$, or numerically $\lambda = 1 - 0.0038701\, t + 0.000009009\, t^2$, λ being the conductivity at temperature $t°$ C. From this equation, the resistivity (the reciprocal of λ), retaining the terms necessary for accuracy in the fifth digit, at the limit of 100° C. becomes,

$$\rho = 1 + 3.8701\, t \times 10^{-3} + 5.968\, t^2 \times 10^{-6} - 1.177\, t^3 \times 10^{-8} - 9.93\, t^4 \times 10^{-11} - 2.769\, t^5 \times 10^{-13}.$$

The graph of this equation, taking values of ρ as vertical ordinates from a horizontal axis of temperatures as abscissæ, yields a curve bending upwards, so that the temperature variation increases with the temperature, the increase in resistance per degree centigrade being at 0° C. 0.387 per cent, and at 100° C. 0.50 per cent of the resistivity at zero C. Matthiessen points out that this bending upwards of the curve is distinctly indicated by his results, and that no straight line can represent them. Not only the 36 observations on copper wires, but more than 200 quoted observations on other metals, all point to a temperature coefficient augmenting with temperature, and negative the supposition of a simple linear relationship between the variables.

Dr. Siemens' researches formed the subject of his Bakerian lecture in 1871. They were undertaken with the object of obtaining a practically reliable scale for the electropyrometer which bears his name, rather than for the direct purposes of scientific research. After pointing out that Matthiessen's formula can on its own evidence be only fairly applicable between the limits of 0° and 100° C., Dr. Siemens proceeds to advance some interesting, although arbitrary, hypotheses for the law of temperature variation in metallic resistances, and then shows that his experimental observations on copper and four other metals are capable of close representation by the empirical formula so obtained. These observations were made on wires heated in air, and also in oil, up to 350° C. with mercury thermometers, and in one series up to 850° C. with a platinum ball pyrometer, an instrument whose indications assumed a constant specific heat in the platinum ball throughout the range of temperature employed.

Dr. Siemens' formula for copper is $\rho = 0.026577\sqrt{T} + 0.0031443\,T - 0.29751$, where ρ is the ratio of the resistivity at any absolute temperature, T, to that at zero centigrade, or $T = 273°$. For 100° C., or $T = 383°$, $\rho = 1.3885$, and the rate of increase of resistivity is at 0° C. 0.394 per cent, and at 100° C. 0.383 per cent of the resistivity at zero C. The graph of the equation is a curve bending slowly downwards towards the axis of temperatures, and the temperature variation diminishes as the temperature rises. All the 170 observations recorded in the paper indicate that the curve bends downward, while all the 250 observations in Matthiessen's 1862 paper make the curve bend upwards. The discrepancy between these two series of results becomes very noticeable between 70° and 100° C.

Professors Dewar and Fleming have published in the *Philosophical Magazine* for October, 1892, a number of observations on the resistivity of metals and alloys at temperatures between $-200°$ C. and $+100°$ C., one series for copper being included. The resistivity of the copper wire is stated to have been 1353 C. G. S. U. at 0.7° C., equivalent to 1349 at 0° C.; and since Matthiessen's standard is 1594 at zero C., this represents a conductivity 18 per cent higher than Matthiessen's standard. Aside, however, from this remarkable and perhaps debatable statement, the graph of the observed values of resistance with respect to temperature is very nearly a straight line throughout the whole range, the resistivity at 100° C. being 1.424 times greater than that at 0° C., and the temperature coefficient being approximately 0.424 per cent per degree centigrade for all temperatures between $-200°$ and $+100°$ C.

Messrs. Cailletet and Bouty, in the *Comptes Rendus* for 1885, give an observed temperature coefficient for copper of 0.418 per cent per degree C., expressed by $\rho_t = \rho_0\ (1 + 0.00418\,t)$ between zero and $-58°$ C., and 0.425 per cent from $-69°$ to $123°$ C., $\rho_t = \rho_0\ (1 + 0.00425\,t)$.

In Poggendorf's *Annalen* for June, 1858, Herr Arndtsen quotes a uniform temperature coefficient of 0.369 per cent per degree from zero to 200° C. He gives, however, one series of observed resistance with a copper wire (containing 0.15 per cent of iron) between 0° and 100° C., showing a linear relation, or a temperature coefficient of 0.394 per cent per degree C., $\rho_t = \rho_0\ (1 + 0.00394\,t)$, and he points out that the divergences from the straight line are within the limits of the observation error.

In view of the discrepancies existing between these best known measurements of the temperature coefficient of copper, Arndtsen, Cailletet, and Bouty giving results practically represented by straight lines, Siemens' results with the line bending distinctly downward and Matthiessen's results with the line bending as distinctly upward, the writers of this paper made a number of measurements in the spring of 1890 upon a sample of copper wire. These measurements were made with great care, and repeated until similar results were obtained in successive series. The wire tested was sealed within the bulb of an air thermometer, so that there could be no appreciable variation between the temperature of the wire itself and the temperature indicated by the pressure of the air in the bulb it occupied. The final results, after full corrections for expansion of the bulb, etc., indicate a linear relation between the resistance and temperature of the wire between the limits of 20° C. and 250° C., represented by the equation $\rho_t = \rho_0\ (1 + 0.00406\,t)$, indicating a uniform temperature coefficient of 0.406 per cent per degree C. throughout the range, the maximum observed being 0.4097 per cent, and the minimum 0.399 per cent at any point. The details of these measurements are here submitted in the form of an appendix, not only in support of the statements made concerning them, but also because the experimental arrangements of apparatus finally successful were the outcome of a series of experimental failures, and it is believed that the details of construction may be of service to those who desire to adopt the same method of measurement.

Concerning the conclusions that may be drawn, we feel only justified in saying that good commercial copper can be found in which a linear relationship holds between resistance and temperature between 20° C. and 250° C., and within the range of small observation errors. It is, of course, possible that in different samples of wire the temperature coefficient may increase or diminish with the temperature. In other words, the second differential coefficient of resistance with respect to temperature may perhaps in some samples be either positive or negative, but it seems desirable that fresh measurements should be made, and evidence collected to settle this point, and we submit the view that the best experimental means of measuring the temperature, and to ensure the coincidence between the measured temperature and that of the tested wire, is to enclose the latter in the bulb of an air thermometer in the manner here described.

II.

General Outline of Apparatus.

Within the cylindrical glass tube of an air thermometer was enclosed about 240 cm. of fine copper wire. Short platinum wires, sealed into the glass bulb, brought this copper wire into communication with apparatus for measuring its resistance. The bulb rested in an oil bath heated electrically, and the height of a mercury column required to balance the pressure of the internal air was measured by cathetometer, at the moment that the resistance of the copper wire was noted. The bulb was thus operated as an air thermometer at constant volume, and corrections were applied to the expansion of the glass walls of the bulb, and for the variation of barometric pressure during the period of observations; also electrically for the resistance of the leads up to and including the platinum seals.

Thermometer Bulbs and Contents.

A vertical section of the thermometer bulb and its accessories is shown to approximately one-sixth true scale in Fig. 1. *AB* is the cylindrical glass bulb 15 cm. long, and 3.15 cm. external, 2.85 cm. internal diameter. The capacity of the chamber so enclosed was approximately 75 c.c. Three separate platinum wires, 2 cm. long and 0.048 cm. in diameter, were sealed in at *D*, and welded with three exterior copper wires, *E*, *F*, and *G*, forming the leads to the apparatus. The central wire was also welded within the bulb to a copper wire, 10 cm. long and 0.08 cm. in diameter, and the seal supported this wire axially along the bulb to its termination at *H*.

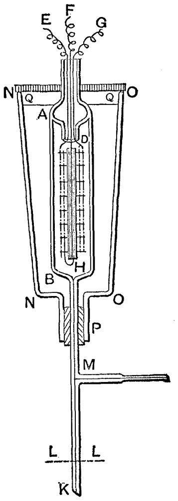

Fig. 1.

This copper wire *DH* was entirely covered by a glass tube, 10 cm. long and 0.25 cm. in external diameter, closely threaded, on which were beads or short cylinders cut from glass tubing, each bead being 1 cm. long, 0.28 cm. internal and 0.4 cm. external diameter. The beads served as sleeves or washers to clamp between them at their junctions circular disks of mica, 2.15 cm. in diameter and 0.02 cm. thick, with a hole in the centre, 0.25 cm. in diameter. Two circles of holes were drilled concentrically around this disk, each circle having fifteen holes. The diameters of these circles were 1.0 cm. and 1.65 cm. respectively. A plan view of a mica disk is shown in Fig. 2, to full scale.

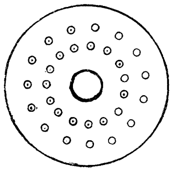
Fig. 2.

The copper wire tested was of good commercial quality, 0.002″ (0.0051 cm.) in diameter, and double silk covered. The silken covering was dissolved off with hot caustic soda, diluted so as not to oxidize the wire, and the bare wire threaded up and down through the mica disk, and parallel to the axis of the bulb; first filling up the inner cylinder of holes, and then the outer. The inside end was soldered to platinum wire, No. 1, connected with the lead *E*; the final exterior end soldered to platinum wire No. 2, from lead *G*; and the junction or midway point connecting the inner and outer cylinders was connected to the copper wire at *H*, which communicated with lead *F*. By this means two separate loops of wire of nearly equal length were provided within the bulb, arranged in two cylinders, one within the other. The outer cylinder between leads *F* and *G* had a diameter of 1.65 cm.; the inner, between *E* and *F*, the diameter of 1.0 cm. The object of this arrangement was to furnish a check by duplicate, upon the observed resistance of the tested wire, and also to ascertain what difference in temperature, if any, existed between the air at radius 0.5 cm. and the air at radius 0.8 cm. from the axis of the bulb, the source of heat being entirely external. Had the mean temperature of the inner and outer cylinders differed at any time in virtue of gradient of temperature within the bulb, by one-tenth of one degree centigrade, it would have been within the range of observation; and had it amounted to one-fifth of a degree, it could not have escaped detection. No appreciable difference was at any time discovered, the increase of resistance in the two cylinders being always proportionally coincident, so that finally the two loops were combined into one for facility in observing, and only divided for an occasional check. The diffusion and convection of the air within the bulb must have been sufficiently rapid to practically equalize the temperature within the air space. The course of the wire within the bulb is indicated by the dotted lines in Fig. 1.

The bulb communicated with the mercury through a glass tube *DK*, round which a hair, secured 15.2 cm. below the bulb at *LL*, formed the fiducial mark. The diameter of this tube was 0.6 cm. externally and 0.1 cm. internally, and the volume of its bore from bulb to fiducial mark, including the offset tube at *M*, was 0.3 c.c. or $\frac{1}{250}$ of the volume of the bulb.

The bulb was held vertically upon the axis of a glass percolator, *NNOO*, the lower tube passing through a rubber stopper at *P*. The space between the bulb and walls of the percolator was filled with boiled linseed oil up to the level *QQ*, and a cylindrical grid of platinoid wire, not shown in the figure, was immersed in the oil. A steady current of from 2 to 3 amperes through this platinoid wire of 16 ohms resistance served to raise the temperature of the oil and immersed bulb at a convenient rate. A disk of asbestos cloth, *NO*, rested as a cover upon the percolator, and more of the same material was wrapped around the exterior surface *NNOO*, in order to impede the escape of heat from the chamber.

When the apparatus was in position, a vertical wooden board, rising above a wooden trough set to catch any mercury that might escape, supported a long vertical tube in front of the cathetometer. A bottle of mercury and an equilibrating bottle of sand were supported by a cord running over pulleys fixed into the ceiling. Lowering the sand bottle raised the bottle of mercury and the attached rubber tube, bringing the level of the mercury in the index tube (allowing for capillarity) up to the same elevation.

Electrical Measuring Instruments. — The resistance of each cylindrical loop of wire within the bulb was about 10 ohms at the normal or initial temperature, in the vicinity of 20° C., making 20 ohms in all, and at the highest temperature reached in the measurements, these resistances doubled, so that the total range of observed change in resistance amounted to 10 ohms in each loop, or 20 when the loops were in series.

In the first trial, the resistances were measured by Wheatstone's bridge. This method was found to be unsatisfactory both in respect to swiftness and precision. In swiftness because the readjustment of the balance required some seconds to effect, and during that time the temperature of the wire might have altered; and in precision, since the resistance of the leads had to be subtracted, and these were likely to vary appreciably with the temperature of the room.

Later, two differential galvanometers were employed. One balanced the two loops within the bulb against each other to detect variations of temperature within the bulb, and the other compared the resistance of both loops in series against a fixed standard in platinoid wire (30 ohms).

Finally, the first galvanometer fell into disuse, since no variation could be detected between the resistances of the two loops, and measurements were confined to the second differential galvanometer, with one wheatstone bridge reading as a check at the outset, and another at the culminating temperature of the series.

The differential galvanometer method is very convenient for such measurement. It enables the variations of the tested resistance to be constantly watched, and balance can be quickly readjusted with a dead-beat instrument by rheostat thrown into the circuit of preponderating influence.

The main circuit consisted of the bulb with its two loops in series, a standard resistance of 30 ohms in platinoid, and an additional resistance of 50 ohms in a rheostat. A single Edison-Lalande cell delivered a steady current of from 6 to 7 milliamperes through this circuit. The two coils of the differential galvanometer had about 2700 ohms each; one was connected to the terminals of the standard 30 ohms, and the other to the terminals of the bulb. A change of resistance in circuit of either coil, amounting to 1 ohm, could be plainly observed on the scale.

Modus Operandi. — The bulb was repeatedly exhausted through its attached tube, then heated to 200° C. under a vacuum for an hour, to expel all moisture, and finally filled with dried air. It was then tightly connected with the mercury apparatus and index tube by double rubber pipes well jointed. The offset tube was opened through a chamber containing calcium chloride so as to acquire internally the pressure of the air, which was noted by mercurial barometer. The level of the mercury was raised at the same time to the fiducial mark, the temperature of the oil bath around the bulb observed, and the resistance of the enclosed copper wire measured. The temperature of the air close to the index tube was also taken. The offset tube was then sealed off with a blow-pipe, and the circuit of the platinoid grid in the oil bath closed. The level of the mercury was then raised in the index tube about 3.5 cm. to cathetometer observation, by lifting the mercury bottle through that distance, driving mercury into the bulb tube above the fiducial mark. The increasing temperature and pressure of the air within the bulb slowly forced the mercury in this column back to the fiducial mark, leaving the index elevation practically unaltered. At the moment that the mercury crossed the fiducial mark, the resistance for balance at the differential galvanometer was noted. The level of the mercury in bottle and index tube would then be raised again another 3.5 cm., the mercury column elevated above the fiducial mark, the cathetometer reading taken, and the resistance balance when the mercury again crossed the line. This process was repeated step by step until the maximum temperature was reached. Meanwhile readings were kept of the barometer pressure in the room, and the temperature of the air by the side of the index tube.

No appreciable time lag existed in the bulb, owing to its small thermal capacity. After the highest desired temperature had been attained in a series, the differential galvanometer would indicate a lowering in the resistance of the copper wire, within ten seconds of the interruption of the heating current. The reduction in temperature could be detected electrically ten or fifteen seconds before the mercury could be observed to retreat. This lag in the mercury was traced, principally, at least, to the influence of fluid friction in the narrow tube. Tapping this tube with the finger was found to accelerate the mercurial movement. Later, the bulb tube whose internal diameter was 1 mm., in order to have as little volume of unheated air-space as possible, was welded into one of larger caliber (0.2 cm.) just above the fiducial mark, increasing volume of air-space outside the bulb to 0.4 c.c., but materially diminishing the fluid friction, so that tapping the tube was scarcely necessary.

A similar series of cathetometer and resistance readings was obtained as the oil bath and bulb cooled down. An entire set of observations generally lasted four hours.

The mercury employed was filtered and kept clean. The internal diameter of the index tube was 0.6 cm., but as its capillarity error entered equally into all the readings, no correction was required on this account.

The coefficient of expansion of the glass forming the bulb was measured by taking two globes blown from the same tubing as the bulb, drawing out their necks to a fine bore, filling them with a measured mass of mercury, at normal temperature, heating them up to their necks in mercury over a sand bath to 157° C. till they ceased to overflow, and the mass of mercury remaining was observed after cooling down.

The mean cubical expansion of the glass was

$$3\beta = (2800 \pm 113) \times 10^{-8},$$

and this value was assumed in all voluminal corrections.

Results. — The first four series of observations were rejected. When computed and plotted, with ordinates of resistance to abscissas of temperature, they showed curves bending slightly upwards, after the manner of Matthiessen's, but the descending curve was distinctly below the ascending branch, so that the diagram appeared to form a loop as though the wire had a lower resistance for a given temperature when cooling than when heating. The cause of this error is not known, but may have been due to fluid friction of the mercury in the bulb tube. With each successive series the curvature of the line diminished, the ascending and descending branches approaching one another. The fifth series was considered to be satisfactory. Its graph is practically a straight line with a slight divergence between upward and downward branches. The reduced observations are given in the accompanying

TABLE I.

	Temperature °C. by air thermometer.	**Linear coefficient per °C.**	**Weighted mean.**	**Discrepancy.**
	29.8	0.004037	0.004059	0.000022
	67.0	0.004076		0.000017
	79.8	0.004066		0.000007
	92.9	0.004085		0.000026
	104.7	0.004063		0.000004
	113.0	0.004055		– 0.000004
	131.4	0.004062		0.000003
	143.0	0.004058		– 0.000001
	155.0	0.004031		0.000022
	168.6	0.004037		0.000023
	181.7	0.004110		0.000051
Maximum	196.2	0.004084		0.000025
	169.0	0.004062		0.000003
	135.3	0.003999		– 0.000060
	123.6	0.003995		– 0.000064
	108.0	0.004010		– 0.000049
	92.0	0.004017		– 0.000042

The sixth series appeared to be the most reliable. Its graph is also practically a straight line up to the maximum temperature of 255 C. with no appreciable deviation between the ascending and descending branches, the latter being carried in this instance no lower than 207° C. The reduced observations referred to linear coefficients are given in the following

Table II.

	Temperature of copper wire in ° C.	Linear coefficient of increase per degree.	Weighted mean.	Discrepancy.
	27.8	0.004007	0.004065	– 0.000058
	42.64	0.003984		– 0.000081
	56.55	0.004038		– 0.000027
	72.25	0.004027		– 0.000038
	87.11	0.004063		– 0.000002
	105.07	0.004172		0.000107
	123.99	0.004080		0.000015
	139.48	0.004141		0.000076
	154.17	0.004143		0.000078
	169.94	0.004082		0.000017
	183.71	0.004028		– 0.000037
Accidental fall in temperature	181.68	0.003968		– 0.000097
	197.03	0.003990		– 0.000075
	215.53	0.004022		– 0.000043
	230.59	0.004022		– 0.000043
	244.87	0.004049		– 0.000016
	255.26	0.004070		0.000005
Maximum	255.26	0.004074		0.000009
	255.26	0.004088		0.000023
	235.44	0.004097		0.000032
	207.52	0.004088		0.00023

Between the fifth and sixth series the apparatus was taken apart, the mercury refiltered, the bulb re-exhausted, and then replaced. The method of measuring the temperature of the copper wire here described and advocated involves considerable more arithmetic labor in computing the results, owing to corrections for barometric pressure, temperature of the mercury column in the index tube, and expansion in the bulb, but it eliminates all doubt as to the coincidence between the temperature of the wire and the temperature indicated by thermometer, and avoids all differences between the true thermometric scale by air thermometer based upon Boyle's law, and the slightly divergent scale of the ordinary thermometer based upon the expansion of mercury.

Tests of the Copper Wire used. — The resistivity of the wire in the bulb was observed to be 1637 C.G.S. units at 0° C. from its mass and resistance, allowing a specific gravity of 8.90. Several different observations did not agree very closely, however, owing probably to the small diameter of the wire, and its liability to become stretched and variable in diameter.

An analysis of the copper wire used was made by Mr. McCoy of the Purdue Chemical Laboratory and one of the writers. The results are as follows: —

Antimony Arsenic	less than 0.01 per cent;	more than 0.0025 per cent.
Iron	less than 0.025 per cent;	more than 0.0025 per cent.
Zinc	less than 0.03 per cent;	more than 0.0025 per cent.

III.

Note on the Temperature of Lowest Visible Red Heat.

A few measurements were made of the resistance of copper wires enclosed in exhausted glass tubes and gradually raised to just visible red heat, by gradually increasing the current strength through them. The tubes were 30 cm. long, and 2 cm. external, and 1.8 cm. internal diameter. The platinum wires were sealed into the glass at each end, and connected with the copper wire, — one to carry the heating current, and the other to act as "pressure wire," in order to eliminate the resistance of the first, or platinum electrode. The copper wire stretched along the axis of the exhausted tube was 30.4 cm. long, and 0.0015' (0.0038 cm.) in diameter. Measuring the resistance of these wires at normal temperature with a very feeble current, they were then raised to redness, and their resistance observed under that condition in a darkened room.

Reckoning back with the linear temperature coefficient of 0.00406 from the normal temperature to zero centigrade, the resistance of the wire was found to be three times that at zero when visible luminosity was just attained, the mean calculated ratio being, in fact, 3.001. If the same linear temperature coefficient be assumed throughout that whole range, the corresponding temperature of lowest visible luminosity becomes 493° C. in this instance.

The method is very sensitive in application, and repeated trials with the same wire and same observer would usually fall within two degrees centigrade by resistance valuation. There was, however, a systematic variation between the observations when the observers were exchanged, amounting to about three degrees of centigrade; and since the criterion of appreciable visibility is merely physiological, it is perhaps impossible to accurately define it. From the sensations experienced in observing, it might be supposed that habit or physical condition would appreciably influence the range of visual appreciation, after the manner of a personal equation.

In conclusion, we desire to express our acknowledgments to Mr. Thomas A. Edison, in whose laboratory the above research was conducted.