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Review

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of bridges. Most teachers may have noticed that it is comparatively easy to excite an interest in the personality of the inventor of a theorem or a law. Who would not prefer the name "Euler line" to "CONG line" for example? To study a curve in detail, to see how the various properties are brought to light, to realise how the history of the curve has been influenced by the progress of discoveries which perhaps had nothing to do with the art of curve tracing, to see where and why one man failed and his friend and rival, it may be, succeeded—this cannot but prove a stimulus to the young and enquiring mind, and may prove the germ of what in later years will ripen into efflorescence. But for even a short lecture such as is here suggested the teacher cannot rely for information on the ordinary text-books. Take for instance Frost's Curve Tracing. It contains, we verily believe, but two names in all its two hundred pages—Newton and De Gua. Think how much more interesting that admirable work might have been made by even the most elementary references to the history of the art. The teacher has no longer the excuse that he does not know whence to draw his material. This translation into German of Dr. Loria's encyclopaedic work will be far more than is necessary for such a modest programme as is permitted to the teacher when other claims are considered. It is monumental. From cover to cover it teems with interest. A copy should be on the shelves of every College Library, for "the sight of means to do *good* deeds make *good* deeds done." The student who wishes to carry further his researches will find all that he requires in the shape of references from the literature of the earliest days up to the most recent memoirs. The amount of patient labour which these two volumes with their 750 pages represent is colossal.

The Foundations of Geometry, by D. HILBERT. Authorised translation, by E. J. TOWNSEND. Pp. viii, 142. 4s. 6d. net. (Open Court Co.: Kegan Paul.) 1902.

A translation of Hilbert's fascinating *Grundlagen der Geometrie* is heartily welcome in this country, and the volume under notice is further enriched by the author's additions, which appeared in the French translation which M. Laugel published some years ago (Gauthier-Villars). It also contains a summary of a memoir embodying Hilbert's latest researches, which has probably already appeared in the *Math. Ann.*

The first attempt to prove the concurrence in the plane of lines which are not parallel was made by Legendre. He showed that if any one triangle has the sum of its angles equal to two right angles, then the sum of the angles of all triangles will be two right angles; but he failed in his endeavour to prove the existence of one such triangle. Saccheri in 1733, and later Gauss, Lobatchewsky, and Bolyai attacked the same problem, but on different lines. They started with the negation of the axiom of parallels, and to the great surprise and alarm of Saccheri (v. Russell, *Foundations of Geometry*, p. 8) the result was more than one Geometry to the logical basis of which no objection could be found. Their success led to further investigations as to the axioms in general. The conception of space as a manifold of numbers gave Riemann, Helmholtz, and Lie the opportunity of establishing on an analytical basis both the non-Euclidean system of Lobatchewsky, and the system in which Euclid's "straight line" is avoided. In the former the sum of the angles of a triangle is always less, and in the latter always greater, than two right angles. On the other hand, we have the purely geometrical investigations of Veronese and Hilbert. How then are the researches of Hilbert to be placed with reference to the analytical researches of other workers in the same field? Helmholtz showed that Euclid's propositions were in disguise but the laws of motion of solid bodies. The non-Euclidean propositions were in the same manner the laws to which are subject bodies analogous to solid bodies, but with no physical existence. Lie went further. Combining all the possible transformations of a figure he calls the total a group. To each of these groups he attached a geometry; all these geometries have common properties; but the generality of his conclusions is impaired by the fact that all his groups are continuous. His space is a *Zahlenmannigfaltigkeit*. His geometries are subject to the forms of analysis and arithmetic. Now, as M. Poincaré points out (*Bull. des Sciences Mathématiques*, Sept. 1902), this is exactly where Hilbert comes in. His spaces are not *Zahlenmannigfaltigkeiten*. The objects he calls point, line, or plane are purely

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logical conceptions. The most important of the additions to be found in the French translation are due to the investigations of Dr. Dehn, of which mention has been made already in our columns (No. 29, Oct. 1901, p. 94). The geometry which he constructs is one in which the sum of the angles of any triangle is two right angles; in which similar non-congruent triangles exist; and in which an infinite number of parallels to any straight line may be drawn through any point.

As a translation the volume before us cannot be said to be entirely successful. It has been unmercifully and somewhat undeservedly gibbeted by Prof. Halsted in *Science*, Aug. 22, 1902. A sober and detailed criticism by Dr. Hedrick of both this and the French translation will be found in the *Bull. of the American Math. Soc.*, Dec. 1902, to which considerations of space compel us to refer the reader, and in which will be found a long list of errors and misprints. We have carefully compared the French and English translations, and we find that Dr. Hedrick has omitted no point of any consequence, and that in our opinion his strictures are quite justified and necessary to the clear understanding of the text. With his list of errata the translation in question may be read by the student who can appeal to an expert for guidance, but on the whole we should prefer to place M. Laugel's book in his hands.

PROBLEMS.

469. [K. 20. b.] If e be so small that its cube and higher powers may be neglected, and $\phi-\theta+2e\sin\theta=\frac{3e^2}{4}\sin 2\theta$, then $\theta-\phi-2e\sin\phi=\frac{5e^2}{4}\sin 2\phi$.

ANON.

470. [D. 2. b.] If successive terms of a series are connected by the relation $u_n u_{n-3}-u_{n-1}u_{n-2}=2a$, and the first three terms are unity, prove that (i.) all the terms are integral functions of a, (ii.) any even term is the arithmetic mean of the adjacent terms; and find an expression for the nth term.

W. H. LAVERTY.

471. [K. 10. c.] Prove that the general form of the approximation to the length L of a circular arc when powers of $\frac{L}{R}$ beyond the $2(n+1)$th are neglected, R being the radius of the circle, is given by L_n where

$$L_n a_1 a_2 a_3 \ldots a_n = l_0 - l_1\frac{a_n}{a_1}2^{b_1} + l_2\frac{a_n a_{n-1}}{a_1 a_2}2^{b_2} - \ldots + (-1)^n l_n 2^{b_n},$$

where $$a_r = 1-4^r;\quad b_r = r(r+2);\quad l_r = \text{chord of arc } \frac{L}{2^r}.$$

(Huygens's approximation is given by $n=1$.)

R. M. MILNE.

472. [L.[1] 7. d.] Given the focus and directrix of a parabola, find by Euclidean methods the points in which the parabola cuts a given straight line. Solve also the same problem for the ellipse.

R. F. MUIRHEAD.

473. [L.[1] 17. e.] A family of conics have their axes along given lines and pass through a given point; show that the locus of the centre of curvature at the given point is a cubic whose asymptotes are all real.

C. F. SANDBERG.

474. [K. 2. e.] In a triangle ABC are inscribed three squares, of which the sides parallel to AB, BC, CA are DE', EF', FD'. Prove that the circles AEF', BFD', CDE' touch one another, and that the radius of the smaller circle which touches all three is $R/(4\cot\omega+7)$.

C. E. YOUNGMAN.

475. [K. 8. a.] The angles α, β, γ, δ of a quadrilateral satisfy the relation

$$\Sigma\cos^2\frac{\alpha}{2}+2\Pi\cos\frac{\alpha}{2}=2+2\Pi\sin\frac{\alpha}{2}.$$

Durham, 1902.

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