

THE PHILOSOPHY OF LOGICAL ATOMISM.

VII. THE THEORY OF TYPES AND SYMBOLISM: CLASSES.

BEFORE I begin to-day the main subject of my lecture, I should like to make a few remarks in explanation and amplification of what I have said about existence in my previous two lectures. This is chiefly on account of a letter I have received from a member of the class, raising many points which, I think, were present in other minds too.

The first point I wish to clear up is this: I did not mean to say that when one says a thing exists, one means the same as when one says it is possible. What I meant was, that the fundamental logical idea, the primitive idea, out of which both those are derived is the same. That is not quite the same thing as to say that the statement that a thing exists is the same as the statement that it is possible, which I do not hold. I used the word "possible" in perhaps a somewhat strange sense, because I wanted some word for a fundamental logical idea for which no word exists in ordinary language, and therefore if one is to try to express in ordinary language the idea in question, one has to take some word and make it convey the sense that I was giving to the word "possible," which is by no means the only sense that it has but is a sense that was convenient for my purpose. We say of a propositional function that it is possible, where there are cases in which it is true.

That is not exactly the same thing as what one ordinarily means, for instance, when one says that it is possible it may rain to-morrow. But what I contend is, that the ordinary uses of the word "possible" are derived from this notion by a process. E. g., normally when you say of a proposition that it is possible, you mean something like this: first of all it is implied that you do not know whether it is true or false, and I think it is implied; secondly, that it is one of a class of propositions, some of which are known to be true. When I say, e. g., "It is possible that it may rain to-morrow,"—"It will rain to-morrow" is one of the class of propositions "It rains at time t ," where t is different times. We mean partly that we do not know whether it will rain or whether it will not, but also that we do know that that is the sort of proposition that is quite apt to be true, that it is a value of a propositional function of which we know some value to be true. Many of the ordinary uses of "possible" come under that head, I think you will find. That is to say, that if you say of a proposition that it is possible, what you have is this: "There is in this proposition some constituent, which, if you turn it into a variable, will give you a propositional function that is sometimes true." You ought not therefore to say of a proposition simply that it is possible, but rather that it is possible in respect of such-and-such a constituent. That would be a more full expression.

When I say, for instance, that "Lions exist," I do not mean the same as if I said that lions were possible; because when you say "Lions exist," that means that the propositional function " x is a lion" is a possible one in the sense that there are lions, while when you say "Lions are possible" that is a different sort of statement altogether, not meaning that a casual individual animal may be a lion, but rather that a *sort* of animal may be the *sort* that we call "lions." If you say "Unicorns are possible," e. g., you

would mean that you do not know any reason why there should not be unicorns, which is quite a different proposition from "Unicorns exist." As to what you would mean by saying that unicorns are possible, it would always come down to the same thing as "It is possible it may rain tomorrow." You would mean, the proposition "There are unicorns" is one of a certain set of propositions some of which are known to be true, and that the description of the unicorn does not contain in it anything that *shows* there could not be such beasts.

When I say a propositional function is possible, meaning there are cases in which it is true, I am consciously using the word "possible" in an unusual sense, because I want a single word for my fundamental idea, and cannot find any word in ordinary language that expresses what I mean.

Secondly, it is suggested that when one says a thing exists, it means that it is in time, or in time and space, at any rate in time. That is a very common suggestion, but I do not think that really there is much to be said for that use of the words; in the first place, because if that were all you meant, there would be no need for a separate word. In the second place, because after all in the sense, whatever that sense may be, in which the things are said to exist that one ordinarily regards as existing, one may very well wish to discuss the question whether there are things that exist without being in time. Orthodox metaphysics holds that whatever is really real is not in time, that to be in time is to be more or less unreal, and that what really exists is not in time at all. And orthodox theology holds that God is not in time. I see no reason why you should frame your definition of existence in such a way as to preclude that notion of existence. I am inclined to think that there are things that are not in time, and I should be sorry to use the word existence in that sense when you

have already the phrase "being in time" which quite sufficiently expresses what you mean.

Another objection to that definition is, that it does not in the least fit the sort of use of "existence" which was underlying my discussion, which is the common one in mathematics. When you take existence-theorems, for instance, as when you say "An even prime exists," you do not mean that the number two is in time but that you can find a number of which you can say "This is even and prime." One does ordinarily in mathematics speak of propositions of that sort as existence-theorems, i. e., you establish that there is an object of such-and-such a sort, that object being, of course, in mathematics a logical object, not a particular, not a thing like a lion or a unicorn, but an object like a function or a number, something which plainly does not have the property of being in time at all, and it is that sort of sense of existence-theorems that is relevant in discussing the meaning of existence as I was doing in the last two lectures. I do, of course, hold that that sense of existence can be carried on to cover the more ordinary uses of existence, and does in fact give the key to what is underlying those ordinary uses, as when one says that "Homer existed" or "Romulus did not exist," or whatever we may say of that kind.

I come now to a third suggestion about existence, which is also a not uncommon one, that of a given particular "this" you can say "This exists" in the sense that it is not a phantom or an image or a universal. Now I think that use of existence involves confusions which it is exceedingly important to get out of one's mind, really rather dangerous mistakes. In the first place, we must separate phantoms and images from universals; they are on a different level. Phantoms and images do undoubtedly exist in that sense, whatever it is, in which ordinary objects exist. I mean, if you shut your eyes and imagine some visual scene, the

images that are before your mind while you are imagining are undoubtedly there. They are images, something is happening, and what is happening is that the images are before your mind, and these images are just as much part of the world as tables and chairs and anything else. They are perfectly decent objects, and you only call them unreal (if you call them so), or treat them as non-existent, because they do not have the usual sort of relations to other objects. If you shut your eyes and imagine a visual scene and you stretch out your hand to touch what is imaged, you won't get a tactile sensation, or even necessarily a tactile image. You will not get the usual correlation of sight and touch. If you imagine a heavy oak table, you can remove it without any muscular effort, which is not the case with oak tables that you actually see. The general correlations of your images are quite different from the correlations of what one chooses to call "real" objects. But that is not to say images are unreal. It is only to say they are not part of physics. Of course, I know that this belief in the physical world has established a sort of reign of terror. You have got to treat with disrespect whatever does not fit into the physical world. But that is really very unfair to the things that do not fit in. They are just as much there as the things that do. The physical world is a sort of governing aristocracy, which has somehow managed to cause everything else to be treated with disrespect. That sort of attitude is unworthy of a philosopher. We should treat with exactly equal respect the things that do not fit in with the physical world, and images are among them.

"Phantoms," I suppose, are intended to differ from "images" by being of the nature of hallucinations, things that are not merely imagined but that go with belief. They again are perfectly real; the only odd thing about them is their correlations. Macbeth sees a dagger. If he tried to touch it, he would not get any tactile sensation, but that

does not imply that he was not *seeing* a dagger, it only implies that he was not *touching* it. It does not in any way imply that the visual sensation was not there. It only means to say that the sort of correlation between sight and touch that we are used to is the normal rule but not a universal one. In order to pretend that it is universal, we say that a thing is unreal when it does not fit in. You say "Any man who is a man will do such-and-such a thing." You then find a man who will not, and you say, he is not a man. That is just the same sort of thing as with these daggers that you cannot touch.

I have explained elsewhere the sense in which phantoms are unreal.¹ When you see a "real" man, the immediate object that you see is one of a whole system of particulars, all of which belong together and make up collectively the various "appearances" of the man to himself and others. On the other hand, when you see a phantom of a man, that is an isolated particular, not fitting into a system as does a particular which one calls an appearance of the "real" man. The phantom is in itself just as much part of the world as the normal sense-datum, but it lacks the usual correlation and therefore gives rise to false inferences and becomes deceptive.

As to universals, when I say of a particular that it exists, I certainly do not mean the same thing as if I were to say that it is not a universal. The statement concerning any particular that it is not a universal is quite strictly nonsense—not false, but strictly and exactly nonsense. You never can place a particular in the sort of place where a universal ought to be, and *vice versa*. If I say "*a* is not *b*," or if I say "*a* is *b*," that implies that *a* and *b* are of the same logical type. When I say of a universal that it exists, I should be meaning it in a different

¹ See *Our Knowledge of the External World*, Chap. III. Also Section XII of "Sense-Data and Physics" in *Mysticism and Logic*.

sense from that in which one says that particulars exist. E. g., you might say "Colors exist in the spectrum between blue and yellow." That would be a perfectly respectable statement, the colors being taken as universals. You mean simply that the propositional function " x is a color between blue and yellow" is one which is capable of truth. But the x which occurs there is not a particular, it is a universal. So that you arrive at the fact that the ultimate important notion involved in existence is the notion that I developed in the lecture before last, the notion of a propositional function being sometimes true, or being, in other words, possible. The distinction between what some people would call real existence, and existence in people's imagination or in my subjective activity, that distinction, as we have just seen, is entirely one of correlation. I mean that anything which appears to you, you will be mistakenly inclined to say has some more glorified form of existence if it is associated with those other things I was talking of in the way that the appearance of Socrates to you would be associated with his appearance to other people. You would say he was only in your imagination if there were not those other correlated appearances that you would naturally expect. But that does not mean that the appearance to you is not exactly as much a part of the world as if there were other correlated appearances. It will be exactly as much a part of the real world, only it will fail to have the correlations that you expect. That applies to the question of sensation and imagination. Things imagined do not have the same sort of correlations as things sensed. If you care to see more about this question, I wrote a discussion in *The Monist* for January, 1915, and if any of you are interested, you will find the discussion there.

I come now to the proper subject of my lecture, but shall have to deal with it rather hastily. It was to explain the theory of types and the definition of classes. Now

first of all, as I suppose most of you are aware, if you proceed carelessly with formal logic, you can very easily get into contradictions. Many of them have been known for a long time, some even since the time of the Greeks, but it is only fairly recently that it has been discovered that they bear upon mathematics, and that the ordinary mathematician is liable to fall into them when he approaches the realms of logic, unless he is very cautious. Unfortunately the mathematical ones are more difficult to expound, and the ones easy to expound strike one as mere puzzles or tricks.

You can start with the question whether or not there is a greatest cardinal number. Every class of things that you can choose to mention has some cardinal number. That follows very easily from the definition of cardinal numbers as classes of similar classes, and you would be inclined to suppose that the class of all the things there are in the world would have about as many members as a class could be reasonably expected to have. The plain man would suppose you could not get a larger class than the class of all the things there are in the world. On the other hand, it is very easy to prove that if you take selections of some of the members of a class, making those selections in every conceivable way that you can, the number of different selections that you can make is greater than the original number of terms. That is easy to see with small numbers. Suppose you have a class with just three numbers, a , b , c . The first selection that you can make is the selection of no terms. The next of a alone, b alone, c alone. Then bc , ca , ab , abc , which makes in all 8 (i.e., 2^3) selections. Generally speaking, if you have n terms, you can make 2^n selections. It is very easy to prove that 2^n is always greater than n , whether n happens to be finite or not. So you find that the total number of things in the world is not so great as the number of classes

that can be made up out of those things. I am asking you to take all these propositions for granted, because there is not time to go into the proofs, but they are all in Cantor's work. Therefore you will find that the total number of things in the world is by no means the greatest number. On the contrary, there is a hierarchy of numbers greater than that. That, on the face of it, seems to land you in a contradiction. You have, in fact, a perfectly precise arithmetical proof that there are *fewer* things in heaven or earth than are dreamt of in *our* philosophy. That shows how philosophy advances.

You are met with the necessity, therefore, of distinguishing between classes and particulars. You are met with the necessity of saying that a class consisting of two particulars is not itself in turn a fresh particular, and that has to be expanded in all sorts of ways; i. e., you will have to say that in the sense in which there are particulars, in that sense it is not true to say there are classes. The sense in which there are classes is a different one from the sense in which there are particulars, because if the senses of the two were exactly the same, a world in which there are three particulars and therefore eight classes, would be a world in which there are at least eleven things. As the Chinese philosopher pointed out long ago, a dun cow and a bay horse makes three things: separately they are each one, and taken together they are another, and therefore three.

I pass now to the contradiction about classes that are not members of themselves. You would say generally that you would not expect a class to be a member of itself. For instance, if you take the class of all the teaspoons in the world, that is not in itself a teaspoon. Or if you take all the human beings in the world, the whole class of them is not in turn a human being. Normally you would say you cannot expect a whole class of things to be

itself a member of that class. But there are apparent exceptions. If you take, e. g., all the things in the world that are not teaspoons and make up a class of them, that class obviously (you would say) will not be a teaspoon. And so generally with negative classes. And not only with negative classes, either, for if you think for a moment that classes are things in the same sense in which things are things, you will then have to say that the class consisting of all the things in the world is itself a thing in the world, and that therefore this class is a member of itself. Certainly you would have thought that it was clear that the class consisting of all the classes in the world is itself a class. That I think most people would feel inclined to suppose, and therefore you would get there a case of a class which is a member of itself. If there is any sense in asking whether a class is a member of itself or not, then certainly in all the cases of the ordinary classes of everyday life you find that a class is not a member of itself. Accordingly, that being so, you could go on to make up the class of all those classes that are not members of themselves, and you can ask yourself, when you have done that, is that class a member of itself or is it not?

Let us first suppose that it is a member of itself. In that case it is one of those classes that are not members of themselves, i. e., it is not a member of itself. Let us then suppose that it is not a member of itself. In that case it is not one of those classes that are not members of themselves, i. e., it is one of those classes that are members of themselves, i. e., it is a member of itself. Hence either hypothesis, that it is or that it is not a member of itself, leads to its contradiction. If it is a member of itself, it is not, and if it is not, it is.

That contradiction is extremely interesting. You can modify its form; some forms of modification are valid and some are not. I once had a form suggested to me which

was not valid, namely the question whether the barber shaves himself or not. You can define the barber as "one who shaves all those, and those only, who do not shave themselves." The question is, does the barber shave himself? In this form the contradiction is not very difficult to solve. But in our previous form I think it is clear that you can only get around it by observing that the whole question whether a class is or is not a member of itself is nonsense, i. e., that no class either is or is not a member of itself, and that it is not even true to say that, because the whole form of words is just a noise without meaning. That has to do with the fact that classes, as I shall be coming on to show, are incomplete symbols in the same sense in which the descriptions are that I was talking of last time; you are talking nonsense when you ask yourself whether a class is or is not a member of itself, because in any full statement of what is meant by a proposition which seems to be about a class, you will find that the class is not mentioned at all and that there is nothing about a class in that statement. It is absolutely necessary, if a statement about a class is to be significant and not pure nonsense, that it should be capable of being translated into a form in which it does not mention the class at all. This sort of statement, "Such-and-such a class is or is not a member of itself," will not be capable of that kind of translation. It is analogous to what I was saying about descriptions: the symbol for a class is an incomplete symbol; it does not really stand for part of the propositions in which symbolically it occurs, but in the right analysis of those propositions that symbol has been broken up and disappeared.

There is one other of these contradictions that I may as well mention, the most ancient, the saying of Epimenides that "All Cretans are liars." Epimenides was a man who slept for sixty years without stopping, and I believe that it was at the end of that nap that he made the remark that

all Cretans were liars. It can be put more simply in the form: if a man makes the statement "I am lying," is he lying or not? If he is, that is what he said he was doing, so he is speaking the truth and not lying. If, on the other hand, he is not lying, then plainly he is speaking the truth in saying that he is lying, and therefore he is lying, since he says truly that that is what he is doing. It is an ancient puzzle, and nobody treated that sort of thing as anything but a joke until it was found that it had to do with such important and practical problems as whether there is a greatest cardinal or ordinal number. Then at last these contradictions were treated seriously. The man who says "I am lying" is really asserting "There is a proposition which I am asserting and which is false." That is presumably what you mean by lying. In order to get out the contradiction you have to take that whole assertion of his as one of the propositions to which his assertion applies; i. e., when he says "There is a proposition which I am asserting and which is false," the word "proposition" has to be interpreted as to include among propositions his statement to the effect that he is asserting a false proposition. Therefore you have to suppose that you have a certain totality, viz., that of propositions, but that that totality contains members which can only be defined in terms of itself. Because when you say "There is a proposition which I am asserting and which is false," that is a statement whose meaning can only be got by reference to the totality of propositions. You are not saying which among all the propositions there are in the world it is that you are asserting and that is false. Therefore it presupposes that the totality of propositions is spread out before you and that some one, though you do not say which, is being asserted falsely. It is quite clear that you get into a vicious circle if you first suppose that this totality of propositions is spread out before you, so that you can

without picking any definite one say "Some one out of this totality is being asserted falsely," and that yet, when you have gone on to say "Some one out of this totality is being asserted falsely," that assertion is itself one of the totality you were to pick out from. That is exactly the situation you have in the paradox of the liar. You are supposed to be given first of all a set of propositions, and you assert that some one of these is being asserted falsely, then that assertion itself turns out to be one of the set, so that it is obviously fallacious to suppose the set already there in its entirety. If you are going to say anything about "all propositions," you will have to define propositions, first of all, in some such way as to exclude those that refer to all the propositions of the sort already defined. It follows that the word "proposition," in the sense in which we ordinarily try to use it, is a meaningless one, and that we have got to divide propositions up into sets and can make statements about all propositions in a given set, but those propositions will not themselves be members of the set. For instance, I may say "All atomic propositions are either true or false," but that itself will not be an atomic proposition. If you try to say "All propositions are either true or false," without qualification, you are uttering nonsense, because if it were not nonsense it would have to be itself a proposition and one of those included in its own scope, and therefore the law of excluded middle as enunciated just now is a meaningless noise. You have to cut propositions up into different types, and you can start with atomic propositions or, if you like, you can start with those propositions that do not refer to sets of propositions at all. Then you will take next those that refer to sets of propositions of that sort that you had first. Those that refer to sets of propositions of the first type, you may call the second type, and so on.

If you apply that to the person who says "I am lying,"

you will find that the contradiction has disappeared, because he will have to say what type of liar he is. If he says "I am asserting a false proposition of the first type," as a matter of fact that statement, since it refers to the totality of propositions of the first type, is of the second type. Hence it is not true that he is asserting a false proposition of the first type, and he remains a liar. Similarly, if he said he was asserting a false proposition of the 30,000th type, that would be a statement of the 30,001th type, so he would still be a liar. And the counter-argument to prove that he was also not a liar has collapsed.

You can lay it down that a totality of any sort cannot be a member of itself. That applies to what we are saying about classes. For instance, the totality of classes in the world cannot be a class in the same sense in which they are. So we shall have to distinguish a hierarchy of classes. We will start with the classes that are composed entirely of particulars: that will be the first type of classes. Then we will go on to classes whose members are classes of the first type: that will be the second type. Then we will go on to classes whose members are classes of the second type: that will be the third type, and so on. Never is it possible for a class of one type either to be or not to be identical with a class of another type. That applies to the question I was discussing a moment ago, as to how many things there are in the world. Supposing there are three particulars in the world. There are then, as I was explaining, 8 classes of particulars. There will be 2^8 (i. e., 256) classes of classes or particulars, and 2^{256} classes of classes of classes of particulars, and so on. You do not get any contradiction arising out of that, and when you ask yourself the question: "Is there, or is there not a greatest cardinal number?" the answer depends entirely upon whether you are confining yourself within some one type, or whether you are not. Within any given type there is

a greatest cardinal number, namely, the number of objects of that type, but you will always be able to get a larger number by going up to the next type. Therefore, there is no number so great but what you can get a greater number in a sufficiently high type. There you have the two sides of the argument: the one side when the type is given, the other side when the type is not given.

I have been talking, for brevity's sake, as if there really were all these different sorts of things. Of course, that is nonsense. There are particulars, but when one comes on to classes, and classes of classes, and classes of classes of classes, one is talking of logical fictions. When I say there are no such things, that again is not correct. It is not significant to say "There are such things" in the same sense of the words "there are" in which you can say "There are particulars." If I say "There are particulars" and "There are classes," the two phrases "there are" will have to have different meanings in those two propositions, and if they have suitable different meanings, both propositions may be true. If, on the other hand, the words "there are" are used in the same sense in both, then one at least of those statements must be nonsense, not false but nonsense. The question then arises, what is the sense in which one can say "There are classes," or in other words, what do you mean by a statement in which a class appears to come in? First of all, what are the sort of things you would like to say about classes? They are just the same as the sort of things you want to say about propositional functions. You want to say of a propositional function that it is sometimes true. That is the same thing as saying of a class that it has members. You want to say that it is true for exactly 100 values of the variables. That is the same as saying of a class that it has a hundred members. All the things you want to say about classes are the same as the things you want to say about propositional func-

tions excepting for accidental and irrelevant linguistic forms, with, however, a certain proviso which must now be explained.

Take, e. g., two propositional functions such as " x is a man," " x is a featherless biped." Those two are formally equivalent, i. e., when one is true so is the other, and *vice versa*. Some of the things that you can say about a propositional function will not necessarily remain true if you substitute another formally equivalent propositional function in its place. For instance, the propositional function " x is a man" is one which has to do with the concept of humanity. That will not be true of " x is a featherless biped." Or if you say, "So-and-so asserts that such-and-such is a man" the propositional function " x is a man" comes in there, but " x is a featherless biped" does not. There are a certain number of things which you can say about a propositional function which would be not true if you substitute another formally equivalent propositional function. On the other hand, any statement about a propositional function which will remain true or remain false, as the case may be, when you substitute for it another formally equivalent propositional function, may be regarded as being about the class which is associated with the propositional function. I want you to take the words *may be regarded* strictly. I am using them instead of *is*, because *is* would be untrue. "Extensional" statements about functions are those that remain true when you substitute any other formally equivalent function, and these are the ones that may be regarded as being about the class. If you have any statement about a function which is not extensional, you can always derive from it a somewhat similar statement which is extensional, viz., there is a function formally equivalent to the one in question about which the statement in question is true. This statement, which is manufactured out of the one you started with, will be ex-

tensional. It will always be equally true or equally false of any two formally equivalent functions, and this derived extensional statement may be regarded as being the corresponding statement about the associated class. So, when I say that "The class of men has so-and-so many members," that is to say "There are so-and-so many men in the world," that will be derived from the statement that " x is human" is satisfied by so-and-so many values of x , and in order to get it into the extensional form, one will put it in this way, that "There is a function formally equivalent to ' x is human,' which is true for so-and-so many values of x ." That I should define as what I mean by saying "The class of men has so-and-so many members." In that way you find that all the formal properties that you desire of classes, all their formal uses in mathematics, can be obtained without supposing for a moment that there are such things as classes, without supposing, that is to say, that a proposition in which symbolically a class occurs, does in fact contain a constituent corresponding to that symbol, and when rightly analyzed that symbol will disappear, in the same sort of way as descriptions disappear when the propositions are rightly analyzed in which they occur.

There are certain difficulties in the more usual view of classes, in addition to those we have already mentioned, that are solved by our theory. One of these concerns the null-class, i. e., the class consisting of no members, which is difficult to deal with on a purely extensional basis. Another is concerned with unit-classes. With the ordinary view of classes you would say that a class that has only one member was the same as that one member. That will land you in terrible difficulties, because in that case that one member is a member of that class, namely, itself. Take, e. g., the class of "Lecture audiences in Gordon Square."² That is obviously a class of classes, and probably it is a

² [These lectures were given in Gordon Square, London.—Ed.]

class that has only one member, and that one member itself (so far) has more than one member. Therefore if you were to identify the class of lecture audiences in Gordon Square with the only lecture audience that there is in Gordon Square, you would have to say both that it has one member and that it has twenty members, and you will be landed in contradictions, because this audience has more than one member, but the class of audiences in Gordon Square has only one member. Generally speaking, if you have any collection of many objects forming a class, you can make a class of which that class is the only member, and the class of which that class is the only member will have only one member, though this only member will have many members. This is one reason why you must distinguish a unit-class from its only member. Another is that, if you do not, you will find that the class is a member of itself, which is objectionable, as we saw earlier in this lecture. I have omitted a subtlety connected with the fact that two formally equivalent functions may be of different types. For the way of treating this point, see *Principia Mathematica*, p. 20, and Introduction, Chap. III.

I have not said quite all that I ought to have said on this subject. I meant to have gone a little more into the theory of types. The theory of types is really a theory of symbols, not of things. In a proper logical language it would be perfectly obvious. The trouble that there is arises from our inveterate habit of trying to name what cannot be named. If we had a proper logical language, we should not be tempted to do that. Strictly speaking, only particulars can be named. In that sense in which there are particulars, you cannot say either truly or falsely that there is anything else. The word "there is" is a word having "systematic ambiguity," i. e., having a strictly infinite number of different meanings which it is important to distinguish.

DISCUSSION.

.....Could you lump all those classes, and classes of classes, and so on, together?

Mr. Russell: All are fictions, but they are different fictions in each case. When you say "There are classes of particulars," the statement "there are" wants expanding and explaining away, and when you have put down what you really do mean, or ought to mean, you will find that it is something quite different from what you thought. That process of expanding and writing down fully what you mean, will be different if you go on to "There are classes of classes of particulars." There are infinite numbers of meaning to "there are." The first only is fundamental, so far as the hierarchy of classes is concerned.

.....I was wondering whether it was rather analogous to spaces, where the first three dimensions are actual, and the higher ones are merely symbolic. I see there is a difference, there are higher dimensions, but you can lump those together.

Mr. Russell: There is only one fundamental one, which is the first one, the one about particulars, but when you have gone to classes, you have traveled already just as much away from what there is as if you have gone to classes of classes. There are no classes really in the physical world. The particulars are there, but not classes. If you say "There is a universe" that meaning of "there is" will be quite different from the meaning in which you say "There is a particular," which means that "the propositional function ' x is a particular' is sometimes true."

All those statements are about symbols. They are never about the things themselves, and they have to do with "types." This is really important and I ought not to have forgotten to say it, that the relation of the symbol to what it means is different in different types. I am not now talking about this hierarchy of classes and so on, but the relation of a predicate to what it means is different to the relation of a name to what it means. There is not one single concept of "meaning" as one ordinarily thinks there is, so that you can say in a uniform sense "All symbols have meaning," but there

are infinite numbers of different ways of meaning, i. e., different sorts of relation of the symbol to the symbolized, which are absolutely distinct. The relation, e. g., of a proposition to a fact, is quite different from the relation of a name to a particular, as you can see from the fact that there are two propositions always related to one given fact, and that is not so with names. That shows you that the relation that the proposition has to the fact is quite different from the relation of a name to a particular. You must not suppose that there is, over and above that, another way in which you could get at facts by naming them. You can always only get at the thing you are aiming at by the proper sort of symbol, which approaches it in the appropriate way. That is the real philosophical truth that is at the bottom of all this theory of types.

VIII. EXCURSUS INTO METAPHYSICS: WHAT THERE IS.

I come now to the last lecture of this course, and I propose briefly to point to a few of the morals that are to be gathered from what has gone before, in the way of suggesting the bearing of the doctrines that I have been advocating upon various problems of metaphysics. I have dwelt hitherto upon what one may call philosophical grammar, and I am afraid I have had to take you through a good many very dry and dusty regions in the course of that investigation, but I think the importance of philosophical grammar is very much greater than it is generally thought to be. I think that practically all traditional metaphysics is filled with mistakes due to bad grammar, and that almost all the traditional problems of metaphysics and traditional results—supposed results—of metaphysics are due to a failure to make the kind of distinctions in what we may call philosophical grammar with which we have been concerned in these previous lectures.

Take, as a very simple example, the philosophy of arithmetic. If you think that 1, 2, 3, and 4, and the rest of the numbers, are in any sense entities, if you think that there

are objects, having those names, in the realm of being, you have at once a very considerable apparatus for your metaphysics to deal with, and you have offered to you a certain kind of analysis of arithmetical propositions. When you say, e. g., that 2 and 2 are 4, you suppose in that case that you are making a proposition of which the number 2 and the number 4 are constituents, and that has all sorts of consequences, all sorts of bearings upon your general metaphysical outlook. If there has been any truth in the doctrines that we have been considering, all numbers are what I call logical fictions. Numbers are classes of classes, and classes are logical fictions, so that numbers are, as it were, fictions at two removes, fictions of fictions. Therefore you do not have, as part of the ultimate constituents of your world, these queer entities that you are inclined to call numbers. The same applies in many other directions.

One purpose that has run through all that I have said, has been the justification of analysis, i. e., the justification of logical atomism, of the view that you can get down in theory, if not in practice, to ultimate simples, out of which the world is built, and that those simples have a kind of reality not belonging to anything else. Simples, as I tried to explain, are of an infinite number of sorts. There are particulars and qualities and relations of various orders, a whole hierarchy of different sorts of simples, but all of them, if we were right, have in their various ways some kind of reality that does not belong to anything else. The only other sort of object you come across in the world is what we call *facts*, and facts are the sort of things that are asserted or denied by propositions, and are not properly entities at all in the same sense in which their constituents are. That is shown in the fact that you cannot name them. You can only deny, or assert, or consider them, but you cannot name them because they are not

there to be named, although in another sense it is true that you cannot know the world unless you know the facts that make up the truths of the world; but the knowing of facts is a different sort of thing from the knowing of simples.

Another purpose which runs through all that I have been saying is the purpose embodied in the maxim called Occam's Razor. That maxim comes in, in practice, in this way: take some science, say physics. You have there a given body of doctrine, a set of propositions expressed in symbols—I am including words among symbols—and you think that you have reason to believe that on the whole those propositions, rightly interpreted, are fairly true, but you do not know what is the actual meaning of the symbols that you are using. The meaning they have *in use* would have to be explained in some pragmatic way: they have a certain kind of practical or emotional significance to you which is a datum, but the logical significance is not a datum, but a thing to be sought, and you go through, if you are analyzing a science like physics, these propositions with a view to finding out what is the smallest empirical apparatus—or the smallest apparatus, not necessarily wholly empirical—out of which you can build up these propositions. What is the smallest number of simple undefined things at the start, and the smallest number of undemonstrated premises, out of which you can define the things that need to be defined and prove the things that need to be proved? That problem, in any case that you like to take, is by no means a simple one, but on the contrary an extremely difficult one. It is one which requires a very great amount of logical technique; and the sort of thing that I have been talking about in these lectures is the preliminaries and first steps in that logical technique. You cannot possibly get at the solution of such a problem as I am talking about if you go at it in a straightforward

fashion with just the ordinary acumen that one accumulates in the course of reading or in the study of traditional philosophy. You do need this apparatus of symbolical logic that I have been talking about. (The description of the subject as symbolical logic is an inadequate one. I should like to describe it simply as logic, on the ground that nothing else really is logic, but that would sound so arrogant that I hesitate to do so.)

Let us consider further the example of physics for a moment. You find, if you read the works of physicists, that they reduce matter down to certain elements—atoms, ions, corpuscles, or what not. But in any case the sort of thing that you are aiming at in the physical analysis of matter is to get down to very little bits of matter that still are just like matter in the fact that they persist through time, and that they travel about in space. They have in fact all the ordinary every-day properties of physical matter, not the matter that one has in ordinary life—they do not taste or smell or appear to the naked eye—but they have the properties that you very soon get to when you travel toward physics from ordinary life. Things of that sort, I say, are not the ultimate constituents of matter in any metaphysical sense. Those things are all of them, as I think a very little reflection shows, logical fictions in the sense that I was speaking of. At least, when I say they are, I speak somewhat too dogmatically. It is possible that there may be all these things that the physicist talks about in actual reality, but it is impossible that we should ever have any reason whatsoever for supposing that there are. That is the situation that you arrive at generally in such analyses. You find that a certain thing which has been set up as a metaphysical entity can either be assumed dogmatically to be real, and then you will have no possible argument either for its reality or against its reality; or, instead of doing that, you can construct a logical fiction

having the same formal properties, or rather having formally analogous formal properties to those of the supposed metaphysical entity and itself composed of empirically given things, and that logical fiction can be substituted for your supposed metaphysical entity and will fulfil all the scientific purposes that anybody can desire. With atoms and the rest it is so, with all the metaphysical entities whether of science or of metaphysics. By metaphysical entities I mean those things which are supposed to be part of the ultimate constituents of the world, but not to be the kind of thing that is ever empirically given,—I do not say merely not being itself empirically given, but not being the *kind* of thing that is empirically given. In the case of matter, you can start from what is empirically given, what one sees and hears and smells and so forth, all the ordinary data of sense, or you can start with some definite ordinary object, say this desk, and you can ask yourselves, "What do I mean by saying that this desk that I am looking at now is the same as the one I was looking at a week ago?" The first simple ordinary answer would be that it *is* the same desk, it is actually identical, there is a perfect identity of substance, or whatever you like to call it. But when that apparently simple answer is suggested, it is important to observe that you cannot have an empirical reason for such a view as that, and if you hold it, you hold it simply because you like it and for no other reason whatever. All that you really know is such facts as that what you see now, when you look at the desk, bears a very close similarity to what you saw a week ago when you looked at it. Rather more than that one fact of similarity I admit you know, or you may know. You might have paid some one to watch the desk continuously throughout the week, and might then have discovered that it was presenting appearances of the same sort all through that period, assuming that the light was kept on all through the night. In that way you could

have established continuity. You have not in fact done so. You do not in fact know that that desk has gone on looking the same all the time, but we will assume that. Now the essential point is this: What is the empirical reason that makes you call a number of appearances, appearances of the same desk? What makes you say on successive occasions, I am seeing the same desk? The first thing to notice is this, that it does not matter what is the answer, so long as you have realized that the answer consists in something empirical and not in a recognized metaphysical identity of substance. There is something given in experience which makes you call it the same desk, and having once grasped that fact, you can go on and say, it is that something (whatever it is) that makes you call it the same desk which shall be *defined* as *constituting* it the same desk, and there shall be no assumption of a metaphysical substance which is identical throughout. It is a little easier to the untrained mind to conceive of an identity than it is to conceive of a system of correlated particulars, hung one to another by relations of similarity and continuous change and so on. That idea is apparently more complicated, but that is what is empirically given in the real world, and substance, in the sense of something which is continuously identical in the same desk, is not given to you. Therefore in all cases where you seem to have a continuous entity persisting through changes, what you have to do is to ask yourself what makes you consider the successive appearances as belonging to one thing. When you have found out what makes you take the view that they belong to the same thing, you will then see that that which has made you say so, is all that is *certainly* there in the way of unity. Anything that there may be over and above that, I shall recognize as something I cannot know. What I can know is that there are a certain series of appearances linked together, and the series of those appearances I shall define

as being a desk. In that way the desk is reduced to being a logical fiction, because a series is a logical fiction. In that way all the ordinary objects of daily life are extruded from the world of what there is, and in their place as what there is you find a number of passing particulars of the kind that one is immediately conscious of in sense. I want to make clear that I am not *denying* the existence of anything; I am only refusing to affirm it. I refuse to affirm the existence of anything for which there is no evidence, but I equally refuse to deny the existence of anything against which there is no evidence. Therefore I neither affirm nor deny it, but merely say, that is not in the realm of the knowable and is certainly not a part of physics; and physics, if it is to be interpreted, must be interpreted in terms of the sort of thing that can be empirical. If your atom is going to serve purposes in physics, as it undoubtedly does, your atom has got to turn out to be a construction, and your atom will in fact turn out to be a series of classes of particulars. The same process which one applies to physics, one will also apply elsewhere. The application to physics I explained briefly in my book on the *External World*, Chapters III and IV (Open Court Publishing Co., 1914).

I have talked so far about the unreality of the things we think real. I want to speak with equal emphasis about the reality of things we think unreal, such as phantoms and hallucinations. Phantoms and hallucinations, considered in themselves, are, as I explained in the preceding lectures, on exactly the same level as ordinary sense-data. They differ from ordinary sense-data only in the fact that they do not have the usual correlations with other things. In themselves they have the same reality as ordinary sense-data. They have the most complete and absolute and perfect reality that anything can have. They are part of the ultimate constituents of the world, just as the fleeting

sense-data are. Speaking of the fleeting sense-data, I think it is very important to remove out of one's instincts any disposition to believe that the real is the permanent. There has been a metaphysical prejudice always that if a thing is really real, it has to last either forever or for a fairly decent length of time. That is to my mind an entire mistake. The things that are really real last a very short time. Again I am not denying that there *may* be things that last forever, or for thousands of years; I only say that those are not within our experience, and that the real things that we know by experience last for a very short time, one tenth or half a second, or whatever it may be. Phantoms and hallucinations are among those, among the ultimate constituents of the world. The things that we call real, like tables and chairs, are systems, series of classes of particulars, and the particulars are the real things, the particulars being sense-data when they happen to be given to you. A table or chair will be a series of classes of particulars, and therefore a logical fiction. Those particulars will be on the same level of reality as a hallucination or a phantom. I ought to explain in what sense a chair is a series of classes. A chair presents at each moment a number of different appearances. All the appearances that it is presenting at a given moment make up a certain class. All those sets of appearances vary from time to time. If I take a chair and smash it, it will present a whole set of different appearances from what it did before, and without going as far as that, it will always be changing as the light changes, and so on. So you get a series in time of different sets of appearances, and that is what I mean by saying that a chair is a series of classes. That explanation is too crude, but I leave out the niceties, as that is not the actual topic I am dealing with. Now each single particular which is part of this whole system is linked up with the others in the system. Supposing, e. g., I take as my par-

ticular the appearance which that chair is presenting to me at this moment. That is linked up first of all with the appearance which the same chair is presenting to any one of you at the same moment, and with the appearance which it is going to present to me at later moments. There you get at once two journeys that you can take away from that particular, and that particular will be correlated in certain definite ways with the other particulars which also belong to that chair. That is what you mean by saying—or what you ought to mean by saying—that what I see before me is a real thing as opposed to a phantom. It means that it has a whole set of correlations of different kinds. It means that that particular, which is the appearance of the chair to me at this moment, is not isolated but is connected in a certain well-known familiar fashion with others, in the sort of way that makes it answer one's expectations. And so, when you go and buy a chair, you buy not only the appearance which it presents to you at that moment, but also those other appearances that it is going to present when it gets home. If it were a phantom chair, it would not present any appearances when it got home, and would not be the sort of thing you would want to buy. The sort one calls real is one of a whole correlated system, whereas the sort you call hallucinations are not. The respectable particulars in the world are all of them linked up with other particulars in respectable, conventional ways. Then sometimes you get a wild particular, like a merely visual chair that you cannot sit on, and say it is a phantom, a hallucination, you exhaust all the vocabulary of abuse upon it. That is what one means by calling it unreal, because "unreal" applied in that way is a term of abuse and never would be applied to a thing that *was* unreal because you would not be so angry with it.

I will pass on to some other illustrations. Take a person. What is it that makes you say, when you meet

your friend Jones, "Why, this is Jones"? It is clearly not the persistence of a metaphysical entity inside Jones somewhere, because even if there be such an entity, it certainly is not what you see when you see Jones coming along the street; it certainly is something that you are not acquainted with, not an empirical datum. Therefore plainly there is something in the empirical appearances which he presents to you, something in their relations one to another, which enables you to collect all these together and say, "These are what I call the appearances of one person," and that something that makes you collect them together is not the persistence of a metaphysical subject, because that, whether there be such a persistent subject or not, is certainly not a datum, and that which makes you say "Why, it is Jones" is a datum. Therefore Jones is not constituted as he is known by a sort of pin-point ego that is underlying his appearances, and you have got to find some correlations among the appearances which are of the sort that make you put all those appearances together and say, they are the appearances of one person. Those are different when it is other people and when it is yourself. When it is yourself, you have more to go by. You have not only what you look like, you have also your thoughts and memories and all your organic sensations, so that you have a much richer material and are therefore much less likely to be mistaken as to your own identity than as to some one else's. It happens, of course, that there are mistakes even as to one's own identity, in cases of multiple personality and so forth, but as a rule you will know that it is you because you have more to go by than other people have, and you would know it is you, not by a consciousness of the ego at all but by all sorts of things, by memory, by the way you feel and the way you look and a host of things. But all those are empirical data, and those enable you to say that the person to whom something happened yesterday

was yourself. So you can collect a whole set of experiences into one string as all belonging to you, and similarly other people's experiences can be collected together as all belonging to them by relations that actually are observable and without assuming the existence of the persistent ego. It does not matter in the least to what we are concerned with, what exactly is the given empirical relation between two experiences that makes us say, "These are two experiences of the same person." It does not matter precisely what that relation is, because the logical formula for the construction of the person is the same whatever that relation may be, and because the mere fact that you can know that two experiences belong to the same person proves that there is such an empirical relation to be ascertained by analysis. Let us call the relation R . We shall say that when two experiences have to each other the relation R , then they are said to be experiences of the same person. That is a definition of what I mean by "experiences of the same person." We proceed here just in the same way as when we are defining numbers. We first define what is meant by saying that two classes "have the same number," and then define what a number is. The person who has a given experience x will be the class of all those experiences which are "experiences of the same person" as the one who experiences x . You can say that two events are co-personal when there is between them a certain relation R , namely that relation which makes us say that they are experiences of the same person. You can define the person who has a certain experience as being those experiences that are co-personal with that experience, and it will be better perhaps to take them as a series than as a class, because you want to know which is the beginning of a man's life and which is the end. Therefore we shall say that a person is a certain series of experiences. We shall not deny that there may be a metaphysical ego. We shall merely say

that it is a question that does not concern us in any way, because it is a matter about which we know nothing and can know nothing, and therefore it obviously cannot be a thing that comes into science in any way. What we know is this string of experiences that makes up a person, and that is put together by means of certain empirically given relations, such, e. g., as memory.

I will take another illustration, a kind of problem that our method is useful in helping to deal with. You all know the American theory of neutral monism, which derives really from William James and is also suggested in the work of Mach, but in a rather less developed form. The theory of neutral monism maintains that the distinction between the mental and the physical is entirely an affair of arrangement, that the actual material arranged is exactly the same in the case of the mental as it is in the case of the physical, but they differ merely in the fact that when you take a thing as belonging in the same context with certain other things, it will belong to psychology, while when you take it in a certain other context with other things, it will belong to physics, and the difference is as to what you consider to be its context, just the same sort of difference as there is between arranging the people in London alphabetically or geographically. So, according to William James, the actual material of the world can be arranged in two different ways, one of which gives you physics and the other psychology. It is just like rows or columns: in an arrangement of rows and columns, you can take an item as either a member of a certain row or a member of a certain column; the item is the same in the two cases, but its context is different.

If you will allow me a little undue simplicity I can go on to say rather more about neutral monism, but you must understand that I am talking more simply than I ought to do because there is not time to put in all the shadings and

qualifications. I was talking a moment ago about the appearances that a chair presents. If we take any one of these chairs, we can all look at it, and it presents a different appearance to each of us. Taken all together, taking all the different appearances that that chair is presenting to all of us at this moment, you get something that belongs to physics. So that, if one takes sense-data and arranges together all those sense-data that appear to different people at a given moment and are such as we should ordinarily say are appearances of the same physical object, then that class of sense-data will give you something that belongs to physics, namely, the chair at this moment. On the other hand, if instead of taking all the appearances that that chair presents to all of us at this moment, I take all the appearances that the different chairs in this room present to me at this moment, I get quite another group of particulars. All the different appearances that different chairs present to me now will give you something belonging to psychology, because that will give you my experiences at the present moment. Broadly speaking, according to what one may take as an expansion of William James, that should be the definition of the difference between physics and psychology.

We commonly assume that there is a phenomenon which we call seeing the chair, but what I call my seeing the chair according to neutral monism is merely the existence of a certain particular, namely the particular which is the sense-datum of that chair at that moment. And I and the chair are both logical fictions, both being in fact a series of classes of particulars, of which one will be that particular which we call my seeing the chair. That actual appearance that the chair is presenting to me now is a member of me and a member of the chair, I and the chair being logical fictions. That will be at any rate a view that you can consider if you are engaged in vindicating neutral monism.

There is no simple entity that you can point to and say: this entity is physical and not mental. According to William James and neutral monists that will not be the case with any simple entity that you may take. Any such entity will be a member of physical series and a member of mental series. Now I want to say that if you wish to test such a theory as that of neutral monism, if you wish to discover whether it is true or false, you cannot hope to get any distance with your problem unless you have at your fingers' end the theory of logic that I have been talking of. You never can tell otherwise what can be done with a given material, whether you can concoct out of a given material the sort of logical fictions that will have the properties you want in psychology and in physics. That sort of thing is by no means easy to decide. You can only decide it if you really have a very considerable technical facility in these matters. Having said that, I ought to proceed to tell you that I have discovered whether neutral monism is true or not, because otherwise you may not believe that logic is any use in the matter. But I do not profess to know whether it is true or not. I feel more and more inclined to think that it may be true. I feel more and more that the difficulties that occur in regard to it are all of the sort that may be solved by ingenuity. But nevertheless there *are* a number of difficulties; there are a number of problems, some of which I have spoken about in the course of these lectures. One is the question of belief and the other sorts of facts involving two verbs. If there are such facts as this, that, I think, may make neutral monism rather difficult, but as I was pointing out, there is the theory that one calls behaviorism, which belongs logically with neutral monism, and that theory would altogether dispense with those facts containing two verbs, and would therefore dispose of that argument against neutral monism. There is, on the other hand, the argument from emphatic par-

particulars, such as "this" and "now" and "here" and such words as that, which are not very easy to reconcile, to my mind, with the view which does not distinguish between a particular and experiencing that particular. But the argument about emphatic particulars is so delicate and so subtle that I cannot feel quite sure whether it is a valid one or not, and I think the longer one pursues philosophy, the more conscious one becomes how extremely often one has been taken in by fallacies, and the less willing one is to be quite sure that an argument is valid if there is anything about it that is at all subtle or elusive, at all difficult to grasp. That makes me a little cautious and doubtful about all these arguments, and therefore although I am quite sure that the question of the truth or falsehood of neutral monism is not to be solved except by these means, yet I do not profess to know whether neutral monism is true or is not. I am not without hopes of finding out in the course of time, but I do not profess to know yet.

As I said earlier in this lecture, one thing that our technique does, is to give us a means of constructing a given body of symbolic propositions with the minimum of apparatus, and every diminution in apparatus diminishes the risk of error. Suppose, e. g., that you have constructed your physics with a certain number of entities and a certain number of premises; suppose you discover that by a little ingenuity you can dispense with half of those entities and half of those premises, you clearly have diminished the risk of error, because if you had before 10 entities and 10 premises, then the 5 you have now would be all right, but it is not true conversely that if the 5 you have now are all right, the 10 must have been. Therefore you diminish the risk of error with every diminution of entities and premises. When I spoke about the desk and said I was not going to assume the existence of a persistent substance underlying its appearances, it is an example of the case in

point. You have anyhow the successive appearances, and if you can get on without assuming the metaphysical and constant desk, you have a smaller risk of error than you had before. You would not necessarily have a smaller risk of error if you were tied down to *denying* the metaphysical desk. That is the advantage of Occam's Razor, that it diminishes your risk of error. Considered in that way you may say that the whole of our problem belongs rather to science than to philosophy. I think perhaps that is true, but I believe the only difference between science and philosophy is, that science is what you more or less know and philosophy is what you do not know. Philosophy is that part of science which at present people choose to have opinions about, but which they have no knowledge about. Therefore every advance in knowledge robs philosophy of some problems which formerly it had, and if there is any truth, if there is any value in the kind of procedure of mathematical logic, it will follow that a number of problems which had belonged to philosophy will have ceased to belong to philosophy and will belong to science. And of course the moment they become soluble, they become to a large class of philosophical minds uninteresting, because to many of the people who like philosophy, the charm of it consists in the speculative freedom, in the fact that you can play with hypotheses. You can think out this or that which *may* be true, which is a very valuable exercise until you discover what *is* true; but when you discover what is true the whole fruitful play of fancy in that region is curtailed, and you will abandon that region and pass on. Just as there are families in America who from the time of the Pilgrim Fathers onward had always migrated westward, toward the backwoods, because they did not like civilized life, so the philosopher has an adventurous disposition and likes to dwell in the region where there are still uncertainties. It is true that the transferring of a

region from philosophy into science will make it distasteful to a very important and useful type of mind. I think that is true of a good deal of the applications of mathematical logic in the directions that I have been indicating. It makes it dry, precise, methodical, and in that way robs it of a certain quality that it had when you could play with it more freely. I do not feel that it is my place to apologize for that, because if it is true, it is true. If it is not true, of course, I do owe you an apology; but if it is, it is not my fault, and therefore I do not feel I owe any apology for any sort of dryness or dulness in the world. I would say this too, that for those who have any taste for mathematics, for those who like symbolic constructions, that sort of world is a very delightful one, and if you do not find it otherwise attractive, all that is necessary to do is to acquire a taste for mathematics, and then you will have a very agreeable world, and with that conclusion I will bring this course of lectures to an end.

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