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"The Resultant Thrust of Fluid-Pressure in Bend-Pipes."

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The ordinary method for ascertaining the magnitude and direction of the force tending to drive off a bend on a line of pipe subjected to internal fluid-pressure is by the construction of a diagram of forces—the resultant force in the straight pipe on either side of the bend being equal to the area A of the pipe, multiplied by the intensity p of the fluid-pressure, and acting axially; the resultant of these two forces is $A \times p \times 2 \sin \frac{\theta}{2}$, where θ is the angle subtended by the bend.

At first sight, however, a suspicion might be aroused that, since water under pressure acts with equal intensity in every direction, equilibrium might somehow be established by the fluid in the bend, which, being subject to a pressure equal to that in the straight pipe at each end of it, might be expected to offer an equal resistance; further, that an investigation which takes into consideration the force acting in one direction only, might lead to an incorrect result. That this is not so may doubtless be shown in more ways than one; but the Author believes the following demonstration is new.

If the bend be projected in a direction parallel to the resultant force, the ends of the bend appear as two elliptical figures (*Figs. 1*). It is obvious that the pressure of the fluid on these ellipses is unbalanced, and that their area A′ multiplied by the intensity of pressure should give the same result as that obtained above. Let D d be respectively the major and minor axes of the ellipses; then

$$A' = D \times d \times \frac{\pi}{4},$$

but

$$d = D \sin \frac{\theta}{2};$$

therefore

$$A' = D^2 \times \frac{\pi}{4} \times \sin \frac{\theta}{2}$$

$$= A \sin \frac{\theta}{2}.$$

The sum of the areas of the ellipses is $2\ A \sin\frac{\theta}{2}$, and the unbalanced force is $A \times p \times 2 \sin\frac{\theta}{2}$—as previously obtained from the diagram of forces.

The total force tending to thrust off the bend is of course dependent on the angle subtended by the bend and not on the radius of the curved pipe. However, with a large radius, and therefore in practice a longer bend, the intensity of the external resistance necessary would be less if supplied, for instance, by the continuous bearing afforded by the side of the pipe-trench. In some cases it is necessary to anchor the bends; as where a pipe rises out of the ground to pass over a bridge. The prudence of doing this became apparent in several instances on the aqueduct from Thirlmere to Manchester. The straps which surrounded the bends, and to which the anchor-rods were attached, did not always follow truly the surface of the pipe, and a slight space was in some cases observable between the strap and the pipe. When the pipes were charged, however, the inner edge of the strap coincided perfectly with the surface of the pipe—showing that the anchor rods were doing the work for which they had been provided. To illustrate the magnitude of the forces to be resisted, it may be mentioned that in the cases of the 40-inch pipes used on the Thirlmere aqueduct, the unbalanced thrust at a bend of 34°, subjected to an internal pressure of 100 lbs. per square inch (and in some instances the pressures were as high as 180 lbs. per square inch), amounts to nearly 33 tons.

Figs. 1.