THE INVOLUTE OF THE CIRCUMFERENCE OF A CIRCLE.

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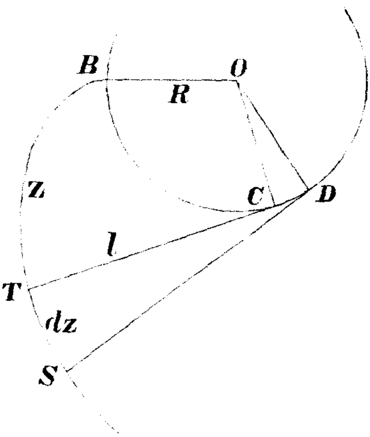
Several years ago I was asked a question like this: Suppose a string 100 feet long to be wound round a vertical cylinder one foot in diameter; if a man takes hold of one end of the string and unwinds it all, by walking around the cylinder, keeping the string stretched, how far will he travel? I solved the general problem as follows:

In the figure let B mark the point where the unwinding begins, and suppose $C\ T$ to represent the part of the string that has been unwound. Denote the length of this by l, denote the radius of the circle by R, and let the *circular measure* of the corresponding angle $B\ O\ C$ be v. Also denote the actual length of the arc $B\ T$ by z. Let $T\ S$ be the differential of z, then the circular measure of the angle $C\ O\ D$ will be dv.



The circular measure of the angle between the tangents $T\ C$ and $S\ D$ is also dv. We shall have

(1) $l = B\ C = R.\ v.$

(2) $d\ z = T\ S = l.\ dv.$ (in the limit.)

By substitution of (1) in (2), $d\ z = R\ v.\ dv.$

By integration,

(3) $$z = \frac{R\,v^2}{2} = \frac{l^2}{2\ R};$$

which is a general formula, easy to remember, that agrees with formula (2) given by Prof. D'Auria, on page 46 of the July number of the current volume of this Journal.

By making $2R = 1$, and $l = 100$, we find the answer to the question proposed at the beginning of this article to be 10,000 feet.

The area of the figure $B\ T\ C$ is found with equal facility, as follows: At the limit we have the area included between $T\ S$, and the lines $T\ C$ and $S\ D$ equal to $\frac{1}{2}\ dz.\ l$, or, denoting the area $B\ T\ C$ by A, we have $d\ A = \frac{1}{2}\ dz.\ l$.

By substituting in this from (2) and (1) we get

$$d\ A = \frac{R^2\ r^2\ dr}{2};$$

By integrating,

$$A = \frac{R^2\ r^3}{6};$$

which agrees with the formula for A given by Prof. D'Auria on p. 47.

Dust Showers.—M. Daubrée has given an account of the dust showers that were observed from the 21st to the 25th of April, 1880, in the departments of the Lower Alps, Isère and Ain. Heavy dark clouds, resembling a dense yellowish mist, traversed the valleys during the day, depositing a reddish dust with a little water. The neighboring mountains were covered with snow, which assumed a rusty hue to the height of about three kilometres (1·864 miles) and above that height it remained white. The snow, upon melting, left a deposit of a yellowish-brown color, a little redder than limonite dust and almost impalpable. When heated in a tube it blackened, with a disengagement of water and some organic matters of a strong odor. With dilute chlorhydric acid it effervesced strongly, showing a considerable proportion of carbonate of lime. Boiling chlorhydric acid destroyed the yellow color and left a residuum which was fusible under the blow-pipe into a white globule. This residuum contained numerous flakes of white and colored mica, together with some particles which appeared to be feldspar. None of the particles were attracted by the magnet, so that the dust was not probably cosmical, but it resembled in its structure the sand of Sahara. F. de Jussieu reports a similar shower at Autun, on the 15th of April, in which there were traces of iron and perhaps also of lead. A shower occurred in Sicily on the 10th of April, which is said to have deposited considerable quantities of metallic iron covered with a thin layer of oxide. Prof. Orazio Silvestri, of the Meteorological Institute in Catania, gives an account of a shower during the night of March 29—30, in which the dust contained particles of iron, small infusoria and organic molecules. Some of the particles were spherical, as if they had been melted.—*Comptes Rendus, Fortsch. d. Zeit.* C.