

WHAT IS THE LABORATORY METHOD?(¹)

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I have given as caption of what I am about to say, a question I am not prepared to answer.

We have recently been discussing here in Chicago various phases of the teaching of mathematics at various times and in various bodies; the term "Laboratory Method" has been used, but not (to my recollection) strictly defined. It has rather been used, I take it, in accordance with that sound pedagogic dictum, *Ideas first, definitions afterward*, as a convenient term for what we are trying to work out; a symbol—the *x-method*—whose meaning we are engaged in fixing.²

In my mind, this *x-method* is somewhat vaguely outlined by various characteristics, no one of which constitutes the method or is peculiar to it.

I have thought that a brief enumeration of these characteristics might make a useful basis for discussion. Perhaps by the free addition of other characteristics and the elimination of any which are improperly here, the discussion may give us a somewhat closer approximation to a common understanding of what we mean in saying "*the laboratory method*."

For brevity, I wish at the outset to prefix what I shall say with a large "*It seems to me*," and to disclaim any dogmatic intention in adopting a more or less dogmatic form for clearness and brevity.

CHARACTERISTICS.

I. A DESIRE TO IMPROVE.

This is the chief single characteristic—the perception by each one of us that his own teaching *can* be improved, and the active purpose to do so.

The *x-method* welcomes and assimilates *whatever* is an im-

(¹) Address delivered before the Mathematics Section of the Central Association of Science and Mathematics Teachers, at Armour Institute of Technology, Chicago, April 11, 1903.

(²) "A laboratory system involving a synthesis and development of the best pedagogic methods at present in use in mathematics and the physical sciences." Moore, On the Foundations of Mathematics, Presidential address: *Bull. Am. Math. Soc.*, 1903, p. 424; also *Science*, 1903, p. 416; also being published in *School Science*.

provement. It recognizes, also, that the determination whether or not any proposed change is an improvement must take into account the purpose of the teaching of mathematics as well as the personality of the teacher and the conditions under which he works.

We are assuming as basis of our discussions a clearly formulated idea of the function of mathematics in secondary education; of the purpose and value of the teaching of mathematics. This is too large a topic to open today, and yet its careful consideration must both logically and actually, precede consideration of methods. No teacher of mathematics can do his best work who has not the ultimate purpose for which he is teaching mathematics, constantly, consciously, in mind, as the final arbiter of *what* he is to teach, and *how* he is to teach it.

Methods are but *means*; the utility of the means can be judged only from the point of view of the *end*.

The *x*-method fosters whatever hastens progress toward the *end*, and accepts nothing else.

It frowns upon *fads*; that is, the exaltation of the means into *ends in themselves*.

All of us are conditioned by our environment. The work we have to do is more or less minutely mapped out for us by curricula, by examination standards, by public opinion in the profession and in the community.

While this frees none of us from the obligation of forming his own clear ideas and ideals of the function of his subject, it may impose on him some specific things to be done. He has certain things to teach, *in fact*, whether these topics are selected by himself in conformity with his own ideals, or by others according to their ideas.

In either case the ultimate end is resolved into a series of mediate ends—the teaching of specific subjects and topics. What should be the teachers attitude as to method under these (not ideal, but very real) conditions? Evidently his task is to lead his pupils to intelligent mastery of the things taught, and withal to develop thought-power which shall be available later. Let him then take as his watchword, dominating all methods, *clearness*;

use the x -method, the y -method or the z -method when (and only when) they aid in attaining clearness.

These things are all truisms—fundamental postulates—but I hope a mathematical audience will pardon the formulation of some fundamental postulates.

In entering into details, I must confine myself very strictly and briefly to points that have been mentioned in the present series of discussions on the laboratory method. Such mention has also been the sufficient credential for the admission of any point.

It has not seemed desirable to attempt to avoid redundancies, the implication of one characteristic by another, or the more or less complete overlapping of topics; the various heads have not even been scrutinized as to compatibility. No attempt has been made to determine or indicate their relative weight or importance.

II. INTEREST.

That we work best only when interested, is generally believed.

A common figure for this is that of food. To be most effectively nutritious, food must be palatable. This has been appreciated for centuries, still the Nobel prize was awarded in 1901 ⁽³⁾ for researches definitely establishing close connection between appetite and digestion, and showing that for best results food should be eaten with interest and enjoyment.

So with us, in mathematics. We have long known that to be most nutritious, mathematics must be palatable, but our present discussions are evidently stimulated by a more vivid consciousness of this truth. We are making a stronger effort to awaken effectively the appetite so essential to mathematical digestion.

III. CHARACTERISTICS RELATING TO SUBJECT MATTER.

1. CORRELATION OF SUBJECTS.

a. The mathematical subjects among themselves. Geometry and algebra (involving and illuminating arithmetic constantly, and developing the elements of trigonometry on occasion) should be taught side by side, and not separately. No “water-tight compartments.”

b. Mathematics with physics. Let physics be taught simul-

⁽³⁾ The *Hospital*, March, 1903.

taneously with mathematics, so that the demand for mathematical treatment of questions may arise in physics, and so that physics may furnish the opportunity for immediate application of mathematical theory.

2. AMALGAMATION OF MATHEMATICS AND PHYSICS.

Teach mathematics no longer; teach physics no longer. Take what has hitherto been taught under these titles, and with suitable eliminations and additions, amalgamate the whole mass into a homeogeneous whole.

This is questionable (*a*) as to *feasibility*. Can the dynamic and static phases be disregarded? Can the making of experiments, the actual manipulation and the quiet thinking, the working out of the theoretic relations be combined? (*b*) as to *desirability*. "I want my roast beef and pudding *correlated*, but I don't want them *amalgamated*."

IV. CHARACTERISTICS RELATING TO MODE OF INSTRUCTION.

1. PROCEED FROM THE CONCRETE TO THE ABSTRACT.

Let experiments be freely used (physical, metrical, graphic, numerical). Let the general definitions and theorems be suggested by concrete special cases and be developed out of them by the pupils if possible. Don't banish the *abstract* from the teaching of mathematics, but change it from a noun (or adjective) to a verb. Never teach *abstract* mathematics but only *abstracted* mathematics and as far as possible let the class abstract its own mathematics. Such work, studying general laws through particular instances, is analogous to that of the physical *laboratory*.

REMARK. The term *concrete* must be taken in a broad sense. *What* is concrete depends upon circumstances. The physically concrete things, the things that can be handled, should be utilized in the mathematical instruction to a much greater extent than heretofore. But other things are concrete also. For example, in algebra much of arithmetic is concrete. In college mathematics (and there is no reason why the main thoughts of what has been said should not be applied in college mathematics) the results of the previous mathematics which have been well grasped would all come under the head *concrete*, used broadly. With such a use of the term, is there any upper limit to the applicability

of the dictum: *Proceed from the concrete to the abstract; from the particular to the general?*

The need of more stress on the concrete, the physical side, is being emphasized in Germany also. In a recent address Professor Klein, of Göttingen, said:

"No teaching in gymnasium and *real schule* is so difficult as that of mathematics, since the large majority of the pupils is decidedly indisposed to allow itself to be harnessed in the rigid framework of logical conclusions. The interest of young people is much more easily won if one sets out from sense-objects and gradually leads on to abstract formulations. It is therefore psychologically quite correct to follow this path. This course commends itself none the less if we inquire concerning the real goal of instruction in mathematics. Sharpening the understanding was formerly regarded as the end. Another chief end is: To make the conviction grow that *correct thought on the foundation of correct premises gives mastery of the external world*. To do this, attention must be directed to the external world from the beginning.

"All this is certainly most true; but there lies a danger in it, and to this danger I wish to call attention today. . . . It is possible that through the mere mass of interesting applications, the real logical training may be crippled, and under no circumstances may this happen, for then the real marrow of the whole is lost. Hence: We desire emphatically, an enlivening of instruction in mathematics by means of its applications, but we desire also that the pendulum which in earlier decades perhaps swung too far in the abstract direction, should not now swing to the other extreme, but we wish to remain in the just mean.

"To preserve the just mean is the problem and the art of the teacher, which should be advanced through an improved *preparation of teachers*."

Note.—It will be remembered that the preparation of the Prussian teachers to whom reference is made, already consists of three years of graduate study of theoretic mathematics at the University, followed by two years of practical training in the art of teaching.

Similar ideas were also presented and discussed at the *Versammlung deutscher Philologen und Schulmänner*, in Strassburg, September, 1901.⁽⁵⁾

2. TEACH THROUGH THE EYE:

- a. By orderly arrangement (laboratory record book, geometry tablets, etc.)
- b. By use of colors.
- c. By careful drawing.
- d. By use of squared paper.

⁽⁴⁾ Klein: *Ueber den Mathematischen Unterricht an den höheren Schulen*. Jahresbericht der deutschen Mathematiker Vereinigung, 1902, pp. 128-140.

⁽⁵⁾ *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht*, 1902, p. 669.

The last is specially valuable (1) to show more vividly and clearly relations running through a mass of data; (2) to give geometric form to algebraic relations.

3. TEACH EVERYTHING IN TWO WAYS:

When one form of presentation is not clear perhaps another will be. Show the different bearings and interpretations of the same truth. All this is subject to the more fundamental dictum: *Teach everything clearly*. Use different ways of expressing the same thought whenever one illustrates and illuminates the other. Due caution to avoid confusion is, of course, needed.

4. WORK WITH A LARGE BODY OF AXIOMS. Assume many things which might be proved. Defer the more philosophic questions.

5. ACCEPT AND ENCOURAGE PROOFS BASED ON INTUITION; PROOFS BASED ON MEASUREMENT.

This is a reaction from the attempt to deny the validity of such proofs. It is not meant that there should not be developed from them (by generalization) the customary abstract proofs, nor that we should overlook one of the special functions of mathematics—training in precision—and the need of such training.

6. LESS STRESS ON EXHIBITION OF WORK done outside; more time given to actual work in the class, thus making the classroom a working place, a *laboratory*.

This does not mean that there is to be *no* exhibition of work.

Societies of learned mathematicians exist, one of whose main purposes is the exhibition of work, and the mathematician will surely not deny to young pupils the benefit and pleasure which he does not forego himself.

It does mean that the teacher helps the pupil to *do* work, rather than *examines* him to find whether or not he has done an allotted task.

7. DEVELOP NEW MATTER INDUCTIVELY WITH THE CLASS. The pupil should seldom, if ever, be allowed to struggle through new theories and processes alone for the first time. Let the teacher develop the theory, with the text in the background for reference.

See President Hadley, Meaning and Purpose of Secondary Education, *School Review*, December, 1902, p. 738.

Such work also bears considerable analogy to that of a physical laboratory.

8. TRAIN ENGINEERS AS WELL AS PROFESSORS. That is, do not proceed on the assumption that every pupil is to be a professional mathematician. A better formulation would be: *Train human beings*; prepare good citizens, equipped with a well rounded education, and not specialists of any sort. Let the university and the technical school train the professor and the engineer.

9. NO DAILY ALLOTMENT OF WORK, the same for all each day. Rather a general allotment, each one working on at the rate best suited to his strength (laboratory idea).

10. PUPILS WORK INDEPENDENTLY. The instructor is a friend, co-worker, not a task master.

11. PUPILS WORK TOGETHER. Mutual assistance and dissemination of results encouraged. This does not contravene what all will agree is one of the chief ends of the teaching of mathematics, viz., to teach the pupil to *think for himself*. We all get suggestions from others, to the benefit rather than impairment of our independence.

12. TWO-HOUR SESSIONS. If the classroom is the workshop and nearly all of the work is done there, a one-hour session is inadequate.

V. SUMMARY.

What are the chief results of what has been said?

1. Close correlation of physics, arithmetic, geometry, algebra, trigonometry.
2. In teaching pass from concrete to abstract.

V. WHAT CAN TEACHERS DO NOW TO ATTAIN THESE ENDS?

This question has been answered so well and in so stimulating a way by the previous papers and by the report of the committee that it would be superfluous at this juncture to say more.

With the motto: *Evolution, not revolution*, let the teacher adopt whatever is good, making some improvement each year, each day, and the evolution will go forward wonderfully well.