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XIX. *On the Friction in Free Surfaces of Liquids.**By A. OBERBECK*.*

1. **I**N his fine investigations on liquids which are withdrawn from the influence of gravity, Plateau† discusses the question why only a few liquids are suitable for the production of thin films, while most of them are totally unsuited for that purpose. According to his view, two properties of liquids play an essential part therein—the tension and the viscosity of their surfaces. The investigation of this second property is the aim of the present memoir.

As is well known, what is designated as the viscosity of a liquid is its deviation from the state of perfect fluidity, and manifests itself by a frictional resistance exerted by different quickly moving neighbouring parts of the liquid upon one another. From experiments which are subsequently to be more fully communicated, Plateau inferred that the amount of this frictional resistance varies according as the motion of the parts takes place in the interior or in the nearest vicinity of the free surface of the liquid; so that we have to distinguish an internal and a superficial viscosity. As the hydrodynamic differential equations with respect to the friction are at present in excellent accordance with all the known facts, and in them the friction has for its expression a single characteristic constant for each liquid (the friction-coefficient), the phenomenon discovered by Plateau might also be expressed in the following form:—The friction-coefficient is indeed a constant in the interior of a liquid; but at very small distances from the free surface it is a function of the distance from the free surface.

All the experiments which have been made upon free liquid surfaces teach that the liquid parts in them are in essentially different conditions from those in the interior; hence it would be quite conceivable that their mutual friction also is different. But it has been held necessary (by Poisson) to assume that the density rapidly changes very near the free surface. Although hitherto neither an experimental proof of this has been produced, nor even theoretically is there any occasion to designate this assumption as a necessary one, yet a variation of the friction-coefficient would include it as a self-evident consequence.

The friction of two different liquids against each other is

* Translated from Wiedemann's *Annalen*, 1880, No. 12, pp. 634–652.

† Plateau, *Mém. de l'Acad. de Belgique*, 1868, xxxvii. pp. 1–102; *Pogg. Ann.* cxli. pp. 44–58 (1870).

essentially different from the friction in the interior of a liquid*. Here also a similar view might be taken—namely, that the friction-coefficient in the interior of the first liquid has a constant value, rapidly changes on approaching nearer to the boundary surface, and on the other side again assumes that value which belongs to it for the second liquid.

These considerations show that the superficial viscosity is absolutely incompatible with the notions we have hitherto entertained of the nature of liquids. Whether it actually exists, and therefore whether it is justifiable to assume a different value of the coefficient of friction at the surface from that in the interior, can of course only be decided by experiment.

2. Plateau's fundamental experiments on surface-viscosity consisted in observing the time taken by a magnetic needle to fall back from a deflection of 90° to the magnetic meridian. For this different times were necessary, according as the motion of the needle took place in the free surface or in the interior of the liquid. If the time in the first case was greater than in the second, Plateau inferred a greater—if the contrary, a less viscosity in the surface. Thus, according to Plateau, water, aqueous salt-solutions, glycerine, and especially solutions of albumen and saponine in water are liquids in which the friction at the free surface is greater than in the interior. Conversely, with alcohol, oil of turpentine, ether, bisulphide of carbon the internal viscosity is greater than the external. Finally, by mixing water and alcohol a liquid could be produced in which the two observed times are equal. Plateau has also endeavoured in a somewhat different manner to ascertain numerical values for the difference between the friction in the interior and at the surface. He observed the angle to which the needle, in falling back to the meridian, moved beyond its original position of equilibrium. From this, on the assumption that the resistance of the liquid is proportional to the square of the velocity, numerical values for the differences of viscosity in different cases can be calculated. Plateau himself, however, admits† that they can make no claim to precision.

After the publication of these experiments the surface-viscosity was investigated by Luvini‡ and Marangoni§; and the result arrived at by the latter was, that a distinction between internal and superficial viscosity is not justified. He

* Cf. O. E. Meyer, Pogg. *Ann.* cxiii. pp. 68, 441 (1861).

† *Mém. de l'Acad. de Belg.* xxxviii. p. 76 (1868).

‡ *Phil. Mag.* [4] xl. pp. 190–197 (1870).

§ *Nuovo Cimento*, [2] v.–vi. pp. 239–273 (1872), [3] iii. pp. 50–68, 97–115, 192–212 (1879).

believes rather that the processes observed by Plateau, especially the retardation of the motion of the magnetic needle at the surface of some liquids, were occasioned partly by capillary actions in consequence of the alteration of the form of the free surface, partly by its being contaminated with foreign substances. In particular, he assumes that the latter form a thin elastic layer on the surface, which, on the motion of a body in the surface, oppose a species of elastic action. In his replies*, Plateau maintains his original view. Instead of entering into a criticism of the conflicting opinions, it seemed to me more important to examine the phenomena in question afresh by a different method. This method had to fulfil the following conditions:—

(a) The effects of friction shall admit of being separated from capillary actions at the surface, or from any surface-elasticity which (according to Marangoni) may be present;

(b) The frictional resistances shall be expressed by numerical values which can really be regarded as a measure for them;

(c) These numerical values shall be comparable, both in the interior and at the surface of the same liquid, and also in different liquids under like circumstances.

It is easy to understand that the usual methods for the determination of the coefficients of friction could not be employed here. In the flow of liquids through capillary tubes the free surface does not at all come into consideration. Moreover even the rotational oscillation of a disk is by no means adapted for the investigation of the phenomena with which we have here to do. If it be caused to oscillate at different distances from the free surface, O. E. Meyer† has observed that the frictional resistances diminish on approaching it; and this is exactly what was to be expected according to the theory of these oscillations unfolded by O. E. Meyer‡. According to it, by far the larger portion of the action upon the disk depends on those liquid particles which are vertically over or under the disk, and not on those which lie in the same horizontal plane with it. If the layer of liquid above the disk is very thin, it follows the oscillations of the disk irrespective of the value of the coefficient of friction.

Lastly, Plateau's method is well suited to demonstrate the phenomenon qualitatively; but it does not permit us to decide whether the longer time taken by the magnet on the surface for its return to the position of equilibrium proceeds from augmented friction or from other counteracting forces.

* *Bull. de l'Acad. de Belg.* [2] xxxiv. pp. 401–419; xlviii. pp. 106–128 (1880).

† Pogg. *Ann.* cxiii. p. 415 (1861).

‡ *Loc. cit.* p. 62–67.

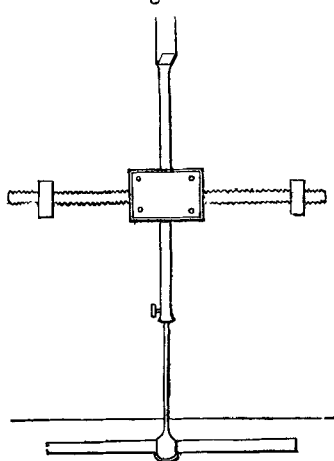
3. Hence, after a long series of preliminary experiments, I employed the following method, which, on the whole, satisfies the requirements above stated. Suspended by two fine, well-annealed platinum wires, a brass cross hangs bifilar, carrying a small mirror for the observation of the oscillations with scale and telescope. The two horizontal arms of the cross are furnished with screw-threads, on which suitable weights can be shifted, so that the oscillation-period can be varied at pleasure.

To the descending part suitable bodies can be fastened, which are destined to execute oscillations within the liquid (see fig. 1). I have used for this purpose chiefly thin plates or cylinders of brass.

The entire apparatus is suspended on a suitable frame, and with the aid of a micrometer-screw could be raised and lowered at pleasure without being at the same time rotated about its vertical axis. On the micrometer-screw a rise amounting to only 0.01 millim. could be read off. Lastly, a small magnetic needle was attached to the apparatus, with the aid of which the above-described mechanism could be set in steady rotatory oscillation. The oscillation-period and the diminution of the oscillations can then, in the well-known manner, be determined with greater precision. The experiments were executed as follows:—The plate was at first entirely immersed in the liquid, and its upper margin was at a determined distance from the free surface; it was afterwards raised till its upper margin was in the free surface or projected a determined distance above it.

In both cases the liquid opposed a certain resistance to the plate, which produced a diminution of the amplitudes. If the friction of the parts of the liquid in the free surface is greater than in the interior, this resistance will also be greater. But in order to observe it in the most favourable form, it was necessary to give to the vessel destined to hold the liquid the most suitable shape possible. It was rectangular, 150 millims. in length, and 30 millims. wide. The long sides consisted of glass plates. Besides, it was so contrived that two other glass

Fig. 1.



plates could be inserted in the vessel, parallel to the first, so as to permit observations to be made with the side plates at different distances. This arrangement was found necessary for the following reasons. If the brass strip turns round a vertical axis in the interior of a laterally unlimited liquid, it first and immediately puts in oscillatory motion that quantity of liquid which fills a cylinder whose diameter and height are determined by the length and height of the strip. It is self-evident that, in consequence of friction, the adjacent liquid also partakes of the motion, although at all events with rapidly diminishing strength. If the upper edge of the strip lies in the free surface of the liquid, a moving disk, of the dimensions indicated, will to a certain extent be separated, which will be chiefly although not exclusively set in motion. Friction in the surface will take place principally at the margin of this disk, where moving and nearly still layers border on one another. Hence it was to be feared that herewith the surface-viscosity that might be present would exercise but little influence. It is different when the liquid is inclosed by two not widely distant plates, to which it adheres. The action of friction, and especially of augmented friction at the surface, must then become evident. Plateau himself, in his experiments cited above, intensified in a similar way the action of the friction. I will subsequently communicate a series of experiments in proof of the correctness of the whole of these speculations.

4. The motion of the liquid under the influence of the oscillating plate is in any case tolerably complicated; and for the moment it seems to me impossible to calculate it from the general hydrodynamic equations. Even its reaction upon the oscillating system probably takes place not according to a law which could be simply rendered by a mathematical expression. I am therefore far from regarding the diminution of the oscillations at once as a measure of the friction-coefficient. But if two experiments be made, differing only in this, that in one of them the immersed plates cut the free surface with their upper margins, while in the other they are sunk a fraction of a millimetre deeper, then the resistances to the motion cannot be different unless the surface-layers exert a special influence.

As in all the experiments I confined myself to oscillations of small amplitude, it might readily be supposed that the liquid would chiefly produce a resistance proportional to the angular velocity of the apparatus, so that also for the oscillation of the system when the plate was immersed the simple equation

$$\frac{d^2\phi}{dt^2} + 2a \frac{d\phi}{dt} + b \cdot \phi = 0 \quad . \quad . \quad . \quad (I.)$$

would hold good.

This assumption can easily be tested. It follows therefrom that oscillation-period and decrement are independent of the amount of the amplitude. This is not precisely correct; only the oscillation-period is constant, while the decrements diminish slowly with the magnitude of the amplitudes. I have not pursued the more precise law of this decrease further; the knowledge of the mean value of the decrements suffices perfectly for the decision of the question put; and I have taken care to arrange the experiments so that the decrements correspond to average amplitudes.

If the margin of the plate projects above the free surface, another disturbing circumstance comes in: the surface between the plate and the glass side is perceptibly curved, and in general raised somewhat above the level. During the oscillations this elevation is no longer the same on both sides; consequently attractive, and sometimes also repellent forces appear, which lengthen or shorten the oscillation-period. Here, however, with small amplitudes the oscillation-period is independent of their magnitude; so that the supervening forces must be regarded as proportional to the angle ϕ . The alteration of the oscillation-period is an excellent means of measuring accurately the forces mentioned. I have instituted a series of experiments in this direction, which I intend to communicate in a separate paper. Lastly, if in the surface a thin layer which (according to Marangoni's assumption) is elastic were present, a displacement or bending of it must at all events produce also a resistance, which, in a first approximation, would be proportional to the angle of deviation.

It follows from these considerations that, when the plate is removed from the interior of the liquid to the surface, in equation (I.) both a and b change. But as the oscillation-period and decrement were always observed, the two variations can be found separately. If we put

$$\phi = \phi_0 \cdot e^{-\frac{\lambda t}{T}} \cos\left(2\pi \frac{t}{T}\right),$$

then

$$a = \frac{\lambda}{T}, \quad b = \frac{4\pi^2 + \lambda^2}{T^2}. \quad . \quad . \quad . \quad . \quad (II.)$$

Particularly, the approximate measure of the friction is therefore the quantity a , or the ratio of the decrement to the oscillation-period.

Of course the apparatus suffers also resistance by the air. This was specially observed repeatedly, but was constantly found very little in comparison with the resistance of the liquid to the motion. As the question here is not about absolute, but

only about relative measurements, I have not thought it necessary to take account of it separately.

5. I communicate first those experiments which concern the testing of the method employed. As was remarked above, it was requisite to determine how far the oscillation-period and the logarithmic decrement are dependent on the quantity of the amplitude. The following table (Table I.) contains the results of three series of experiments. The above-described apparatus having been set oscillating, and having already executed a series of oscillations, the turnings towards both sides were noted, and therefrom the initial amplitudes determined; at the end of four oscillations the final amplitude was obtained in the same manner: both numbers are given in the first column. From them the decrement λ was calculated. At the same time the periods of the four oscillations were ascertained and the mean taken, which is returned under T. Here the above-described rectangular vessel was employed, in which the movable plates faced each other at a distance of 15 millims. The vessel was filled with distilled water, into which dipped a rectangular brass plate 100 millims. long, 5 millims. high, and 0.5 millim. thick. During series A the plate was entirely in the interior of the liquid; during B the upper edge exactly intersected the free surface, while during C it projected 0.5 millim. above. The oscillation-period T is given in seconds. The quantity λ is the difference of the ordinary logarithms.

TABLE I.

A.				B.			
Amplitudes.		T.	λ .	Amplitudes.		T.	λ .
424	378	11.50	0.01247	420	321	11.85	0.02919
337	303	11.65	0.01155	353	271	11.50	0.02870
280	252	11.45	0.01144	244	190	11.80	0.02716
227	205	11.50	0.01132	Mean ... 11.72			
185	167	11.50	0.01086				
Mean ...		11.52					

C.			
Amplitudes.		T.	λ .
502	390	12.50	0.02741
324	256	12.30	0.02558
216	172	12.70	0.02475
146	117	12.30	0.02379
Mean		12.45	

As we see, the oscillation-period (the determination of which, as the mean of only four oscillations, was not very exact) is constant in each individual series—*i. e.* independent of the magnitude of the amplitude. It increases, when series A, B, C are compared, with the projection of the upper edge out of the free surface. There can be no doubt that this is to be ascribed to the action of attraction-forces in consequence of the curvature of the surface. The decrements slowly lessen when the amplitudes become smaller; and this diminution is pretty uniform in all the series. I have therefore not attempted to search out its more precise law, but have contented myself with taking, in all the further experiments I have to communicate, a mean value of the logarithmic decrement in each case from ten oscillations, in which the initial and final amplitudes lay in the interval from 400 to 100 scale-divisions. The great difference in the absolute values of the decrement between series A on the one hand and series B and C on the other, is to be attributed to the influence of the free surface.

I have above referred to the necessity of inclosing the liquid to be studied in a narrow vessel with parallel sides. The proof of this is given by the experiments communicated in Table II. Here also distilled water was experimented with, and the same brass plate as before employed. Under h in this, as in all the following tables, the distance of the upper edge of the plate from the free surface of the liquid is to be understood. If the edge lay below the same, and therefore within the interior of the liquid, the positive sign is employed; if it projected above, the negative.

The distance between the displaceable glass plates is indicated by the quantity e ; $e = \infty$ signifies a vessel of such dimensions that any influence of the side walls was inconceivable. The oscillation-period T is given here also, in order to show the influence of the attracting forces. Lastly, the quantity $\frac{\lambda}{T}$ (which was always multiplied by 1,000,000 to avoid decimal-places) serves as the measure of the frictional resistances.

The values of the oscillation-period are in the first two series nearly constant; in the third series the attraction of the side-walls makes itself perceptible; and this influence comes out still more strongly in the last series.

TABLE II.

h .	$e = \infty$.		$e = 30$ millims.	
	T.	$\frac{\lambda}{T}$	T.	$\frac{\lambda}{T}$
millim.				
+5	11.28	1172	11.22	1085
+0.5	11.28	1245	11.58	1119
0	11.31	1248	11.56	1632
-0.5	11.22	1222	11.36	1754
h .	$e = 20$ millims.		$e = 15$ millims.	
	T.	$\frac{\lambda}{T}$	T.	$\frac{\lambda}{T}$
millim.				
+5	11.00	1143	11.42	1188
+0.5	11.00	1277	11.44	1567
0	11.24	2355	11.82	2649
-0.5	11.84	2451	12.68	2784

The different series of experiments were made at different times; and hence, in the first place, only the numerical values of each series are comparable among themselves. Nevertheless the frictional resistances show for the motion at the greater depth ($h=5$ millims.) a sufficiently good accordance; they are at all events not considerably influenced by the shape of the vessel. This influence, however, becomes already prominent as soon as the plate approaches near to the free surface ($h=0.5$ millim.), and becomes very considerable when the upper edge cuts the free surface. While in the first series the resistances remain the same, in the rest a sudden increase takes place when the plate is raised to the small extent of 0.5 millim.; and is the greater the nearer the bounding plates are. Of this great influence of the limitation of the vessel upon the resistance, the causes of which I have previously explained, I sought to convince myself in still another way.

After the resistance of a surface of pure water in the vessel in which there is no perceptible influence of the sides had been once more determined ($h=0$), the surface was covered with a thin layer of oil spread as uniformly as possible by stirring, and the resistance again investigated. Finally, the same experiment was repeated; but onto the pure water surface a thin layer of oil of turpentine was brought in a

similar manner. The following slightly differing values were obtained:—

Pure water surface	1248
The same with oil	1301
The same with turpentine . .	1119

while on using the narrowly separated plates ($e=15$) a scarcely visible layer of oil sufficed to make the resistance to the motion so great that the movable system returned, without oscillations, to its position of equilibrium. According to this, the use of a laterally narrowly limited vessel is found to be absolutely necessary with the oscillation-method employed.

6. This previous question being settled, I proceed to the comparison of the resistance of the interior and of the surface with different liquids. Most of the experiments were executed with the most favourable distance of 15 millims. between the plates, by means of the repeatedly mentioned brass plate of 100 millims. length; but I have not omitted to make some experiments also with the greater distance of 20 millims., as well as to use instead of the brass plate a brass cylinder of the same length and about 5 millims. in diameter.

The following Tables III., IV., V. (which are quite intelligible after the previous explanations) give only the values of $\frac{\lambda}{T}$ as measuring the resistance. For the last table I may further remark that f signifies a previous filtration, and that the numbers standing with the names of the liquids represent their specific gravities.

TABLE III. Rectangular brass plate.

Distance between the glass sides 20 millims.

Liquid.	$h=10$ mm.	$h=5$ mm.	$h=0.5$ mm.	$h=0$ mm.	$h=-0.5$ mm.
Distilled water	1106	1310	2045	2026
Distilled water } filtered	1081	1143	1277	2355	2451
Alcohol	1021	1155	1024	1016

TABLE IV. Brass cylinder.

Distance between the side plates 15 millims.

Liquid.	$h=5$ mm.	$h=0.5$ mm.	$h=0$ mm.	$h=-0.5$ mm.
Distilled water 1	1131	1614	3388	4074
Distilled water 2	1574	3301	3613
Alcohol	1331	1512	1396	1293
Oil of turpentine	1875	2327	2107	1899

TABLE V. Rectangular brass plate.

Distance between the glass sides 15 millims.

Liquid.	$h=$ 10 mm.	$h=$ 5 mm.	$h=$ 0.5 mm.	$h=$ 0 mm.	$h=$ -0.5 mm.
Distilled water	1188	1567	2649	2784
Distilled water f	1115	1145	1483	2545	2637
KNO_3 in water f , 1.1223	1091	1103	1444	2529	2428
Na_2SO_4 in water f , 1.169	1499	2310	4013	4088
K_2CO_3 in water f , 1.367	2175	3692	5947
CaCl_2 in water f , 1.348	3171	5292	8155
Alcohol 1	1185	1463	1312
Alcohol 2	1225	1453	1256	1145
CuCl_2 in alcohol f , 0.878	1566	2120	1938
CaCl_2 in alcohol f , 0.995	6028	7338	6942	6373
Sulphide of carbon	870	1144	2001	1476	1362
Oil of turpentine	2073	2276	2711	2368	2229
Water and alcohol f , 0.9708	1346	1943	2133
Water and alcohol f , 0.9274	1736	2216	1988	1848

From the comparison of the last three Tables it is evident that the motion-resistances, in spite of the somewhat varying conditions of the experiments, show on the whole the same course. I will therefore content myself with annexing all further inferences to the numerical values of the last table. For the quicker survey of the frictional resistances, I have represented them by delineations in figures 2 and 3. For water and alcohol the means of the but little differing series of experiments have been used.

Fig. 2.

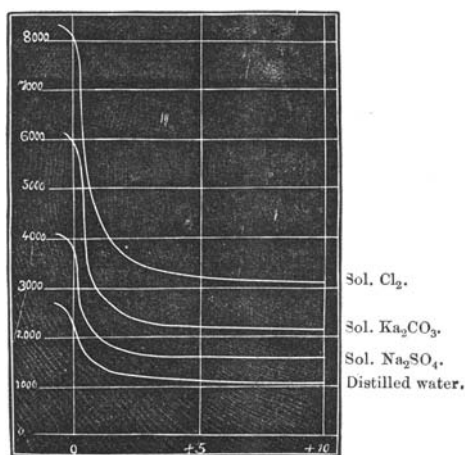
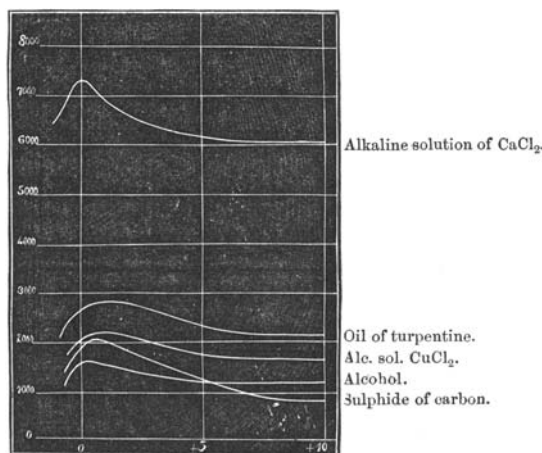


Fig. 3.



From the numerical values of the last table, as well as from these drawings, the following results are obtained:—

(a) In all the liquids studied, the resistance to motion is not considerably augmented when the plate approaches the free surface.

(b) On a further elevation of the plate causing its upper edge to enter the free surface, a characteristic difference between the various liquids becomes manifest, and we find, exactly as Plateau states,

I. Liquids which exhibit a considerable augmentation of the resistance—water and aqueous salt-solutions;

II. Liquids in which a diminution of the resistance occurs—alcohol, alcoholic solutions, sulphide of carbon, oil of turpentine;

III. Mixtures of water and alcohol, which join the one or the other group according to the proportion of the constituents.

(c) When the resistances of the liquids at greater depths are compared, that of sulphide of carbon is the least; water and alcohol follow, with nearly equal resistances; lastly, oil of turpentine with a considerably greater resistance. The addition of a salt produces in most cases a considerable augmentation of the resistance. It is likewise to be noted as remarkable, that mixtures of water and alcohol show a greater resistance than either constituent by itself.

One portion of these results admits of a simple explanation. In the first place, that in all liquids the resistance increases as the surface is approached is not surprising; for the liquid particles put in motion by the plate give way not only in a

horizontal direction, but also downward and upward. In close proximity to the free surface giving way in the last direction is impossible, or at least more difficult; for in this case the liquid particles would have to overstep the upper horizontal boundary plane; and with that both gravity and the capillary forces would counteract their motion. It is moreover possible that, in consequence of the evaporation taking place at the free surface, its temperature is somewhat below the mean temperature of the liquid, so that the friction, quickly changing with the temperature, is there somewhat greater than in the deeper layers. By this is probably to be explained especially the striking behaviour of sulphide of carbon, the resistance of which at greater depths is much less than those of alcohol and water, while in close proximity to the surface ($h = +0.5$) it considerably exceeds them. In spite of the precautions which were taken, with sulphide of carbon the evaporation was so considerable that its temperature had fallen during the experiments some degrees below that of the enclosure. At all events, in these and similar experiments it is necessary to take into account the behaviour of the free surface, and therefore to bear in mind that, under some circumstances, it opposes an increased resistance to a motion taking place close beneath it.

7. To come now to the chief object of the investigation, the behaviour of the free surface, it follows unequivocally from the experiments above given that in water the resistance suddenly increases very considerably as soon as the upper edge of the plate comes into the free surface. In consequence of the entire arrangement of the experiments it appears to me beyond question that this is caused by increased friction in the surface layer. The aqueous salt-solutions studied behave very similarly to distilled water. The increase of the resistance on passing from the last position of the plate within the liquid to its entrance into the surface amounts:—

	Per cent.
In distilled water	to 60.9
In solution of KNO_3	75.1
" " Na_2SO_4	73.7
" " K_2CO_3	61.0
" " CaCl_2	54.1

These numbers are, on the whole, of the same order of magnitude. The resistance in the free surface might, of course, be compared also with that at greater depths. The differences are then considerably greater still. The correctness of the facts, under the conditions of the experiments, surely cannot after this be doubted. It is true that the question is not thereby yet decided whether we have here to do with

a peculiarity of homogeneous liquids, free from any foreign mixture, or whether substances coming from the atmosphere or from the sides of the vessel into the free surface cooperate. As water and salt-solutions generally possess but little different and proportionally high capillary-constants*, the possibility is given of an outspreading of substances possessing less tension.

The investigation of free water-surfaces which, not only during the experiments, but also previously, have been protected from all contact with the air might be extraordinarily difficult; at all events the arrangements necessary for it were wanting to me.

The precautions employed by me—filtering the water (heated to boiling) into a narrow-necked flask, covering the glass vessel during the experiment with wet filtering-paper—had almost no influence at all upon the numerical values obtained. Also water from the conduit did not show any essential difference from distilled water. I tried still another means in order to ascertain the action of the contact of the air or of the particles suspended in it. If that is the chief reason for the peculiar behaviour of the water, the action must increase with the time which has elapsed since the free surface was formed. Respecting this I made the following experiments. As quickly as possible after pouring the liquid into the vessel the state of the free surface was examined; after a longer time the examination was repeated. Finally it was attempted by mechanical means (stirring with a clean platinum plate, shortly before heated to incandescence) to remove foreign bodies from the surface. Some of these experiments are collected in the following table. The numbers given are,

as before, the quotients $\frac{\lambda}{T}$. The brass plate, as well as the brass cylinder, were placed so that their upper extremity exactly touched the free surface.

TABLE VI.

Rectangular plate : distance between the sides 20 mm.		Rectangular plate : distance between the sides 15 mm.		Cylinder : distance between the sides 15 mm.	
h. m.		h. m.		h. m.	
1 20	2045	11 7	2649	9 30	3388
2 45	2132	12 5	2744	10 45	3360
After stirring	1909	After 24 hours	∞	Stirred	3374
		Stirred	2539		

* Quincke, Pogg. Ann. clx. p. 337-375, 560-588 (1877).

Phil. Mag. S. 5. Vol. 11. No. 66. Feb. 1881.

The first two series show a small increase of the resistance in the interval of an hour; in the last series it remains constant. After a period of 24 hours the resistance has become so great that it could not be determined by the method employed. After stirring, however, it falls back to only a little less than the initial value. Although, therefore, it cannot be disputed that longer contact with the air increases the resistance, just as little can it be doubted that a special surface-resistance is present immediately after the formation of the free surface. Hence we must conclude either that a very great surface-resistance belongs to the free water-surface, or that a pure water-surface in contact with the air has no existence at all. We should then have, in the method of observation employed, a fine means of judging of the condition of a water-surface.

While in water and aqueous salt-solutions the increase of resistance at the surface is very considerable, the decrease of the same in the other liquids is proportionally small. It amounts in

	Per cent.
Alcohol	to 11·9
Alcohol solution of CuCl_2	8·6
CaCl_2	5·4
Oil of turpentine	12·6
Sulphide of carbon	26·3

The behaviour of the last liquid is (as was previously found to be the case) abnormal in consequence of the strong evaporation; so that a proportionally large diminution cannot be surprising. In the rest of the liquids, moreover, the resistance at the surface (contact with it by the upper edge of the plate) is always still greater than on the motion of the plate at a greater distance from the free surface. Hence I do not think we are entitled to infer from these experiments a diminution of the coefficient of friction at the surface.

In solutions of water in alcohol we get:—

At the sp. gr. 0·9708, increase	9·8 per cent.
" " 0·9274, decrease	10·3 "

From this it follows that even a trifling addition of alcohol to water considerably lessens its characteristic property, while on a further addition the mixture behaves nearly like pure alcohol. This appears to me to stand in the closest connexion with the rapid diminution of the cohesion of water by the addition of alcohol. For it Quincke* gives the following values:—

* Pogg. *Ann.* clx. p. 368 (1877).

σ 0.9973	0.9852	0.9110	0.7904
α 8.000	5.657	2.947	2.354

where σ denotes the specific gravities, and α the cohesion-constants.

In order to see on what the resistances to the motion of the plate in the interior of the liquid chiefly depend, I determined, with the aid of an oscillating brass disk, the coefficients of friction of some of the liquids employed, according to the formulæ and rules given by O. E. Meyer*. They are collected in the following table, in which I have confined myself to giving their relative values referred to water. The temperature amounted to 21° C.; only with sulphide of carbon was it lower, and that in consequence of the hardly to be avoided evaporation. Since the absolute values of η for distilled water,

According to O. E. Meyer†, at 21° 6 0.01190,

According to Grottrian‡, at 21° 5 0.01250, at 21° 58 0.01236,

are known, the friction-coefficients of the other liquids can easily be calculated from the following table.

TABLE VII.

	ρ .	$\eta \cdot \rho$.	η .	$\frac{\lambda}{T}$.
Sulphide of carbon ...	1.293	0.4262	0.3297	0.782
Oil of turpentine	0.870	2.030	2.333	1.859
Alcohol	0.7937	1.055	1.329	1.063
"	0.8720	2.282	2.617	
"	0.9023	2.720	3.014	
"	0.9737	1.721	1.767	
Water.....	1.000	1.000	1.000	1.000

A comparison of the values of $\eta \cdot \rho$ with the ratios of the resistances $\frac{\lambda}{T}$ shows, as was to be expected, no perfect agreement. Much rather, in the resistances the specific gravity of the liquid has also some influence. The peculiar behaviour of the mixtures alcohol and water, which possess a maximum of η , has moreover been already observed by Poiseuille§.

Halle a. d. S., Aug. 1, 1880.

* Pogg. *Ann.* cxiii. 1861.

† Ibid. clx. p. 242 (1877).

‡ Ibid. cxiii. p. 399.

§ Ibid. lviii. p. 437 (1843).