

Henri Poincaré

The Passing of a Great Mathematician and Philosopher

By J. W. N. Sullivan

THE nineteenth century has been aptly called the age of science. It is not that that century surpassed all others in the magnitude of its discoveries, but that scientific results and scientific methods were then, for the first time, brought home to the minds of the people. The general interest taken in the theory of evolution, consequent on the publication of the "Origin of Species," extended to other branches of science. One great cause which contributed to the general spread of scientific ideas at that time was the part taken by some of the foremost scientific men in presenting their results in a form devoid of technicalities, and in such a way as to be readily understood by the average man. Since that day we have had innumerable popular expositions of every branch of science. Indeed, the writing of popular accounts of scientific discoveries has almost taken rank as a distinct profession. Such writers usually confine their efforts to explaining recent positive discoveries. But of late years something far more interesting than any isolated discovery, however wonderful, has taken place. We are in the midst of a general criticism of scientific procedure, and of the value of scientific results. The scientist has grown uneasy. He is no longer sure of the very foundations on which the whole vast superstructure of modern science is erected. To present these intricate questions, these subtle and far-reaching issues, in a form which renders them readily understood of the average man, requires the hand of a master.

In the domain of mathematics and physics, at any rate, the master appeared. In the late H. Poincaré we had a genius of the first order, a man whose accomplishments in his chosen sphere earned him a foremost place among his contemporaries. By many competent authorities he was considered to have been the greatest mathematician of our day. It would be impossible in the space at our disposal to give an adequate account of Poincaré's work in mathematics, mathematical physics, and dynamical astronomy. We here wish to call attention to his three books, "The Value of Science," "Science and Hypothesis," and "Science and Method."

These works are, in a large measure, critical rather than expository. They contain the profound reflections of their distinguished author on questions of the first importance, and yet are written with such clearness and charm of style that they may be read with pleasure and profit by any ordinary thinking man.

Take, for instance, mathematics, the most impeccable of the sciences. Poincaré sees that its very existence appears paradoxical. We start from definitions and axioms, and deduce consequences which are absolutely necessary results of our premises. We introduce no fresh facts. We make no appeal to experiment. How then can we discover new truths? It would seem that all mathematical reasoning must reduce to a gigantic tautology, and that all the mathematical theorems which fill so many imposing works are simply various ways of saying that A is A .

Poincaré analyzes this paradox and shows that the very essence of mathematical reasoning, and that which alone makes it valuable, is the principle of mathematical induction. This great principle affirms that if a theorem is true of the number 1, and if we can further assert that, being true for the number n , it is true for $n + 1$, when we have proved that the theorem is true for all integer numbers. This principle is, in one way, a kind of short cut. It enables us to dispense with the labor of verifying the theorem for every particular case, and is the fundamental characteristic of mathematical reasoning. It alone makes generalization possible.

Again, we all know what great changes have occurred in our way of looking at the fundamental concepts of geometry. Since Lobachevski and Riemann have shown us that it is possible to construct perfectly logical and consistent geometries quite unlike Euclid's, the question has arisen in some minds, which geometry is true? For instance, according to Euclid's geometry the three interior angles of a triangle are together equal to two right angles; according to Lobachevski they are less than two right angles, and according to Riemann they are greater than two right angles, and these differences increase with the area of the triangle. Suppose then we effect some astronomical measurements by measuring the angles of the very large triangle formed by the

diameter of the earth's orbit and the lines joining its extreme points to a fixed star. If we make our measurements with sufficient accuracy, it would seem that we should be in a position to answer the question, which geometry is true? Poincaré shows us that this is not the case. In making our measurements we have implicitly assumed that light is propagated in straight lines. If we found that the sum of our measured angles was not equal to two right angles, we should not change our geometry, *we should simply conclude that light is not propagated in perfectly straight lines.* We should do this because it would be more *convenient.* This is the key-note to Poincaré's treatment of this subject. It is meaningless to talk about the *truth* of geometric theorems. It is as meaningless as to talk about the *truth* of the metric system. The theorems are necessary consequences of the preliminary hypotheses, and these hypotheses are *arbitrary.* We have selected those we have in preference to others because



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they most nearly and most simply accord with the observed facts relating to solid bodies and are, therefore, the most convenient.

We have not space here to give an account of Poincaré's profound remarks on the subjects of space and time, those old bugbears of metaphysics. Suffice it to say that he treats all such questions with unrivaled insight and ability. On the subject-matter of physics proper his views are equally striking and original. He points out that in the accepted theory of matter, the electron theory, Newton's third law, asserting the quality of action and reaction, does not hold unless we assume a reaction on the ether which gives the necessary compensation. This fact is of fundamental importance. On analyzing our concept of energy, one of the most important concepts in the whole range of physics, he succeeds in showing that except in a few comparatively simple cases it is very hard to define, and that a general definition simply reduces to saying that in any physical phenomenon there is something which remains constant. This notion of energy, in fact, belongs to a class of hypotheses which are distinguished by the fact that they can never be contradicted by experiment. Then there are two classes of hypotheses in science—those which may be submitted to the test of experiment and judged by their behavior under that test, and those which serve us as general controlling principles, but which are outside the range of experiment. Poincaré illustrates this difference very clearly,

and an interesting example, taken from biology, has been given by Prof. Royce in his introduction to the American translation of "L'Hypothèse et la Science." This extremely inadequate sketch of the subject-matter of Poincaré's popular expositions will, we hope, induce the reader to make the acquaintance of the volumes mentioned. He will find himself amply repaid for his trouble.

Like many other great men, Poincaré manifested his extraordinary powers at a very early age. He was born in Nancy in 1854, the son of a French physician. As a boy he was rather delicate, and disinclined for the society of other boys of his age. He seems to have been a very engaging child and wonderfully intelligent. He was trained first as an engineer, and received his doctorate in mathematics at the age of twenty-five. But he had very early given proof of his extraordinary mathematical powers. It is related that he never took notes of the lectures which he attended, nor did he ever read the copies of the lectures which were distributed among the students. He simply made a mental note of the results given by the professor, and when called upon, could always supply a proof of his own. Like many other people who are in the habit of indulging in intense mental concentration, he did very peculiar things. He was very fond of walking while engaged in thinking out his problems, and had a habit of "fiddling" with something in his hand. This was so well known at the college where he was a student that the professor gave him a bunch of keys to carry as he paced the corridors in meditation. On one occasion he was discovered on one of the principal streets of Paris carrying a large new wicker-work basket in his hand. He suddenly became alive to the fact, and retracing his steps, found that he had unconsciously purloined the basket from a store he had passed some time previously. It is also related that he was once found packing his trunk with the bed-linen of the house where he was a guest under the impression that he was packing his handkerchiefs. The histories of other great mathematicians, such as Gauss and Lagrange, show that intense mental concentration is often accompanied by this peculiar state of oblivion to the external world. In fact, at times, as in the case of Sir Isaac Newton, it has given rise to temporary suspicions regarding their sanity. Poincaré was a well-known figure in Parisian society, and was by no means the retired mathematical recluse of popular imagination. He was a member of the French Academy of Literature, an extraordinary honor for a scientific man, and a member of a great number of scientific societies in various countries. His published works embrace nearly every branch of pure and applied mathematics. His work in dynamical astronomy is of great value. Among other things, he has made contributions to the famous problem of three bodies which are of the first importance. In general function theory

he has created a new type of functions. Together with Klein he has applied some of the concepts of the new geometries to the integration of linear differential equations, and he has greatly helped to develop the branch of mathematics known as Analysis Situs.

It is of the greatest interest to know the way in which the mind of a great man works. On this point Poincaré has given us some most illuminating information. There appear to be three stages in the solution of a problem: first, a period of intense mental effort, usually unsuccessful; secondly, a complete mental rest from that particular problem; and thirdly, a sudden revelation of the solution, followed by a period of conscious effort, trying to find the proofs of the answer revealed. We say *conscious* effort, because Poincaré himself believes that the period of apparent mental rest is in reality a period of intense activity, the activity of the subconscious self.

The first period of intense mental activity seems analogous to setting a number of things in motion, which then proceed to form combinations among themselves. The question why the correct combination, and no other, is presented to the conscious mind, does not permit of any very satisfactory answer. But it is significant that the right combination is the one which most appeals to the aesthetic sense of the mathematician, and there is probably a deep relation between the correctness of a theorem and its mathematical beauty.