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9.

General perturbations of the rectangular coordinates of Parthenope by Jupiter and Saturn in units of the 7th decimal, and determination of the orbit by means of them.

By *E. Schubert*.

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A. Perturbations by 4.

$i,$	i'	$r^0 \delta r$		ξ'		η'		ζ'	
		\cos	\sin	\cos	\sin	\cos	\sin	\cos	\sin
0,	0	+ 38,92 t		— 160,13 t		— 1314,95 t		+ 42,18 t	
1,	0	— 260,81 t	— 1283,60 t	— 8,02 t	+ 485,94 t	— 537,96 t	— 13,67 t	— 282,69 t	+ 7,95 t
2,	0	— 12,89 t	— 112,94 t	+ 52,32 t	+ 486,88 t	— 489,25 t	+ 52,05 t	— 13,97 t	+ 0,39 t
3,	0	— 0,96 t	— 8,37 t	+ 7,81 t	+ 70,43 t	— 70,62 t	+ 7,78 t	— 1,04 t	+ 0,03 t
4,	0	— 0,08 t	— 0,73 t	+ 1,04 t	+ 9,18 t	— 9,18 t	+ 1,02 t	— 0,09 t	
5,	0		— 0,07 t	+ 0,11 t	+ 1,17 t	— 1,13 t	+ 0,11 t		
0,	0	+ 3928		+ 57		+ 107		+ 11	
1,	0	+ 98	+ 81	+ 1636	— 6	— 10	+ 1537	+ 2	+ 91
2,	0	+ 64	+ 23	+ 151	— 71	+ 70	+ 150	+ 4	+ 14
3,	0	+ 5	+ 1	0	— 9	+ 9	0	0	+ 1
4,	0			— 1	— 1	+ 1	— 1		
5,	0			— 1	0				
— 4,	— 1			0	+ 2	+ 2	0		
— 3,	— 1	+ 1	— 2	+ 1	+ 20	+ 20	— 1		
— 2,	— 1	— 1	— 18	+ 13	+ 168	+ 169	— 12	+ 4	— 1
— 1,	— 1	+ 48	— 402	+ 246	+ 1522	+ 1516	— 249	+ 79	— 6
0,	— 1	— 1283	— 3019	+ 6071	+ 9242	+ 9331	— 6005	+ 206	— 148
1,	— 1	+ 7800	+ 12346	+ 229	— 1086	+ 1240	+ 316	— 30	+ 77
2,	— 1	+ 738	+ 1427	— 2809	— 4446	+ 4422	— 2792	+ 90	— 63
3,	— 1	+ 40	+ 83	— 371	— 617	+ 615	— 370	+ 4	— 2
4,	— 1	+ 4	+ 6	— 47	— 78	+ 79	— 47		
5,	— 1			— 5	— 9	+ 9	— 5		
6,	— 1			— 1	— 1	+ 1	— 1		
— 3,	— 2			+ 4	— 13	— 13	— 4		
— 2,	— 2	— 4	+ 9	+ 35	— 108	— 108	— 35	0	— 1
— 1,	— 2	— 29	+ 124	+ 397	— 1052	— 1052	— 398	— 8	— 10
0,	— 2	— 1163	+ 3095	+ 6945	— 12727	— 12692	— 6907	— 153	— 123
1,	— 2	+ 4123	— 6567	+ 6201	— 15217	— 15325	— 5555	+ 385	+ 175
2,	— 2	+ 11975	— 30180	— 4345	+ 7740	— 7390	— 4124	+ 45	— 9
3,	— 2	+ 469	— 1402	— 1953	+ 4666	— 4645	— 1945	+ 26	+ 8
4,	— 2	+ 32	— 106	— 297	+ 736	— 734	— 296	+ 2	0
5,	— 2	+ 2	— 8	— 40	+ 102	— 99	— 39		
6,	— 2			— 5	+ 12	— 12	— 5		

$r^0 \delta r$		ξ'		η'		ζ'	
$i,$	i'	\cos	\sin	\cos	\sin	\cos	\sin
-4,	-3			-	8	-	29
-3,	-3	+	3	-	45	-	192
-2,	-3	+	33	-	405	-	1660
-1,	-3	+	443	-	3682	-	14985
0,	-3	+	9106	-	38324	-	153988
1,	-3	+	5888	-	37831	-	36488
2,	-3	-	71792	+	36822	-	145754
3,	-3	+	205	+	15765	-	26419
4,	-3	-	47	+	1962	-	3466
5,	-3	-	10	+	224	-	420
6,	-3	+	1	+	28	-	52
7,	-3			+	3	-	6
-2,	-4				0	-	1
-1,	-4	+	1	-	3	-	15
0,	-4	+	21	+	11	-	108
1,	-4	-	167	+	1019	-	623
2,	-4	+	1311	-	243	-	873
3,	-4	+	319	+	133	+	14
4,	-4	+	679	+	70	-	177
5,	-4	+	69	+	41	-	5
6,	-4	+	6	-	9	+	1
7,	-4			-	2		0
-1,	-5				0	-	3
0,	-5		0	+	4	-	23
1,	-5	-	2	+	70	-	228
2,	-5	+	73	-	167	-	179
3,	-5	-	540	+	323	-	198
4,	-5	+	338	-	196	+	54
5,	-5	-	9	+	239	-	9
6,	-5	+	5	+	31	+	5
7,	-5		0	+	3	+	1
-1,	-6			+	2	+	7
0,	-6	-	4	+	14	+	67
1,	-6	-	72	+	57	+	674
2,	-6	+	94	+	418	+	251
3,	-6	-	620	+	40	-	553
4,	-6	-	42	+	163	+	170
5,	-6	+	111	+	11	+	29
6,	-6	-	61	+	3	+	6
7,	-6	-	9	+	1	+	2
8,	-6	-	1	+		+	
1,	-7	+	1	-	4	-	3
2,	-7	+	6	-	14	-	21
3,	-7	-	17	+	6	+	4
4,	-7	-	27	+	19	-	17
5,	-7	+	26	+	4	+	17
6,	-7	+	6	-	12	+	6
7,	-7	-	21	-	3	+	2
8,	-7	-	6	-	1		0
3,	-8	+	2	+	5	+	4
4,	-8	-	12	+	13	-	5
5,	-8	-	5	+	8	-	4
6,	-8	+	16	+	6	+	5
7,	-8	-	13	+	3	+	4
8,	-8	-	5	-	0	+	0

B. Perturbations by η .

$i,$	i''	$r^0 \delta r$		ξ'		η'	
		\cos	\sin	\cos	\sin	\cos	\sin
0,	0	+ 0,14 t		- 0,67 t		- 27,35 t	
1,	0	- 0,94 t	- 96,31 t	+ 0,47 t	+ 213,78 t	- 210,79 t	+ 0,41 t
2,	0	- 0,05 t	- 4,76 t	+ 0,22 t	+ 30,06 t	- 29,97 t	+ 0,22 t
3,	0		- 0,35 t	+ 0,03 t	+ 3,76 t	- 3,75 t	+ 0,03 t
4,	0				+ 0,40 t	- 0,38 t	
0,	0	+ 163		- 19			
1,	0	- 28	0	+ 64			+ 61
2,	0	- 3	0	+ 13			+ 13
3,	0			+ 1			+ 1
-2,	-1	+ 3	0	- 13	- 2	- 1	+ 14
-1,	-1	+ 63	- 16	- 53	- 61	- 59	+ 54
0,	-1	+ 71	- 30	- 298	+ 295	+ 332	+ 357
1,	-1	- 504	+ 453	+ 38	+ 90	- 106	+ 28
2,	-1	- 31	+ 21	+ 128	- 111	+ 110	+ 128
3,	-1	- 2	+ 1	+ 17	- 15	+ 16	+ 19
4,	-1			+ 2	- 2	+ 2	+ 2
-2,	-2			- 1	+ 4	+ 4	+ 1
-1,	-2	+ 10	- 8	- 21	+ 38	+ 41	+ 21
0,	-2	+ 20	- 121	+ 2	+ 482	+ 482	+ 2
1,	-2	- 10	+ 603	+ 43	+ 221	+ 231	+ 17
2,	-2	+ 24	+ 517	+ 4	- 199	+ 200	+ 3
3,	-2	+ 2	+ 29	- 2	- 70	+ 71	- 2
4,	-2	0	+ 1	0	- 11	+ 11	0
-2,	-3	- 7	+ 1	0	+ 1	+ 1	0
-1,	-3	- 21	- 18	+ 12	0	0	- 12
0,	-3	+ 30	+ 135	+ 25	+ 130	+ 130	- 29
1,	-3	+ 58	+ 178	+ 9	+ 96	+ 66	- 42
2,	-3	- 28	- 18	- 29	- 72	+ 45	+ 1
3,	-3	- 3	- 1	- 8	- 25	+ 25	- 7
4,	-3			+ 1	- 3	+ 2	0
0,	-4	0	- 1	- 3	- 5	- 5	+ 3
1,	-4	- 4	- 2	+ 16	+ 25	+ 27	- 16
2,	-4	+ 32	+ 45	- 9	- 6	- 3	+ 8
3,	-4	- 17	- 7	- 1	- 6	+ 5	- 6
4,	-4	+ 5	- 1				

The computation of these perturbations has been based upon the following elements derived by Dr. *Luther* of Bilk:

1850 Jan. 0 Berlin M. T.

$$M = 251^{\circ} 35' 12'' 9$$

$$\pi = 316 \ 39 \ 4,0$$

$$\Omega = 124 \ 58 \ 50,3 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{M. Eq. Ep.}$$

$$i = 4 \ 36 \ 58,8'$$

$$\phi = 5 \ 41 \ 17,3$$

$$\mu = 924'' 1155.$$

Now for the purpose of correcting this set of elements by means of the above perturbations it was necessary, in the first place, to form 14 normal-places for the 14 observed apparitions from 1850 to 1868. The Normals, referred to the mean equinox 1850,0, are:

Berlin M. T.		α	δ
1850 May	31,0	225° 57' 33'' 5	- 9° 48' 27'' 9
1851 Oct.	22,0	28 46 48,3	+ 3 20 40,6
1853 Febr.	7,0	141 50 9,9	+ 16 50 37,7
1854 June	3,0	252 46 36,7	- 16 0 53,0
1855 Nov.	9,0	49 49 12,4	+ 10 32 47,2
1857 Febr.	20,0	158 10 19,5	+ 12 58 7,7
1858 June	18,0	281 16 13,4	- 18 50 29,8
1859 Dec.	2,0	67 40 6,8	+ 15 31 7,3
1861 March	13,0	172 44 43,7	+ 8 55 54,5
1862 Juli	26,0	306 21 31,2	- 19 3 26,1
1863 Dec.	8,0	88 56 39,5	+ 18 36 38,2
1865 March	28,0	189 4 52,7	+ 3 18 58,3
1866 Aug.	24,0	334 30 24,6	- 14 35 2,7
1868 Januar	3,0	103 56 14,8	+ 20 3 8,3

With four of these normals approximate corrections of the elements were computed, the are:

$$\begin{aligned} dM &= -3039''8 \\ d\phi &= +253,7 \\ d\pi &= +884,7 \\ d\Omega &= +160,3 \\ di &= +35,0 \\ d\mu &= -0,2888. \end{aligned}$$

Hence

Corrected Elements I.

1850 Jan. 0,0 Berlin Mean Time.

$$\begin{aligned} M &= 250^\circ 44' 33''1 \\ \pi &= 316 \ 53 \ 48,7 \\ \Omega &= 125 \ 1 \ 30,6 \\ i &= 4 \ 37 \ 34,7 \\ \phi &= 5 \ 45 \ 31,0 \\ \mu &= 923''8267 \end{aligned} \left. \vphantom{\begin{aligned} M \\ \pi \\ \Omega \\ i \\ \phi \\ \mu \end{aligned}} \right\} \text{M. Eq. Ep.}$$

$$\begin{aligned} \pi' &= 317^\circ 40' 8'' \\ \Omega' &= 10 \ 33,6 \\ i' &= 21 \ 7,6 \end{aligned} \left. \vphantom{\begin{aligned} \pi' \\ \Omega' \\ i' \end{aligned}} \right\} \text{referred to the equator.}$$

$$\cos(x, x) = 9,863070 \quad \cos(x, y) = 9,792801n \quad \cos(x, z) = 9,458476n$$

$$\cos(y, x) = 9,832957 \quad \cos(y, y) = 9,844801 \quad \cos(y, z) = 9,337492$$

$$\cos(z, x) = 8,819873 \quad \cos(z, y) = 9,549400n \quad \cos(z, z) = 9,969783$$

The computation of the Normals with Corrected Elements I. and the above perturbations gives comp. minus obs.:

$\Delta \alpha \cos \delta$	$\Delta \delta$
+335''2	-87''5
+216,4	+77,2
+198,5	-62,0
+219,9	-26,6
+128,9	+28,4
+152,9	-67,1
-2,6	+17,0
+76,6	-6,1
+125,6	-61,6
-184,4	-3,5
+16,0	-21,7
+48,7	-25,9
-405,4	-101,4
+15,0	-28,3

The perturbations were now rigidly computed for all 14 Normals with the thus corrected Arguments, they are in units of the 6th decimal and referred to the equator:

ξ	η	ζ
+31069	-15254	-8387
-8002	+23444	+9852
-14335	-18393	-5926
+35666	-6017	-5177
-15661	+20265	+9164
-7038	-22092	-7945
+40918	+4629	-1471
-17168	+9777	+5340
-1192	-25521	-9684
+21275	+9914	+2034
-17731	+650	+1942
+5517	-23837	-9593
+14101	+17259	+5389
-14481	-4420	-254

The following logarithms are constants for the transformation of the perturbations from the place of the orbit to the equator. (Berliner Jahrbuch for 1857, p. 391 and 392.)

from which were derived by the method of least squares:

$$\begin{aligned} dM &= +232''1 \\ d\phi &= +9,4 \\ d\pi &= -422,4 \\ d\Omega &= -166,7 \\ di &= -2,6 \\ d\mu &= +0,04360 \end{aligned}$$

and therewith

Corrected Elements II.

1850 Jan. 0 Berlin Mean Time.

$$\begin{aligned} M &= 250^\circ 48' 25''2 \\ \pi &= 316 \ 46 \ 46,3 \\ \Omega &= 124 \ 58 \ 43,9 \\ i &= 4 \ 37 \ 32,1 \\ \phi &= 5 \ 45 \ 40,4 \\ \mu &= 923''8703. \end{aligned} \left. \vphantom{\begin{aligned} \pi \\ \Omega \\ i \end{aligned}} \right\} \text{M. Eq. Ep.}$$

The Normals are represented thus:

$\Delta \alpha \cos \delta$	$\Delta \delta$
+21.0	— 3.1
+19.9	+ 8.6
—89.4	+28.8
+34.8	— 9.7
+ 5.7	+ 3.5
—53.0	+16.7
— 5.2	— 3.4
+14.0	— 0.3
+ 7.1	— 4.2
+ 5.8	+ 1.5
+10.8	— 1.0
+31.1	— 9.2
—64.5	—18.9
+65.8	— 7.3

Another correction of the elements by means of these residuals is impossible since $[na]$, $[nb]$ etc. come out very near Zero; we have, therefore, here before us in the greater numbers the combined effect of the neglected pertur-

bations of the second order by Jupiter. In the class $i' = 3$, where the perturbations of the first order are very great on account of the small divisor $\mu - 3\mu' = +0.04733$, the variations of the coefficients with the Arguments $-3M'$, $M - 3M'$ and $2M - 3M'$ by the perturbations of the second order must be considerable. The Argument $M - 3M'$ is of long period and the effect of it mixed up with the mean daily motion, but the Arguments $-3M'$ and $2M - 3M'$ are of short periods, and the above greater residuals must be the effect of the neglected variations of their Cosinus- and Sinus-coefficients. It is now of some interest to examine the differential-coefficients for those variations of the great terms. If we designate the variations of the coefficients of the cosinus and sinus for ξ and η' with the Arguments $-3M'$ and $2M - 3M'$ respectively with A, B, C, D , and A', B', C', D' , assume for unity the sixth decimal and put $\frac{1}{100}A$ etc. we shall have, denoting by dx'' , dy'' and dz'' the variations of the equatorial-coordinates in seconds, putting $206265 \times 0.000100 = K$, $-3M' = I$ and $2M - 3M' = II$:

$$\begin{aligned} \frac{dx''}{A} &= K \cos(x, x) \cos I = a & \frac{dy''}{A} &= K \cos(x, y) \cos I = a' & \frac{dz''}{A} &= K \cos(x, z) \cos I = a'' \\ \frac{dx''}{B} &= K \cos(x, x) \sin I = b & \frac{dy''}{B} &= K \cos(x, y) \sin I = b' & \frac{dz''}{B} &= K \cos(x, z) \sin I = b'' \\ \frac{dx''}{C} &= K \cos(y, x) \cos I = c & \frac{dy''}{C} &= K \cos(y, y) \cos I = c' & \frac{dz''}{C} &= K \cos(y, z) \cos I = c'' \\ \frac{dx''}{D} &= K \cos(y, x) \sin I = d & \frac{dy''}{D} &= K \cos(y, y) \sin I = d' & \frac{dz''}{D} &= K \cos(y, z) \sin I = d'' \end{aligned}$$

Putting now

$$\begin{aligned} -\sin \alpha \frac{1}{\Delta} &= \cos \delta \frac{d\alpha}{dx''} = e & -\sin \delta \cos \alpha \frac{1}{\Delta} &= \frac{d\delta}{dx''} = g \\ \cos \alpha \frac{1}{\Delta} &= \cos \delta \frac{d\alpha}{dy''} = f & -\sin \delta \sin \alpha \frac{1}{\Delta} &= \frac{d\delta}{dy''} = h \\ & & \cos \delta \frac{1}{\Delta} &= \frac{d\delta}{dz''} = i \end{aligned}$$

we get finally:

$$\begin{aligned} \cos \delta \frac{d\alpha}{A} &= ae + a'f & \frac{d\delta}{A} &= ag + a'h + a''i \\ \cos \delta \frac{d\alpha}{B} &= be + b'f & \frac{d\delta}{B} &= bg + b'h + b''i \\ \cos \delta \frac{d\alpha}{C} &= ce + c'f & \frac{d\delta}{C} &= cg + c'h + c''i \\ \cos \delta \frac{d\alpha}{D} &= de + d'f & \frac{d\delta}{D} &= dg + d'h + d''i \end{aligned}$$

For A', B', C', D' we have only to introduce II . instead of I .

After the numerical computation of these differential-coefficients the trial has been made to derive by the method of least squares A, B etc. together with new corrections of the elements, that is to determine 14 unknown quantities from 28 equations of condition.

Equations of condition.

0 =	+21,0	+1,6347	-3,2109	+1,5864	+0,0848	+0,4617	+ 2,1374		
	+19,9	+1,7468	+3,1338	+1,6161	-0,1140	+0,5839	+ 11,6000		
	-89,4	+1,2595	-0,0929	+1,5325	-0,3472	+0,0865	+ 14,3144		
	+34,8	+1,9321	-3,2240	+1,7464	+0,0868	+0,2373	+ 31,2794		
	+ 5,7	+1,6126	+3,2171	+1,6032	+0,0318	+0,4541	+ 34,6189		
	-53,0	+1,2525	-0,8712	+1,5084	-0,3919	+0,2292	+ 32,7538		
	- 5,2	+2,0918	-2,3767	+1,7762	-0,0599	-0,0042	+ 64,9871		
	+14,0	+1,5086	+2,9392	+1,6023	+0,0888	+0,2670	+ 54,6725		
	+ 7,1	+1,2826	-1,5654	+1,5038	-0,3616	+0,3913	+ 52,4730		
	+ 5,8	+2,2261	-0,8902	+1,8200	-0,2917	-0,0105	+102,2267		
	+10,8	+1,4126	+2,3996	+1,5869	+0,0124	-0,0631	+ 72,1262		
	+31,1	+1,3478	-2,2749	+1,5143	-0,2745	+0,5153	+ 75,0746		
	-64,5	+2,1443	+0,9052	+1,7540	-0,4463	+0,2285	+130,4331		
	+65,8	+1,3583	+1,7264	+1,5907	-0,0768	-0,0131	+ 89,3559		
		dM	$d\varphi$	$d\pi$	$\frac{1}{10}d\Omega$	di	$100d\mu$		
-7,437	-11,630	- 0,052	-0,081	-1,567	+13,715	- 0,011	+0,097		
+4,748	-12,675	- 1,496	+3,995	+8,537	+10,504	- 2,691	-3,311		
+0,480	- 0,064	-11,602	+1,551	+0,483	- 0,033	-11,678	+0,806		
-8,402	-10,949	- 4,262	-5,654	-7,523	+11,570	- 3,816	+5,869		
+2,795	-13,282	+ 0,210	-0,996	+2,034	+13,420	+ 0,152	+1,006		
+3,613	- 0,969	-10,596	+2,843	+3,384	+ 1,594	- 9,925	-4,674		
-6,883	- 6,786	- 9,359	-9,226	-8,592	+ 4,431	-11,682	+6,025		
+0,425	-12,157	+ 0,171	-4,893	-4,786	+11,184	- 1,926	+4,501		
+5,966	- 2,708	- 8,835	+4,008	+3,619	+ 5,461	- 5,359	-8,087		
-3,223	- 1,935	-14,188	-8,520	-3,718	- 0,553	-16,369	-2,437		
-0,601	- 9,422	- 0,544	-8,529	-7,283	+ 6,007	- 6,593	+5,438		
+7,956	- 5,026	- 6,344	+4,008	+0,632	+ 9,389	- 0,504	-7,487		
+3,968	+ 1,400	-14,890	-5,303	+3,151	+ 2,795	-11,828	-10,542		
-1,693	- 6,691	- 2,541	-9,980	-6,850	+ 0,869	-10,216	+1,297		
		$\frac{1}{100}A$	$\frac{1}{100}B$	$\frac{1}{100}C$	$\frac{1}{100}D$	$\frac{1}{100}A'$	$\frac{1}{100}B'$	$\frac{1}{100}C'$	$\frac{1}{100}D'$
0 =	- 3,1	-0,4809	+0,9735	-0,4749	+0,4596	+1,5022	- 0,5034		
	+ 8,6	+0,6490	+1,1089	+0,5907	+0,1492	-1,6001	+ 4,1512		
	+28,8	-0,3088	+0,0067	-0,3758	-1,1878	+0,3527	- 3,4836		
	- 9,7	-0,3224	+0,5916	-0,2991	+0,8562	+1,3940	- 5,0888		
	+ 3,5	+0,4819	+0,9444	+0,4697	-0,3065	-1,5617	+10,2062		
	+16,7	-0,4005	+0,2482	-0,4835	-1,0552	+0,7169	-10,4212		
	- 3,4	+0,0043	+0,0575	-0,0009	+1,2281	+0,9393	+ 0,2415		
	- 0,3	+0,3074	+0,6114	+0,3197	-0,7199	-1,3375	+11,0374		
	+ 4,2	-0,4656	+0,5251	-0,5494	-0,8327	+1,0701	-18,9553		
	+ 1,5	+0,3526	-0,1365	+0,2882	+1,4579	+0,0664	+16,2005		
	- 1,0	+0,1123	+0,2158	+0,1210	-0,9761	-1,0566	+ 5,6428		
	- 9,2	-0,5162	+0,8337	-0,5863	-0,5576	+1,3324	-28,6456		
	-18,9	+0,6549	+0,2152	+0,5345	+1,2606	-0,7508	+39,7542		
	- 7,3	-0,0271	-0,0244	-0,0335	-1,1827	-0,6128	- 1,8258		
		dM	$d\varphi$	$d\pi$	$\frac{1}{10}d\Omega$	di	$100d\mu$		
+2,283	+3,571	-0,366	-0,595	+0,481	-4,210	-0,077	+0,675		
+1,027	-4,346	-0,837	+2,287	+2,927	+3,601	-1,540	- 1,895		
+0,048	-0,003	+2,849	-0,381	+0,017	-0,001	+2,868	-0,198		
+1,643	+2,141	+0,298	+0,388	+1,461	-2,263	+0,267	-0,410		
+1,036	-3,909	-0,102	+0,483	+0,598	+3,950	-0,074	-0,488		
-0,909	+0,244	+3,471	-0,931	-0,851	-0,401	+3,251	+1,531		
+0,344	+0,339	-0,317	-0,313	+0,429	-0,221	-0,296	+0,204		
+0,092	-2,638	+0,015	-0,442	-1,038	+2,427	-0,174	+0,406		
-1,821	+0,826	+3,469	-1,573	-1,104	-1,666	+2,104	+3,175		
-0,477	-0,286	-2,255	-1,354	-0,550	-0,082	-2,602	-0,387		
-0,064	-1,003	-0,017	-0,269	-0,775	+0,689	-0,208	+0,171		
-2,745	+1,734	+2,856	-1,804	-0,218	-3,239	+0,217	+3,370		
+0,727	+0,259	-4,656	-1,658	+0,577	+0,512	-3,697	-3,295		
-0,011	-0,041	+0,085	+0,385	+0,139	+0,005	+0,343	-0,044		
		$\frac{1}{100}A$	$\frac{1}{100}B$	$\frac{1}{100}C$	$\frac{1}{100}D$	$\frac{1}{100}A'$	$\frac{1}{100}B'$	$\frac{1}{100}C'$	$\frac{1}{100}D'$

from which was obtained:

$$\begin{aligned} dM &= -191''6 \\ d\phi &= -3,5 \\ d\pi &= +19,9 \\ d\Omega &= +3,1 \\ di &= -6,6 \\ d\mu &= +0,06645 \\ A &= -2770 & A' &= +3809 \\ B &= +1818 & B' &= +2151 \\ C &= +779 & C' &= -1794 \\ D &= +3260 & D' &= +4162 \end{aligned}$$

units of the 6th decimal.

The substitution into the equations of condition leaves:

$\Delta \alpha \cos \delta$	$\Delta \delta$
- 7''3	- 0''1
- 2,1	+ 11,7
+ 0,8	+ 4,0
+ 11,6	- 4,9
+ 0,2	- 0,9
- 4,7	- 0,7
- 8,2	+ 0,8
+ 0,7	- 3,8
+ 5,1	- 3,5
+ 2,6	+ 1,9
+ 2,5	- 6,9
- 0,8	+ 5,1
+ 1,7	- 8,9
+ 0,4	- 7,7

A, B etc. have evidently come out too great, the solution of the problem is practically very precarious; there must be a great uncertainty since the whole system of the corrections depends upon $D' = -\frac{[n h', 13]}{[h' h', 13]} = \left(\frac{-5.744}{+0.138}\right)$. The system that appears if D' is diminished even by 1000 leaves still very nearly the same residuals.

That the greater residuals left by set II. are really the effect of the neglected greater terms of the second order is evidently shown by a comparison of the residuals for

1853	-89,4	+28,8	
	+36,4	-12,1	
1857	-53,0	+16,7	
	+60,1	-20,9	
1861	+ 7,1	- 4,2	

with the corresponding differential-coefficients. For 1853 six of these coefficients are nearly Zero and only those of C and C' are very great both in α and δ , and the residuals are chiefly the combined effect of C and C' . Since the coefficients of C and C' are equal in α as well as in δ but the signs different in α and δ , the signs of the residuals must be different too and the residuals quantitatively nearly in the proportion 4 : 1; and so it is indeed. Looking at the variations of the residuals we find very nearly $36:60 = 12:21$. In 1857, 1861 and so on the differential-

coefficients of C and C' in α decrease and those of the other variations increase; in δ the coefficients of C and C' remain nearly the same and the other coefficients increase too. The variation of the combined effect upon the residuals from 1853 to 1861 is $+96''5$ and $-33''0$.

It will, therefore, be safest to rely on the elements II. and construct approximate tables upon them since the effect of the neglected second order upon the determination of the elements can not have been great on account of the 18 years interval of time. The combined effect of the neglected second order will in most cases be moderate and always within such limits that in the next fifty years there will be no difficulty to find the planet.

From 3 observations made at Bilk and 2 observations made at Berlin this year a normal-place has been formed; it is reduced to the mean equinox 1850,0:

1869 April 14,5 Berlin Mean Time.

$$206^{\circ}9'28''3 \quad -2^{\circ}44'25''6$$

and represented by set II.:

$\Delta \alpha \cos \delta$	$\Delta \delta$
+18''4	- 8''5

The elements II. + corrections with A, B etc. leave
-31''6 +17''8

Dr. Luther has computed for 1870 Sept. 20,5 from his elements and special perturbations:

App. α	App. δ
$0^h 0^m 2^s 95$	$-7^{\circ}9'10''1$
elements II. give 0 0 0,31	-7 9 39,8

Finally the attention of astronomers is called to a very remarkable circumstance. In this case of Parthenope an augmentation of the perturbations (the mass of Jupiter) is throughout nearly identical with an augmentation of the epoch M . Making $\Delta m' = \frac{1}{25000}$ we get

$\cos \delta \frac{d\alpha}{dM}$	$\cos \delta \frac{d\alpha}{dM}$	$\frac{d\delta}{dM'}$	$\frac{d\delta}{dM}$
+1''5	+1''6	-0''6	-0''5
+1,5	+1,7	+0,5	+0,6
+1,1	+1,3	-0,3	-0,3
+2,2	+1,9	-0,4	-0,3
+1,4	+1,6	+0,4	+0,5
+1,1	+1,3	-0,4	-0,4
+2,6	+2,1	0,0	0,0
+1,0	+1,5	+0,2	+0,3
+1,1	+1,3	-0,5	-0,5
+1,6	+2,2	+0,2	+0,4
+1,0	+1,4	+0,1	+0,1
+1,2	+1,3	-0,5	-0,5
+1,4	+2,1	+0,5	+0,7
+0,8	+1,4	0,0	0,0

In δ the concordance is striking. For 1869 was found:

+1''2	+1''3	-0''4	-0''5
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