



# LII. On graphic solution of dynamical problems

Lord Kelvin

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states are absent, and these are the very rare people with vision of light and shade only.

Donders even thinks that he can trace, within the ocular area, a vestige of a difference of the kind existing between red and green blindness, the former having a shortened spectrum, and the latter being a stage nearer perfect vision, which, if it were established, would be an additional element in the analogy. The idea of a complete system of evolution for colours might then be sketched out somewhat as follows :—

1. Achromic vision (light and shade only).
2. Dichromic imperfect vision (called “Red-blindness” : short spectrum, low sensitiveness to the long-waved rays).
3. Dichromic perfect vision (called “Green-blindness” : longer spectrum, full sensitiveness to the long-waved rays).
4. Trichromic imperfect vision (as pointed out by Lord Rayleigh), with low sensitiveness to certain colours.
5. Trichromic perfect vision.

These classes would be subject to intermediate gradations, as in other evolutionary development.

Looked upon in this way, colour-blindness would be only an imperfect development of normal vision, not springing out of it, as the Young-Helmholtz explanation would suggest, but antecedent to it. It would be a system whose two energies resulted independently from the decomposition of white light, and, therefore, would be complementary to each other.

Donders also cites, as favouring this view, the peculiar mode of hereditary transmission of the defect, according to the unanimous testimony of experts. A patient transmits it, not to his sons, but to his grandsons through a daughter, who is free from it herself : thus causing it to skip over one generation.

Athenæum Club, S.W.  
October, 1892.

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LII. *On Graphic Solution of Dynamical Problems.*  
*By* LORD KELVIN\*.

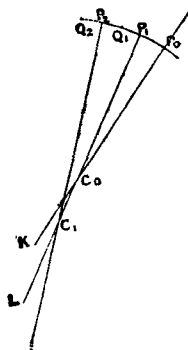
THE method of drawing meridional curves of capillary surfaces of revolution, described in ‘Popular Lectures and Addresses,’ vol. i., 2nd edition, pp. 31–42, and illustrated by woodcuts made from large scale curves, worked out according to it with great care and success by Professor Perry when a student in the Natural Philosophy Class of Glasgow

\* Communicated by the Author.

University, suggests a corresponding method for the solution of dynamical problems.

In dynamical problems regarding the motion of a single particle in a plane, it gives the following plan for drawing any possible path under the influence of a force of which the potential is given for every point of the plane. Suppose, for example, it is required to find the path of a particle projected, with any given velocity, in any given direction through any given point  $P_0$  (fig. 1). Calculate the normal component force at this point; and divide the square of the

Fig. 1.



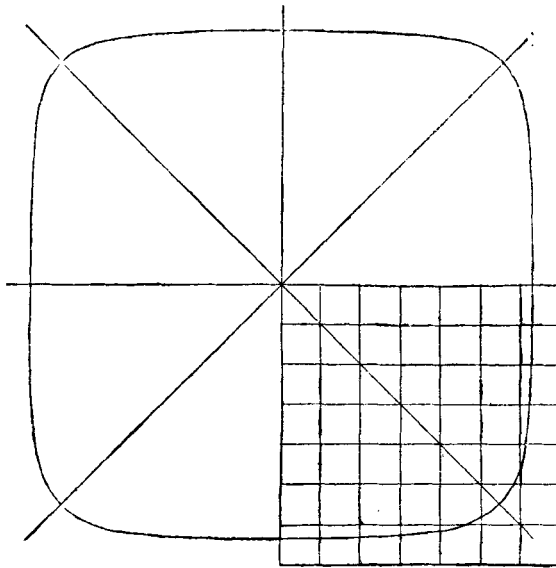
velocity by this value, to find the radius of curvature of the path at that point. Taking this radius on the compasses, find the centre of curvature,  $C_0$ , in the line,  $P_0K$ , perpendicular to the given direction through  $P_0$ , and describe a small arc,  $P_0P_1Q_1$ , making  $P_1Q_1$  equal to about half the length intended for the second arc. Calculate the altered velocity for the position  $Q_1$ , according to the potential law; and, as before for  $P_0$ , calculate a fresh radius of curvature for  $Q_1$  by finding the normal component force for the altered direction of normal and for the velocity corresponding to the position of  $Q_1$ . With this radius, find the position of the centre of curvature,  $C_1$ , in  $P_1C_0L$ , the line of the radius through  $P_1$ . With this centre of curvature, and the fresh radius of curvature, describe an arc  $P_1P_2Q_2$  making  $P_2Q_2$  equal to about half the length intended for the third arc; calculate radius of curvature for position  $Q_2$ ; draw an arc  $P_2P_3Q_3$ ; and continue the procedure. This process is well adapted for finding orbits by the 'trial and error' method described in my article "On Some 'Test Cases' of the Maxwell-Boltzmann Doctrine regarding Distribution of Energy," sect. 13; Proc. Royal Soc., June 11, 1891.

The accompanying curve (fig. 2) has been drawn with great care, and with very interesting success, in the 'trial and error' method of finding the first and simplest orbit, by my secretary, Mr. Thomas Carver, for the case of motion defined by the equations

$$\frac{d^2x}{dt^2} = -yx^2.$$

$$\frac{d^2y}{dt^2} = -xy^2.$$

Fig. 2.



The initial point  $P_0$  was taken on one of the lines cutting the axes of  $x$  and  $y$  at  $45^\circ$ , and at first at a random distance from the origin. A trial curve was worked according to the method described above, and was found to cut the axis of  $x$  at an oblique angle. Other trial curves, with unchanged energy-constant, were worked from initial points at greater or less distances from the origin, until a curve was found to cut the axis of  $x$  perpendicularly. This curve is one-eighth part of the orbit; and is shown in fig. 2 repeated eight times in order to complete the orbit, which is symmetrical on the two sides of the axes of  $x$  and  $y$ .

As an interesting case of motion related to the Lunar Theory, suppose the mass of the moon be infinitely small in comparison with the mass of the earth; and the earth and sun to have uniform motions in circles round their centre of

gravity. Let  $(x, y)$  be coordinates of the moon relative to  $OX$  in line with the sun, outwards, and  $OY$  perpendicular to it in the direction of the earth's orbital motion. The well-known equation of motion relatively to revolving coordinates gives, for the equations of the moon's motion, if  $a$  denote the distance from  $O$  (the earth) of the centre of gravity of the sun and earth,

$$\frac{d^2x}{dt^2} - 2\omega \frac{dy}{dt} - \omega^2(a+x) = -\frac{dV}{dx}, \quad . . . \quad (1)$$

$$\frac{d^2y}{dt^2} + 2\omega \frac{dx}{dt} - \omega^2y = -\frac{dV}{dy}, \quad . . . \quad (2)$$

where  $V$  is the potential of the attractions of the sun and earth on the moon, and  $\omega$  the angular velocity of the earth's radius-vector. From this we find, for the relative-energy equation

$$\frac{1}{2} \left( \frac{dx^2}{dt^2} + \frac{dy^2}{dt^2} \right) = E + \frac{1}{2} \omega^2 (x^2 + y^2) - V, \quad . . . \quad (3)$$

where  $E$  denotes a constant; and for the relative-curvature equation we find

$$\frac{dx d^2y - dy d^2x}{(dx^2 + dy^2)^{\frac{3}{2}}} = -2\omega \frac{dt}{(dx^2 + dy^2)^{\frac{3}{2}}} + \frac{N dt^2}{dx^2 + dy^2}, \quad . . . \quad (4)$$

where  $N$  denotes the component perpendicular to the path, of the resultant of  $(X, Y)$  with

$$X = \omega^2(x+a) - \frac{dV}{dx}, \quad . . . . . \quad (5)$$

$$Y = \omega^2y - \frac{dV}{dy} . . . . . \quad (6).$$

Hence if  $q$  denote moon's velocity and  $\rho$  the radius of curvature of her path, relatively to the revolving plane  $XOY$ , we have

$$\frac{1}{2} q^2 = E + \frac{1}{2} \omega^2 (x^2 + y^2) - V, \quad . . . . . \quad (7)$$

and

$$\frac{1}{\rho} = \frac{-2\omega}{q} + \frac{N}{q^2} . . . . . \quad (8).$$

Calling  $S$  the sun's mass, and  $a$  his distance from the earth, and supposing the earth's mass infinitely small in comparison with the sun's, we have

$$\frac{S}{a^2} = \omega^2 a, \quad . . . . . \quad (9)$$

and therefore

$$-V = \frac{\omega^2 a^3}{[(a+x)^2 + y^2]^{\frac{3}{2}}} + \frac{m}{r}, \quad . . . . . \quad (10)$$

where  $m$  denotes the earth's mass, and  $r = \sqrt{(x^2 + y^2)}$ .

Hence

$$-V = \frac{1}{2}\omega^2(2a^2 - 2ax + 2x^2 - y^2) + \frac{m}{r} \quad . \quad . \quad (11).$$

With this, and with  $\omega=1$  and  $m=b^3$ , for simplicity in the numerical work which follows, we have

$$\frac{d^2x}{dt^2} - 2\frac{dy}{dt} = X = x\left(3 - \frac{b^3}{r^3}\right), \quad . \quad . \quad (12)$$

$$\frac{d^2y}{dt^2} + 2\frac{dx}{dt} = Y = -y\frac{b^3}{r^3}, \quad . \quad . \quad . \quad (13)$$

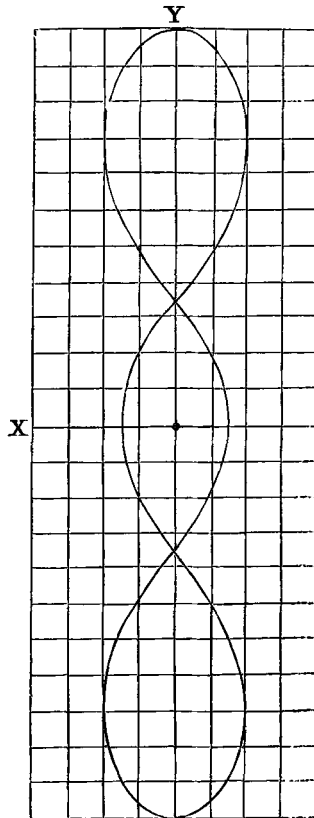
$$q^2 = 2E + 3x^2 + \frac{2b^3}{r}, \quad . \quad . \quad . \quad (14)$$

and

$$\rho = \frac{q^2}{N - 2q} \quad . \quad . \quad . \quad (15).$$

From equations (12) and (13), G. W. Hill has, with four different values of  $E$ , found  $x$  and  $y$  explicitly in terms  $t$ , for the particular solution in each case which gives the simplest orbit (relatively to the revolving plane XOY); of which the one which presents the greatest deviation from the well-known 'variational' oval of the elementary lunar theory is a symmetrical curve with two outwardly projecting cusps corresponding to the moon in quadratures. He supposed this to be the most extreme deviation from the variational oval possible for an orbit surrounding the earth. Poincaré, in his *Méthodes Nouvelles de la Mécanique Céleste*, p. 109 (1892), admiring justly the manner in which Hill has thus 'si magistralement' studied the subject of finite closed lunar orbits, points out that there are solutions corresponding to *looped* orbits, transcending Hill's, wrongly supposed extreme, cusped orbit. Mr. Hill

Fig. 3.



tells me that he accepts this criticism. The labour of working out a fairly accurate analytical solution for any of Poincaré's looped orbits, by Hill's method, would probably be very great. I have therefore thought it might interest others besides ourselves to apply my graphic method to the drawing of at least one of Poincaré's looped orbits, in our Physical (and Arithmetical) Laboratory in the University of Glasgow. Figure 3 represents a looped orbit, which has been worked out accordingly by Mr. Magnus Maclean, Chief Official Assistant of the Professor of Natural Philosophy, from the equations (14) (15) above. The initial values used for obtaining the curve, were  $x=2$ ;  $y=0$ ;  $b=10$ ;  $2E=-130$ ; and  $\therefore q_0^2=882$  and  $\rho_0=4.8$ .

### LIII. *Notices respecting New Books.*

#### *Organic Dyestuffs.*

*Chemistry of the Organic Dyestuffs.* By R. NIETZKI, Ph.D.  
Translated, with additions, by A. COLLIN, Ph.D., and W. RICHARDSON. (London: Gurney & Jackson, 1892.)

THE German editions of this little volume are so well known to all chemists who interest themselves in the tar colouring-matters that Messrs. Collin & Richardson have done good service by presenting Dr. Nietzki's work in an English form. The author, it is perhaps needless to state, is Professor in the University of Basle, and is best known in the chemical world as one of the most successful investigators into the constitution of the complex organic colouring-matters which science furnishes to technology. Coming from the pen of such a recognized authority as Dr. Nietzki, no special commendation is necessary to assure English students and technologists that they have received a most important and valuable contribution to their literature. The translators have done their part of the work also with commendable skill, and have fairly well expressed the author's meaning throughout.

One special feature of the present work is its purely scientific treatment of a subject which is necessarily intimately connected with manufacturing processes. There are already in Germany several exhaustive works on the technology of coal-tar colouring-matters, notably those of Schultz and Mühlhäuser, but while these are replete with manufacturing details and reprints of bulky patent specifications, Dr. Nietzki concerns himself more especially with the classification and constitution of the compounds, and his work appeals therefore to the purely scientific chemist as well as to the technologist. Only sufficient technology is introduced to make the scientific discussion coherent and intelligible. Many of the