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KINEMATICS AND DYNAMICS.

An Elementary Treatise on Kinematics and Dynamics. By James Gordon MacGregor, M.A., D.Sc., &c., Munro Professor of Physics, Dalhousie College, Halifax, N.S. (London: Macmillan and Co., 1887.)

THE logical order of arrangement has been carefully attended to in this book: Part I., on "Kinematics," building up a new subject on the foundation of Euclid's axioms in conjunction with the idea of the variables, such as velocity and acceleration, due to the flow of time; while Part II., on "Dynamics," requires three new axioms —Newton's Laws of Motion—to make a fresh start and connect mechanical effects with their causes.

But it is doubtful if the strictly logical order is the best order for the student to make his first acquaintance with a new mathematical subject: the ideas must grow in his brain by accretion round simple fundamental problems. A student would master the present treatise more easily by reading Part II. first, and referring back to Part I. as occasion required, for the explanation of the details of the mathematical calculations. There is nothing to prevent this order of study here, although the author has, from logical considerations, placed the kinematical part first.

One defect of the logical system is that it places some of the most difficult parts of the subject in the way of beginners: for instance, the theory of the change of units, a theory of which the importance can only be appreciated by those who have made considerable progress in the subject.

In Part I., "Kinematics," the treatment is simple and concise, but we should like to see more examples of phenomena on a large scale, such as those of physical astronomy, or even of railway-train problems.

In questions involving the size of the earth (pp. 74 and 80) it is the circumference and not the diameter which should be given in metres, the circumference being 40,000,000 metres, a kilometre being a centesimal minute of latitude. Or, if the size of the earth is given in miles, it is the nautical mile which should be used, the circumference of the earth being $360 \times 60 = 21{,}600$ nautical miles, a nautical mile being a sexagesimal minute of latitude.

The expression "knots an hour" (p. 60) is irritating to a sailor, as emanating from the engine-room; the proper nautical expression is "knot" simply, a speed of 10 knots being 10 nautical miles an hour.

The formula $\frac{1}{2}v^2 = \frac{1}{2}v_0^2 + as$ is to be preferred to that on p. 34, $v^2 = v_0^2 + 2as$; in all cases the factor $\frac{1}{2}$ should go with the v^2 in the equation of energy, so that the objectionable expression "*vis viva*" may finally be stamped out from all dynamical treatises.

In dealing with rotation, in Chapter V., the author would do well to study Maxwell's geometrical representation of the direction by means of the screw, right-handed or left-handed; and to discard all attempts by comparison with a clock-wise or counter-clock-wise rotation, requiring as these do a specification of the aspect of the plane of motion.

Pure homogeneous strain is analyzed in Chapter VII. as far as is possible by simple geometrical methods; such a strain may be produced by the superposition of three linear strains in directions at right angles to one another. In a linear strain the increment of distance of two points in the line of the strain is properly their *elongation;* while the ratio of the elongation to the original distance is called the *extension*, not the *elongation*, as on p. 167.

In Part II., "Dynamics," we find in Chapter I. the discussion on the units of measurement of weight, mass, and force customary in mathematical treatises, and of the usual unsatisfactory nature. The author, disregarding the vernacular use of the word "weight," defines the weight of a body as the force with which it is attracted by the earth, but is at variance with his own definition in the statement of the majority of the subsequent examples, relapsing into the language of ordinary life. A collection of 500 different ways of spelling the name of the town of Birmingham has been made, and a similar collection could be made from the present treatise of different ways of expressing the simple ideas of the pound *weight* and the pound *force*, to use the ordinary language of practical men. The attraction of the earth on a pound is, in the vernacular, "the force of a pound," not the "weight of a pound," the latter implying what the mathematician likes to distinguish as the "mass of a pound." Thus a mathematical precisionist, to express the simple idea of a force of 10 pounds, to be consistent should call it "a force equal to the weight of the mass of 10 pound weights," the absurdity of which is evident.

Again, in straining after the equation $F = ma$, when using the gravitation unit of force, the mathematician in the F.P.S. (foot-pound-second) system of units is obliged to use the variable unit of mass of g pounds to measure the invariable quantity, the mass of the body; while what he calls the weight of the body, and denotes by w, measuring it in pounds, is, although variable with g, always measured by the same number.

Next we have the equation $w = mg$, the source of all the confusion in dynamical teaching, and only to avoid writing the dynamical equation with gravitation units in the form

$$F = w\frac{a}{g}.$$

This terminology culminates in the solecisms that on p. 477 we must suppose pressure to be measured in poundals on the square foot in hydrostatical problems; and that if the equation $w = mg$ is supposed to be used with absolute units, that the weight of a body is measured in poundals; as if a mathematician asked in a shop for "half a poundal of tea, or tobacco." Ordinary people measure weight in pounds, so that if mass is also measured in pounds, then $w = m$.

It is time now, as Prof. Minchin has pointed out, that "the astronomical unit of mass," defined in § 315, should disappear, and that in all problems of physical astronomy the gravitation constant k should be retained, while m, the mass, is measured in terms of the ordinary units.

Although the author does not allow himself the use of the methods and notation of the Calculus, still he has managed to discuss a number of interesting problems in the dynamics of a rigid body, usually proved by the methods of Analytical Mechanics.

Working under these restrictions, he has given elegant elementary proofs of the chief properties of the common catenary; but here, again, it is time that the equation

should be presented in the form $y/a = \cosh x/a$, using the notation of the hyperbolic functions; which might also be employed with advantage in the statement of the results of the examples on p. 302. Chains of 5000 feet span, and 400 feet versed sine, are in existence, providing striking numerical examples in this part of the subject.

Most of the examples are carefully chosen, but the author by diligent search could easily add more interest to the collection, particularly to the examples on parabolic trajectories, and problems concerning the motion of railway-trains. Ex. 85, p. 499, certainly requires careful revision. The diagrams of the simple machines are of the usual academic nature; the author should consult Prof. Kennedy's "Mechanics of Machinery" for better illustrations, especially of the differential pulley, and of pulley tackle in general. If the differential screw is given (p. 435), why not also the integral screw, which is to be met with more commonly in real life—for instance, in railway couplings, and in the rigging of ships.

Except for the parts criticized above, on the units of weight, mass, and force, the present treatise shows that the author has read with profit and discrimination the most recent treatises on dynamics; he has produced a very useful work, suitable for instruction in technical colleges, and likely also to prove a necessary corrective to the very abstract treatment of the subject of mechanics too common in the character of University instruction.

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ATLAS OF THE DISTRIBUTION OF PLANTS. *Atlas der Pflanzenverbreitung.* (Berghaus's "Physikalischer Atlas," Abtheilung V.) Bearbeitet von Dr. Oscar Drude. (Gotha: Justus Perthes, 1887.)

THE history of the science of the distribution of plants begins with Linnæus, who was the first to cite systematically the countries and situations in which the plants he described grew. This we find carefully done in the first edition of the "Species Plantarum," published in 1753. No perceptible advance beyond this was made before the appearance of Humboldt and Bonpland's "Essai sur la Géographie des Plantes" in 1805, which work may be designated the real foundation of the science. It was followed in 1823-24 by the Dane, Schouw's "Grundtræk" and "Plantegeographisk Atlas," the latter containing twenty-two maps illustrating the vegetation of the world, and especially the distribution of plants cultivated for food. There is also a German edition of both the "Outlines" and the "Atlas." From this date onward many of the most eminent botanists investigated distribution in connection with classification of plants, notably R. Brown, A. P. De Candolle, H. C. Watson, C. Darwin, A. De Candolle, J. D. Hooker, Edward Forbes, Von Martius, and Grisebach, to say nothing of the younger botanists. But the results of their labours ars still scattered, or at least only partially elaborated; for Grisebach, in his "Vegetation der Erde," deals with the facts from a peculiarly narrow stand-point.

It is true that both Drude and Engler ("Versuch einer Entwicklungsgeschichte der Florengebiete") have attempted something beyond this, but neither, we suspect, regards his work as more than a preliminary effort. The primary geographical divisions of these two writers are essentially the same, though their nomenclature differs; but, considering the complexity of the subject, probably no two persons would agree exactly on these points; yet it is highly desirable that there should be something approaching uniformity in the names of the divisions. Grisebach designates his primary divisions "Gebiete," and Drude his "Reiche"; whilst Engler's four primary divisions are designated "Reiche," and his secondary ones "Gebiete." Let us now briefly examine the main features of Drude's Atlas. Following the most authoritative English writers on zoological and botanical geography, we will call the primary divisions regions, and the secondary divisions sub-regions.

Drude divides the world into fourteen floral regions, and each of these into a number of sub-regions, indicating by lines and dots the overlapping of the elements of contiguous sub-regions. The regions are: (1) Northern, (2) Central Asia, (3) Mediterranean, (4) East Asia, (5) Middle North America, (6) Tropical Africa, (7) East African Islands, (8) Indian, (9) Tropical America, (10) Cape, (11) Australia, (12) New Zealand, (13) Andes, (14) Antarctic.

While agreeing in the main with the foregoing divisions, we cannot but regard some of them as including too, much or too little, according to the number of primary divisions adopted. We recognize the difficulties of the task, and admit that it is practically impossible to divide the vegetation of the world into regions of equal value and importance, even leaving out of consideration the mountain flora within the tropics. Instead, however, of giving Madagascar and the neighbouring islands the rank of an independent region, we should treat it as a sub-region of the tropical African flora. On the other hand, the Indian region seems too comprehensive, as it includes the whole of tropical India, Malaya, Cochin-China, the Malayan Archipelago, New Guinea, North Australia, and Polynesia, even to the Sandwich Islands. The very extensive recent collections of Madagascar plants, made by various English and French travellers, prove that the flora is really a sub-region of the tropical African flora. With regard to the flora of Polynesia, it is true that the littoral element consists almost exclusively of species common to the Malayan Archipelago and North Australia, many having an even wider range; but the Australian and American affinities of the endemic element are certainly too pronounced, in our opinion, to treat this flora as a sub-region of the Indian; and the Sandwich Island flora is as highly specialized, to say the least, as that of New Zealand. Perhaps it would be more convenient to make it an independent region. Again, the purely Australian types surely predominate largely over the Asiatic in North Australia, especially if we eliminate the widely-dispersed coast plants. Dr. Drude's New Zealand region includes the surrounding islands, except the more southern Macquarie; yet, of the eighteen vascular plants recorded from this island, sixteen are common to the New Zealand group. The Auckland and Campbell groups should be reckoned in the Antarctic region rather than New Zealand; and St. Paul and Amsterdam Islands, as well as the Tristan d'Acunha group, do not belong to the same category. Further, the higher mountain flora of Central America and South Mexico has certainly a greater claim to be included in the Andine region than has that of the Galapagos, though Dr. Drude separates them.

We have called attention to these defects or incon-