

ART. XLI.—*Seiches on the Bay of Fundy*;* by A.
WILMER DUFF.

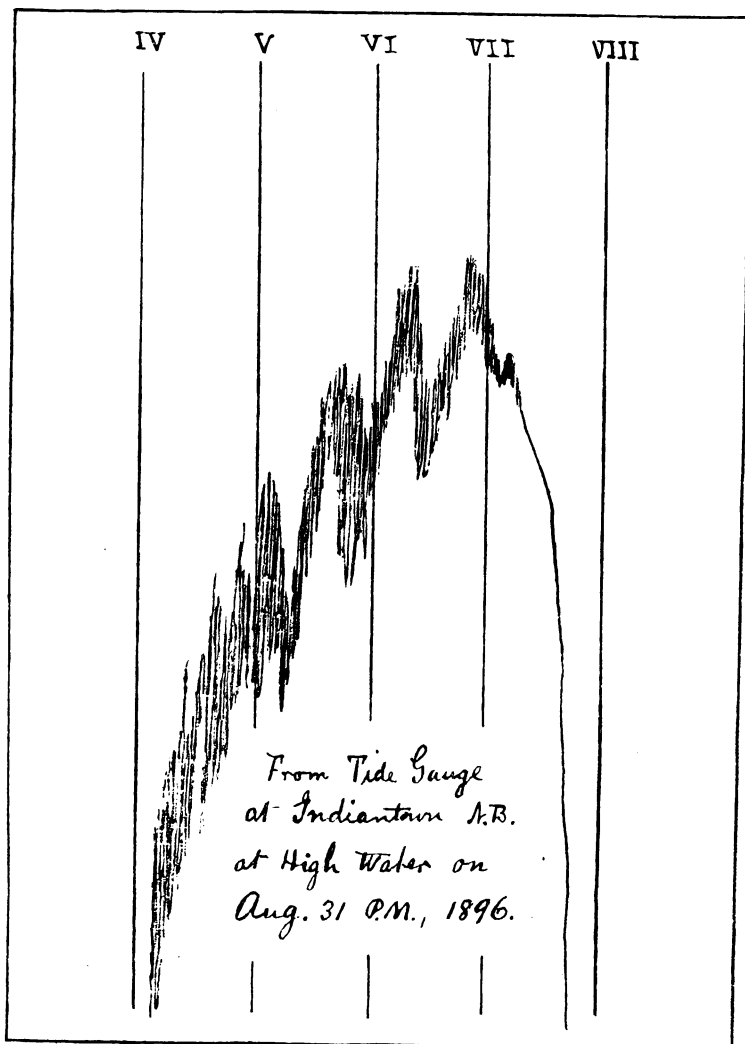
REFERENCE has been made by several writers on tides to so-called "secondary undulations" recorded by self-recording tide-gauges. The most recent mention of the matter is that of Mr. W. Bell Dawson, engineer in charge of the Dominion of Canada Tidal Survey. He notes† these "secondary undulations" on the records of the Kelvin tide-gauge in the harbor of St. John, New Brunswick, on the Bay of Fundy. "They stand," says Mr. Dawson, "in much the same relation to the main tidal wave as a high octave would to a low musical note, when their undulations are recorded graphically. They have an amplitude which is sometimes over a foot, and a period of about forty minutes. It does not seem that any satisfactory explanation has yet been given to account for them."

The same matter came to my attention in the summer of 1896 in such a way as to suggest an explanation. While engaged with a simple form of self-recording tide-gauge in making some tidal observations on the St. John River, I obtained at a point near the mouth of the river the curious trace of high water shown in the adjoining figure. The station at which it was obtained was Indiantown, a part of the city of St. John, immediately above the very narrow outlet (only one hundred yards wide), through which the large river rushes into St. John Harbor. The day on which it was obtained (Aug. 31), was a very calm one. While watching the instrument before high tide I noticed curious little fluctuations of level, having a fairly constant period of thirty-five seconds, as determined by a stop-watch. This period I knew would be sufficient to enable the fluctuations to give a record on the drum of the instrument, whereas mere steamer waves succeeding each other at irregular intervals of only a few seconds could not, owing to the smallness of the opening through which the water obtained access to the float, give any such trace. These 35-second undulations show themselves in the fine tracings on the left of the record. But what was quite unexpected was the series of larger undulations shown in the record and having a period of between thirty and forty minutes. It will be noticed that both series cease at nearly the same time, about half an hour after high water at Indiantown, that is about two and a half hours after high water in St. John Harbor (for the

* Abstract of a paper read before the Natural History Society of New Brunswick.

† Trans. Roy. Soc. Can., vol. i, 1895-96.

narrow gate between the harbor and the river produces a delay of two hours in high water and low water at Indiantown, which is less than a mile from the harbor). This simultaneous ceasing of the two series of undulations seems to suggest a connection between them.



As regards the slower undulations, it seemed very probable, from the similarity of the period, that they were merely the secondary undulations in the harbor, noted by Mr. Dawson, propagated into the river. These secondary undulations have usu-

ally been regarded as having some connection with the tides proper. As regards the 35-seconds undulations such a connection was clearly impossible, when regard was had to their very brief period. They evidently could not be *forced* vibrations, and if not they must be the *free* vibrations of some semi-confined body of water. Now this body could only be the small bay into which the river expands at Indiantown, nearly closed above by a part called the Narrows, and below by the narrow outlet referred to. This suggested at once that the slower undulations in the harbor had a similar origin, namely, that they were the free vibrations in another semi-confined basin. This basin could not be the harbor itself. Its dimensions are about those of the small bay at Indiantown, and its time of free swing could not be sixty times as great. The only other basin available for an explanation was the Bay of Fundy itself, limited on one side by the New Brunswick coast and on the other by the Nova Scotia coast. Hence it seemed probable that the "secondary undulations" in the harbor were the free vibrations of the Bay of Fundy along a line from St. John to the nearest part of the Nova Scotia coast. The rate of such vibrations depends on the dimensions of the basin, and this provides a means of testing the above theory.

Before applying this test, attention may be called to some similar phenomena observed long since but only recently come to my knowledge. For a great many years certain fluctuations of water level at points on Lake Geneva were known and popularly called *seiches*. After many scattered observations by Bertrand, de Saussure, Vaucher and others, a thorough study of the subject was undertaken by Dr. F. A. Forel, whose two articles* give a complete résumé of the subject. Vaucher had found that the *seiches* were most common in changeable weather, with a low barometer, and he considered them to be due to merely local temperature and consequent barometric changes, caused by rifts in clouds and variations of sunshine on the lakes. Forel added the idea that, after the disturbance of equilibrium produced by the above causes, the *seiches* are simply the subsequent vibrations of the whole lake about its equilibrium position. This theory he tested by making observations on several lakes with the use of self-recording gauges and proving the following laws:† (1) At any one place the *seiche* period is somewhat variable, but the mean is fairly constant; (2) While the water is falling at one side of a lake it is rising at the opposite side, and *vice versa*; (3) The period in the different lakes varies as the width, and inversely as the square root of the depth. He even predicted a depressed area in

* *Annales de Chimie et Physique*, ix, 1876.

† *Phil. Mag.*, Series V, vol. ii, 1876.

Lake Wallenstadt because of the *seiche* period not agreeing with (3) and discovered it by soundings. He also found both vibrations parallel to the width and vibrations parallel to the length of certain lakes.

The laws of such vibrations as the above have been established both theoretically and experimentally. If t be the time of vibration, parallel to a side of length l , of the water contained in a shallow rectangular vessel with vertical sides and horizontal bottom, and of depth h , then

$$t = 2 \left(\frac{\pi l}{mg} \coth \frac{m\pi h}{l} \right)^{\frac{1}{2}}$$

in which for fundamental or binodal vibrations $m=1$, for trinodal $m=2$, etc. When h is very small compared with l

$$t = \frac{2l}{m\sqrt{gh}} \text{ nearly.}$$

The above is considered to be a first approximation to a solution when the depth is variable, if h be understood to mean the average depth. Thus Forel's law (3) is completely in accord with theory.

Forel's proof of his theory was accepted as sufficient by Sir G. B. Airy.* The latter in reducing tidal observations from Malta found in them oscillations of a sensibly uniform period of 21 minutes and of magnitude considerably greater, at times, than the tides themselves. They were usually simple harmonic curves but sometimes *notched* at the top by smaller oscillations. Airy ascribed the Malta undulations to vibrations between a certain sand bank on the African coast and another on the Sicilian coast, and states that a rough calculation seemed to verify this. He also refers to similar records at Swansea.

No one else seems to have devoted much time or attention to studying the vibrations of large bodies of water, although a number of random observations have been made. As the subject seems worthy of attention, I give in a foot-note references to the literature of the subject.† In two respects, at least, the

* Phil. Trans. Roy. Soc., 1878.

† Forel, Airy, Dawson—see references already given.

J. H. McFarland (Nature, Mar. 12, 1895), observed *seiches* on Lake Derravaragh.

Charles Rhodes (Science, May 7, 1886), observed fluctuations at Oswego.

J. LeConte (Overland Monthly, 1883), predicted *seiche* period of Lake Tahoe.

Ledyard (Science, Feb. 7, 1890), observed 10-min. pulsations on Lake Cazenovia.

T. D. Graham (Proc. A. A. A. S., 1883), found tides on Lake Michigan but apparently no *seiches*.

Smithsonian Contributions, vol. xii, mention fluctuations on Lake Superior.

M. P. Du Boys (Comptes Rendus, tome xii, No. 21, 25 Mai, 1891).

Century Dictionary (under "*Seiche*"), refers to similar phenomena on the Baltic.

question seems of importance. (1) A determination of *seiche* period will afford an estimate of mean depth and may assist in checking soundings or even discovering areas of depression, such as Forel found in Lake Wallenstadt. (2) Many, at least, of the often noted but (except for Airy's apparently half-forgotten remark) unexplained "secondary undulations" may be merely *seiches* on salt water. The Bay of Fundy offers special advantages for testing this latter point. The care with which it has been sounded and its great tidal range (at St. John 27 feet at springs), should enable us to apply the three following tests: (1) Calculation should give a vibration period agreeing with the period of the "secondary undulations." (2) This period should be greater at low water, owing to the less depth, than at high water. (3) Opposite sides should move in the same or in opposite phase, according as the vibrations have an odd or an even number of nodes.

Let us first make a calculation for the small bay at Indian-town. From the chart of the river we have, for the width at the point of observation (Marble Point), 2,030 feet. This is also about the width for some distance farther up; immediately below, there is a great increase of width, due to a small cove; but as this cove is very shallow while the main basin is unusually deep, it will have no practical effect on the period of vibration. For the mean depth we have the following soundings (in feet), along the shortest line to the opposite bank, 56, 60, 78, 94, 108, 136, 124, 108 the mean of which is 95 feet. Immediately above and below, the depth is slightly greater, in one place, 196 feet. Remembering that the formula for calculation is only approximate, it will suffice if 100 feet be taken as mean depth. Substituting these figures in the first formula, we get for binodal vibrations a period of 72 seconds and for trinodal 37.5 seconds. This seems to point to the 35-second pulsations being trinodal vibrations. (The less accurate formula would have given a period of 35.8 seconds.) As the tidal range is less than two feet this period should not appreciably vary with the tide.

In the case of the "secondary undulations" in the harbor, we have, for the width of the bay along a line from the tide-gauge station at St. John to the nearest point of the Nova Scotia coast, 39.8 miles. There will be a slight error in taking this as the width of the basin, for, while the Nova Scotia coast is nearly straight and unbroken, St. John harbor forms an indentation on the New Brunswick side. As it is uncertain how much deduction should be made on this account (perhaps about two miles), the above figure will be used as it stands. From the chart we obtain the following (low water) soundings between St. John and the opposite coast: 10, 4, 5, 9, 21, 27,

33, 36, 38, 40, 42, 43, 47, 50, 50, 45, 43, 43, 46, 46, 40, 40, 39, 35, 27 fathoms, or a mean of 34.4 fathoms. Reducing to feet and substituting in either formula we have, for binodal vibrations, a period of 87 minutes, and for trinodal, 43.5 minutes. Now the period of the "secondary undulations" is, according to Mr. Dawson, "about 40 minutes." If an allowance of two miles had been made for the depth of the indentation referred to, and the corresponding soundings omitted, the calculated period for low water would become very nearly 40 minutes. On the other hand, it will be shown below that the observed period is possibly more nearly 43 minutes. In any case the evidence that these "secondary undulations" are also trinodal vibrations seems strong if not conclusive.

Next, as to the difference of vibration period at high water and at low water, it can be calculated that for a difference of 20 feet (about the mean range at St. John), the period should be two minutes less at high water than at low water. To enable me to test this point, Mr. Dawson has kindly supplied me traces of all the records of secondary undulations at present available and sufficiently well defined to give a reliable estimate. Unfortunately these can hardly be considered sufficient to establish the point, but such as they are, they are given. For low water the periods are 41, 45, 45, 39, 40, 32, 49, 53, giving a mean of 43 minutes; for high water, 43, 35, 43, 40, 46, giving a mean of 41.4. The former mean is 1.6 minutes greater than the latter, but the data are insufficient to justify much reliance being placed on this argument. The apparent variability of period shown by the above figures may be readily accounted for. In the first place every slight barometric change or change of wind produces a corresponding change of level and this is superposed on the "secondary undulations," producing an apparent variation of period. Secondly, the irregularities of coast line may have an effect. Thirdly, and perhaps most important of all, different modes of vibration may co-exist in different proportions at different times, so that the resultant period may at some time be more nearly that of binodal vibration, at others of trinodal, etc. Thus, a higher mode of vibration co-existing with the prevailing mode might account for the "notches" observed by Airy in the secondary undulations at Malta.

As regards the relation of the phases of vibration at opposite sides of the bay, no information is at hand. St. John is the only point on the bay at which there is a recording tide-gauge. Another at Digby, on the opposite side of the bay, would probably settle the matter in a month; but the cost of erecting a gauge suitable for such work is considerable. I hope on some occasion, when the water is calm and "secondary

undulations" exist at St. John, to be able to obtain a record near Digby by use of a small portable gauge.

If the preceding explanation of "secondary undulations" prove correct, it will be a question of interest why the undulations here treated of are trinodal, while those of the lakes observed by Forel are chiefly binodal. No light seems thrown on the question by present hydrodynamical theory. The explanation may perhaps be found in the fact that a lake is a completely enclosed body of water, while a bay is open at one side or two opposite sides. If, as seems possible, the smaller undulations across the basin at Indiantown are produced in some way by the larger ones of the harbor entering at one end of the small bay, the result would more probably be a simultaneous elevation at opposite sides than an elevation at one and a depression at the other. This suggests that the undulations across the Bay of Fundy may in some similar way be produced by the uprush or downrush of the tides. They do not seem (as in the case of the Swiss *seiches*) to be connected with abnormal conditions of barometer; at least no such connection appears to exist in the cases whose periods are given above.

Addendum. In a recent article (Phil. Mag., Jan. 1897), Mr. C. Davison has pointed out that a considerable error may be made in extending the formula for a basin of uniform depth to one of irregular depth. In fact the true period of the trinodal vibrations considered in the preceding is

$$\int_0^l \frac{dx}{\sqrt{gy}}, \quad (y = \text{depth}).$$

Changing this to the form

$$\frac{l}{\sqrt{g}} \times \left(\text{mean } \frac{1}{\sqrt{y}} \right),$$

I have calculated its value for trinodal vibrations across the Bay of Fundy, at St. John (allowing two miles as before) and find that it gives a period of 42.2 minutes.

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