and the equations thus become

$$2x_4 - \frac{1}{E}\frac{dE}{dq}x_1 - \frac{1}{G}\frac{dG}{dq}x_2 = 0,$$
$$\&c. \qquad \&c. \qquad \&c.,$$

which, in fact, agree with the equations (10 bis) in Lamé's "Leçons sur les Coordonnées curvilignes," Paris (1859), p. 89. The surface will be divisible into squares if only E : G is the quotient of a function of p by a function of q, or say if

$$E = \Theta P, \quad G = \Theta Q,$$

where Θ is any function of (p, q), but P and Q are functions of p and q respectively; we then have

$$\frac{1}{E}\frac{dE}{dq} = \frac{1}{\Theta}\frac{d\Theta}{dq}, \quad \frac{1}{G}\frac{dG}{dp} = \frac{1}{\Theta}\frac{d\Theta}{dp},$$

and the equations for x, y, z are

$$2x_4 - \frac{1}{\Theta}\frac{d\Theta}{dq}x_1 - \frac{1}{\Theta}\frac{d\Theta}{dp}x_2 = 0,$$
$$\&c. \qquad \&c. \qquad \&c.,$$

viz., x, y, z being functions of p, q such that $x_1x_2 + y_1y_2 + z_1z_2 = 0$, and which besides satisfy these equations, or say which each of them satisfy the equation

$$2u_4 - \frac{1}{\Theta}\frac{d\Theta}{dq}u_1 - \frac{1}{\Theta}\frac{d\Theta}{dp}u_2 = 0,$$

then the values of x, y, z in terms of (p, q) determine a surface which has the property in question.

On Professor Cremona's Transformation between Two Planes, and Tables relating thereto. By Samuel Roberts, M.A.

[*Read June* 13*th*, 1872.]

In the transformation theory of Prof. Cremona, we have a point-to-point correspondence between two planes. To a right line in one plane corresponds a unicursal curve of the order n in the other plane, and *vice versâ.* To the right lines in one plane (say P) corresponds a reseau of unicursal curves of the order n in the other plane (say P′). Any two curves of the reseau intersect in a certain number of fundamental points giving the equivalent of n^2-1 intersections, and these points are the same for all the curves of the reseau. The two curves in question meet besides in one point, which corresponds to the intersection of the two right lines, to which those curves themselves correspond. All this holds good when P and P′ are interchanged.

The fundamental or principal points in P may be divided into groups

$$a_p, \ a_q, \ a_r, \ \&c.,$$

where a_p denotes the number of such points which are multiple in the degree p, and so on. In like manner, the principal points in P′ may be divided into groups $a'_{p'}, \ a'_{q'}, \ a'_{r'}, \ \&c.$

The systems satisfy respectively the equations

$$\begin{aligned} \Sigma p \ a_p &= 3n-3, \\ \Sigma p^2 a_p &= n^2-1, \\ \Sigma p' a'_{p'} &= 3n-3, \\ \Sigma p'^2 a'_{p'} &= n^2-1. \end{aligned}$$

Some solutions of these equations are, however, usually inadmissible; and the problem of determining for a given integer n the systems of principal points has not been completely solved. Prof. Cremona has determined them for $n = 1, 2, 3 \ldots 10$, and has also given some of the forms for general values of n.

In many instances, two conjugate systems are the same in form; that is to say, we may write

$$\begin{aligned} a_p &= a'_{p'}, \quad a_q = a'_{q'}, \quad a_r = a'_{r'}, \quad \&c., \\ p &= p', \quad q = q', \quad r = r', \quad \&c. \end{aligned}$$

When this is not the case, the systems still possess this property, that to each group a_p in one system corresponds an equal group $a'_{p'}$ in the other system, and *vice versâ;* that is to say, we may write

$$a_p = a'_{p'}, \quad a_q = a'_{q'}, \quad a_r = a'_{r'}, \quad \&c.,$$

but *not* necessarily $p = p', \quad q = q', \quad r = r', \quad \&c.$

This relation, following from the symmetry of the two systems, was inferred by Prof. Cremona, and rigorously established by Prof. Clebsch (Math. Annalen, Band IV., p. 490).

But when two conjugate systems are determined (and it is to be observed that for $n > 3$ there are several conjugate systems), we have yet to determine the singular correspondents of the principal points themselves.

To the principal points in P corresponds the Jacobian of the reseau which corresponds to right lines in P, and this Jacobian is made up of certain curves which are of the order p, q, r, &c., and are unicursal; that is to say, it is made up of a_p curves C_p, a_q curves C_q, a_r curves C_r, &c., and to each constituent point of the group a_p corresponds a curve C_p, to each constituent point of the group a_q corresponds a curve C_q, and so forth.

We have next to determine the order of multiplicity of the principal points in P′ through which these singular correspondents respectively pass.

This problem being solved, we are able to construct tables which in fact express the character of the Jacobians.

I do not wish to occupy space with more details than are necessary to make the remarks I have to offer intelligible. The general theory will be found in the original memoirs by Prof. Cremona, and in a paper by Prof. Cayley on the rational transformation between two spaces. (Proc. Lond. Math. Soc., Vol. III., p. 127 *et seq.*) I am concerned at present with the form of the tables which for $n = 2, 3 \ldots 6$, and some general cases, are given in the last-named memoir.

In the first place, let us suppose that a table or a pair of tables, where the conjugates are dissimilar, is given, so that we know the corresponding conjugate systems and the character of the Jacobians.

If we take any right line in P′, its correspondent in P is a unicursal curve of the order n, passing p times through each point of the group a_p, q times through each point of the group a_q, and so on. If, however, the line passes through a point of the group $a'_{p'}$, the reduced correspondent is of the order $n-p'$, and the reduction takes place in consequence of the point of the group $a'_{p'}$ having for its singular correspondent a unicursal curve of the order p'. The gross correspondent of the right line still remains of the order n, and passes p times through each point of the group a_p, &c., but breaks up into the nett correspondent $C_{n-p'}$ and the curve $C_{p'}$, which, as corresponding to a point of the group $a'_{p'}$, is invariable for the pencil of right lines passing through that point. What is to be observed is that, if we know the relation of the curve $C_{p'}$ to the principal points in P, we can determine the relation of the curve $C_{n-p'}$ to the same system.

And if the primitive line is a curve $C_{k'}$, and passes through the principal points in P′ in a stated manner, we can in the same way deduce the relation of the correspondent C_k in P to the principal points in P, when we know the nature of the correspondent if $C_{k'}$ does not pass through any of the principal points, and know also the nature of the correspondents of the principal points in P′.

To fix the ideas, suppose that

$$\begin{array}{ccc} a_p, & \beta_q, & \gamma_r \\ a_s, & \beta_t, & \gamma_u, \end{array}$$

are conjugate systems, and that C_s, corresponding to a point of the group a_s, passes

a times through A points of the group a_p,
b times through B points of the group a_q,
c times through C points of the group a_r.

Then to a right line through a point of the group a_s will correspond a curve C_{n-s} which passes

p times through $\alpha_p - A$ points of the group α_p,
$p-a$ times through A points of the group α_p,
q times through $\beta_q - B$ points of the group β_q,
$q-b$ times through B points of the group β_q,
r times through $\gamma_r - C$ points of the group γ_r,
$r-c$ times through C points of the group γ_r.

I will now make use of these considerations in one or two cases of transformation.

For $n=2$, we have the table (Proc., vol. iii. p. 148)

	$\alpha_1 = 3$
$\alpha'_1 = 3$	2

Let the curve $C_{k'}$ pass a'_1 times through a point of the group α'_1, b'_1 times through another point of the same group, and c'_1 times through the remaining point.

Let the correspondent C_k in P pass a_1 times through a point of the group α_1, b_1 times through another point of the same group, and c_1 times through the remaining point.

If the curve $C_{k'}$ did not pass through any principal point, we should have
$$a_1 = b_1 = c_1 = k'.$$
We have then
$$k = 2k' - a'_1 - b'_1 - c'_1,^*$$
$$k' = 2k - a_1 - b_1 - c_1;$$
$$a_1 = k' - b'_1 - c'_1,$$
$$b_1 = k' - a'_1 - c'_1,$$
$$c_1 = k' - a'_1 - b'_1.$$
And since the conjugate systems are similar
$$a'_1 = k - b_1 - c_1,$$
$$b'_1 = k - a_1 - c_1,$$
$$c'_1 = k - a_1 - b_1.$$
These are the known formulæ for a quadric transformation.

For the cubic transformation we have

	$\alpha_1 = 4$	$\alpha_2 = 1$
$\alpha'_1 = 4$	1	1
$\alpha'_2 = 1$	4	1

* The general formula being
$$k = k'n - \Sigma(a'_1 + 2a'_2 + \dots),$$
$$k' = kn - \Sigma(a_1 + 2a_2 + \dots).$$

Let $C_{k'}$ in P′ pass a'_1 times, b'_1 times, c'_1 times, d'_1 times through the respective points of the group a'_1, and a'_2 times through the sole point of the group a'_2. In like manner let k, a_1, b_1, c_1, d_1, a_2 refer to the correspondent C_k in P_1. Then we have

$$k = 3k' - \Sigma a'_1 - 2a'_2,$$
$$k' = 3k - \Sigma a_1 - 2a_2,$$

with

$$\begin{aligned} a_1 &= k' - a'_1 - a'_2, \\ b_1 &= k' - b'_1 - a'_2, \quad a_2 = 2k' - \Sigma a'_1 - a'_2, \\ c_1 &= k' - c'_1 - a'_2, \\ d_1 &= k' - d'_1 - a'_2; \end{aligned}$$

and conversely

$$\begin{aligned} a'_1 &= k - a_1 - a_2, \\ b'_1 &= k - b_1 - a_2, \quad a'_2 = 2k - \Sigma a_1 - a_2, \\ c'_1 &= k - c_1 - a_2, \\ d'_1 &= k - d_1 - a_2. \end{aligned}$$

These are the formulæ for the cubic transformation. In like manner we can write down the formulæ of transformation in any case where the Jacobian tables are completely given.

Thus, for an example of dissimilar conjugates take the last pair of tables for $n=6$.

Using the same notation as before (Proc., vol. iii. p. 149), we have for $C_{k'}$ and its correspondent

$$k = 6k' - \Sigma a'_1 - 2\Sigma a'_2 - 4a'_4,$$
$$k' = 6k - \Sigma a_1 - 2a_2 - 3\Sigma a_3,$$

$$\begin{aligned} a_1 &= k' - a'_2 - a'_4, & a_2 &= 2k' - \Sigma a'_2 - 2a'_4, \\ b_1 &= k' - b'_2 - a'_4, & a_3 &= 3k' - b'_1 - c'_1 - \Sigma a'_2 - 2a'_4, \\ c_1 &= k' - c'_2 - a'_4, & b_3 &= 3k' - a'_1 - c'_1 - \Sigma a'_2 - 2a'_4, \\ d_1 &= k' - d'_2 - a'_4, & c_3 &= 3k' - a'_1 - b'_1 - \Sigma a'_2 - 2a'_4; \end{aligned}$$

and conversely

$$\begin{aligned} a'_1 &= k - b_3 - c_3, & a'_2 &= 2k - a_1 - a_2 - \Sigma a_3, \\ b'_1 &= k - a_3 - c_3, & b'_2 &= 2k - b_1 - a_2 - \Sigma a_3, \quad a'_4 = 4k - \Sigma a_1 - a_2 - 2\Sigma a_3, \\ c'_1 &= k - a_3 - b_3, & c'_2 &= 2k - c - a_2 - \Sigma a_3, \\ & & d'_2 &= 2k - d_1 - a_2 - \Sigma a_3. \end{aligned}$$

For a more general case I take the second pair of dissimilar conjugates for $n=3p$, and obtain in a similar manner

$$k = 3pk' - \underset{4}{\Sigma} a'_1 - 2a'_2 - 3 \underset{2p-2}{\Sigma} a'_3 - (3p-3)\, a'_{2p-2},$$
$$k' = 3pk - \underset{2p-2}{\Sigma} a_1 - (p-1)\, a_{p-1} - p \underset{4}{\Sigma} a_p - 2pa_{2p},$$

$$a_1 = k' - a'_3 - a'_{3p-3}, \qquad a_p = pk' - a'_1 - a'_2 - \sum_{2p-2} a'_3 - (p-1)\, a'_{3p-3},$$

$$b_1 = k' - b'_3 - a'_{3p-3}, \qquad b_p = pk' - b'_1 - a'_2 - \sum_{2p-2} a'_3 - (p-1)\, a'_{3p-3},$$

....

$2p-2$ equations, 4 equations,

$$a_{p-1} = (p-1)\, k' - \sum_{2p-2} a'_3 - (p-2)\, a'_{3p-3}, \qquad a_{2p} = 2pk' - \sum_4 a'_1 - a'_2 - 2\sum_{2p-2} a'_3 - (2p-2)\, a_{3p-3};$$

where the suffixes under Σ denote the numbers of the letter, which the symbols cover.

And we also have

$$a'_1 = k - a_p - a_{2p}, \qquad a'_3 = 3k - a_1 - a_{p-1} - \sum_4 a_p - 2a_{2p},$$

$$b'_1 = k - b_p - a_{2p}, \qquad b'_3 = 3k - b_1 - a_{p-1} - \sum_4 a_p - 2a_{2p},$$

....

4 equations, $2p-2$ equations,

$$a'_2 = 2k - \sum_4 a_p - a_{2p}, \qquad a'_{3p-3} = (3p-3)\, k' - \sum_{2p-2} a_1 - (p-2)\, a_{p-1} - (p-1) \sum_4 a_p - (2p-2)\, a_{2p}.$$

These results might be tabulated in the following manner. The last system gives

a'_1	$k - a_p - a_{2p}$
a'_2	$2k - \sum_4 a_p - a_{2p}$
a'_3	$3k - a_1 - a_{p-1} - \sum_4 a_p - 2a_{2p}$
a_{3p-3}	$(3p-3)\, k - (p-2)\, a_{p-1} - (p-1) \sum_4 a_p - (2p-2)\, a_{2p}$

But, in fact, it is unnecessary to tabulate what is so directly derivable from the Jacobian tables.

It will be observed that if in each of the conjugate systems of principal points there are M groups containing m constituents in all, we have m equations linear in a'_1, &c., to express the values of m quantities, a_1, &c., and *vice versâ*. The one set of equations is derivable from the other, and we need not, in fact, make use of two conjugate tables, since either of them will give the relations sought by solving a linear system.

With an ulterior purpose, I now take an extreme case $k' = 0$, so that $C_{k'}$ represents a point or points.

We shall have in general $k=0$, since there is a point to point correspondence between P and P′.

But let us suppose that $C_{k'}$ is in fact a constituent of the group $a'_{p'}$. Looking at the manner in which the expressions for k, k', a_p &c., a'_p &c., are obtained, we see that, retaining the general formulæ, we shall get, with negative signs, the values of a_p &c. for the singular correspondent of a point of the group $a'_{p'}$, when we put $a'_{p'}=1$, and erase the rest of the accented letters.

For in the case of a general curve $C_{k'}$, we have been subtracting for the effect of the singular correspondents of principal points. We now have $k'=0$, while the singular correspondent $C_{p'}$, which in the general case is deemed extraneous, is no longer so. Thus, in the cubic transformation, putting $a'_1=1$, $b_1=\&c.=k'=0$, we get

$$-a_1=1, \quad -a_2=1;$$

indicating that the right line corresponding to a point of the group a'_1 passes once through a point of the group a_1, and once through a point of the group a_2. Since a knowledge of this was assumed in obtaining the formulæ, we do not gain anything directly by this consideration, except that it becomes plain how we can obtain from a given table its conjugate by the solution of a linear system. In addition to this, however, the results show a remarkable symmetry, which may be regarded from a more general point of view.

I observe that the individual constituents of any group a_p have no preeminence among themselves. From this simple consideration, it follows that if a curve $C_{k'}$ in P′ passes through each point of each group of principal points in P′ a number of times, which does not vary for the same group, then the correspondent C_k in P also passes through each point of each group of principal points in P a number of times which does not vary for the same group. And further, since the number "zero" is included, we may express the conclusion thus,—

If a curve $C_{k'}$ in P′ passes through each point of a certain number of groups of principal points in P′, a number of times which does not vary for the same group, and not otherwise, then the correspondent C_k in P passes through each point of the groups of principal points in P, or some of them, a number of times which does not vary for the same group, and not otherwise.

In what precedes, we have supposed that the different groups of principal points are of different orders of multiplicity. I observe further, however, that this supposition is not intrinsically necessary. For if we divide a group (say $a'_{p'}$) into two or more sub-groups, and also divide the equal corresponding group (say a_p) into two or more corresponding equal groups, the conclusion remains unaltered. To illustrate this, consider the case

$$n=4p+2,$$

$$\begin{aligned} a'_1 &= 3, & a_1 &= 2p-1, \\ a'_2 &= 3, & a_p &= 3, \\ a'_4 &= 2p-1, & a_{p+1} &= 1, \\ a'_{4p-2} &= 1, & a_{2p+1} &= 3. \end{aligned}$$

For $p=1$, these become

$$\begin{vmatrix} a'_1=3, & a_1=1 \\ a'_2=3, & a_1=3 \\ a'_4=1, & a_2=1 \\ a'_2=1, & a_3=3 \end{vmatrix} = \begin{vmatrix} a'_1=3, & a_1=4 \\ a'_2=4, & a_2=1 \\ a'_4=1, & a_3=3 \end{vmatrix}$$

And it is evidently immaterial whether we treat this as a case of four groups or a case of three groups.

Hence we may conclude that what has been stated above with reference to a curve passing through the constituents of groups of different orders of multiplicity, is also true when the groups, or some of them, are of the same degree of multiplicity.

It follows, for instance, that if $C_{k'}$ passes an equal number of times through a certain number K of the constituents of $a'_{p'}$, and we will suppose a different but equal number of times through the remaining $a'_{p'}-K$ constituents, and also through each of the points of the other groups an equal number of times not varying for the same group, then the correspondent C_k in P will pass through K of the constituents of the corresponding group a_p an equal number of times, and through the remainder of the group a_p an equal but generally different number of times, and through the other groups a number of times not varying for the same group. For we may divide $a_p=a'_{p'}$ into a group K and a group $a'_{p'}-K$, so that we still have equal corresponding groups. Applying this reasoning still further, we arrive at the following conclusion:—

If $C_{k'}$ in P′ passes an equal number of times through K *constituents of any group of principal points, then the correspondent* C_k *in* P *passes an equal number of times through* K *constituents of the corresponding equal group.*

And this result is borne out by the formulæ which have been given.

Falling back, then, on groups of different multiplicity, and the formulæ of transformation for that case, we gather that the expressions for a_p, b_p, &c. must contain $a'_{r'}$, $b'_{r'}$, &c. either altogether or not at all; and that they must contain $a'_{p'}$, $b'_{p'}$, &c., relating to the corresponding equal group in such combinations of l letters that

$$\frac{a_p \,.\, a_p-1 \ldots a_p-l+1}{1\,.\,2\,.\,3 \ldots l}=a_p;$$

that is to say, $l=1$ or $l=a_p-1$.

Let us now apply this to the case when $k'=0$, and $C_{k'}$ represents a point of the group $a'_{p'}$. The correspondent will then represent the singular correspondent of a point of the group $a'_{p'}$, except, as has been explained, that the signs will be negative, and this discrepancy of sign can of course be readily removed.

From what has been said, the following conclusion results :—

The numbers (exclusive of indices, according to Prof. Cayley's notation) which enter a table are the numbers of the groups of the principal system or zero, except as to the squares of the table which relate to corresponding equal groups. The numbers in these squares will be either unity, or the number of the corresponding group less 1, *or both of these numbers.*

A difficulty arises when each system contains two or more equal groups. In such a case, I do not know how to determine, *à priori*, the corresponding equal group. It is not difficult, however, practically to determine this. Meanwhile we know that in such cases the numbers in the corresponding square are either zero, the number of the group, unity, or the number of the group less 1. In the case of corresponding equal groups, *both* unity and the number of the group less 1 may occur, with different indices.

Although the indices remain undetermined, a step has been gained, and for values of n of moderate magnitude the tables can be completed without difficulty. The solution is unique when the conjugate systems are given. Otherwise we should have more than one possible correspondent to a principal point.

I have employed this method to complete the tables up to $n=10$, and also for the general forms given by Prof. Cremona.

Since the corresponding equal groups may be so arranged that a diagonal of the table belongs to them (and may be so arranged in various ways), I have adopted this plan so that the numbers on either side of the diagonal are the numbers of the groups. The letters and numbers at the top of the several tables denote the principal groups in one plane; those at the left-hand margin indicate the fundamental groups in the other plane, the correspondents of which pass through the first-mentioned system in the manner indicated by the numbers in the tables. These numbers are the numbers of the points through which the correspondents pass; their indices give the multiplicity.

It is unnecessary to repeat the tables given by Prof. Cayley.

Tables $n=7$.

	α_1 12	α_6 1
α_1 12	1	1
α_6 1	12	1^5

	α_1 2	α_2 3	α_3 2	α_4 1 *
α_3 2	1	3	2	1^2
α_2 3		2	2	1
α_1 2			1	1
α_4 1	2	3	2^2	1^2

	α_2 3	α_3 4
α_2 3	1	4
α_3 4	3	$1^2,3$

	α_1 5	α_3 3	α_4 1
α_2 5	1	3	1
α_1 3		1	1
α_5 1	5	3^2	1^3

	α_1 3	α_2 5	α_5 1
α_3 3	1	5	1^2
α_1 5		1	1
α_4 1	3	5	1^3

Tables $n=8$.

	α_1 14	α_7 1
α_1 14	1	1
α_7 1	14	1^6

	α_1 3	α_2 2	α_3 3	α_5 1
α_3 3	1	2	3	1^2
α_2 2		1	3	1
α_1 3			1	1
α_5 1	3	2	3^2	1^3

	α_1 1	α_2 3	α_3 2	α_4 2
α_1 1				2
α_2 3		1	2	2
α_4 2	1	3	$1^2,1$	2^2
α_3 2		3	2	$1^2,1$

	α_3 7
α_3 7	$6,1^3$

	α_1 3	α_2 3	α_4 3
α_4 3	2	3	3^2
α_2 3		2	3
α_1 3			2

	α_1 3	α_2 6	α_6 1
α_4 3	2	6	1^3
α_1 6		1	1
α_3 1		6	1^2

	α_1 6	α_3 1	α_4 3
α_2 6	1	1	3
α_6 1	6	1^2	3^3
α_1 3			2

* It will be observed that this table is sibi-reciprocal only in a restricted sense, since the suffixes of the corresponding groups are not the same. I have noticed this in other cases where there are several equal groups.

TABLES $n=8$, *continued.*

	a_2 5	a_3 2	a_5 1
a_3 5	4	2	1^2
a_1 2		1	1
a_4 1	5	2^2	1^2

	a_1 2	a_3 5	a_4 1
a_3 2	1	5	1^2
a_2 5		4	1
a_5 1	2	5^2	1^2

TABLES $n=9$.

	a_1 16	a_8 1
a_1 16	1	1
a_8 1	16	1^7

	a_1 4	a_2 1	a_3 4	a_6 1
a_3 4	1	1	4	1^2
a_2 1			4	1
a_1 4			1	1
a_6 1	4	4	4^2	1^4

	a_1 2	a_2 3	a_3 1	a_4 2	a_5 1
a_4 2	1	3	1	2^2	1^2
a_2 3		1	1	2	1
a_3 1		3	1	2	1^2
a_1 2				1	1
a_5 1	2	3	1^2	2^2	1^3

	a_2 4	a_4 4
a_2 4	1	4
a_4 4	4	$1,3^2$

	a_1 3	a_2 7	a_7 1
a_4 3	1	7	1^3
a_1 7		1	1
a_5 1	3	7	1^4

	a_1 7	a_4 3	a_5 1
a_2 7	1	3	1
a_1 3		1	1
a_7 1	7	3^3	1^4

Tables $n=9$ *continued.*

	α_1 1	α_2 4	α_3 3	α_6 1
α_4 1	1	4	3	1^3
α_3 4		3	3	1^2
α_1 3			1	1
α_5 1	1	4	3^2	1^3

	α_1 3	α_3 4	α_4 1	α_5 1
α_3 3	1	4	1	1^2
α_2 4		3	1	1
α_1 1			1	1
α_6 1	3	4^2	1^3	1^3

	α_2 3	α_3 3	α_4 1	α_5 1
α_3 3	2	3	1	1^2
α_4 3	3	$1^2, 2$	1^2	1^3
α_1 1			1	1
α_2 1		3	1	1

	α_1 1	α_2 1	α_3 3	α_4 3
α_4 1	1	1	3	3^2
α_5 1	1	1	3^2	3^3
α_2 3			2	3
α_3 3		1	3	$1^2, 2$

Tables $n=10$.

	α_1 18	α_9 1
α_1 18	1	1
α_9 1	18	1^8

	α_1 5	α_3 5	α_7 1
α_3 5	1	5	1^2
α_1 5		1	1
α_7 1	5	5^3	1^5

	α_1 1	α_2 4	α_4 2	α_5 2
α_1 1				2
α_2 4		1	2	2
α_4 2		4	$1, 1^2$	2^2
α_5 2	1	4	2^2	$1^2, 1^3$

	α_2 2	α_3 2	α_4 3	α_5 1
α_3 2	1	2	3	1^2
α_2 2		1	3	1
α_4 3	2	2	$2^2, 1$	1^2
α_5 1	2	2^3	3^2	1^3

	α_3 7	α_6 1
α_3 7	6	1^2
α_6 1	7^2	1^3

TABLES $n=10$ (*continued*).

	α_1 3	α_2 8	α_8 1
α_5 3	2	8	1^4
α_1 8		1	1
α_4 1		8	1^3

	α_1 8	α_4 1	α_5 3
α_2 8	1	1	3
α_8 1	8	1^3	3^4
α_1 3			2

	α_1 2	α_2 3	α_3 4	α_7 1
α_4 2	1	3	4	1^3
α_2 3		2	4	1^2
α_1 4			1	1
α_6 1	2	3	4^2	1^4

	α_1 4	α_3 3	α_4 2	α_6 1
α_3 4	1	3	2	1^2
α_2 3		2	2	1
α_1 2			1	1
α_7 1	4	3^2	2^3	1^4

	α_1 1	α_2 3	α_3 2	α_4 2	α_6 1
α_2 1			2	2	1
α_3 3		2	2	2	1^2
α_5 2	1	3	$1^2, 1$	2^2	1^3
α_1 2				1	1
α_4 1		3	2	2^2	1^2

	α_1 2	α_2 1	α_3 3	α_4 1	α_5 2
α_4 2	1	1	3	1^2	2^2
α_1 1					2
α_2 3			2	1	1
α_6 1	2	1	3^2	1^2	2^3
α_3 3		1	3	1	$1, 1^2$

	α_1 3	α_2 3	α_4 3	α_6 1
α_5 3	2	3	3^2	1^3
α_2 3		1	3	1
α_1 3			1	1
α_3 1		3	3	1^2

	α_1 3	α_2 3	α_3 1	α_5 3
α_4 3	1	3	1	3^2
α_2 3		1	1	3
α_6 1	3	3	1^2	3^3
α_1 3				2

Tables $n=10$ (*continued*).

	a_1 3	a_4 6
a_5 3	2	6^2
a_2 6		5

	a_2 6	a_5 3
a_4 6	5	3^2
a_1 3		2

	a_2 1	a_3 5	a_5 2
a_1 1			2
a_4 5	1	$1^2, 4$	2^2
a_3 2		5	$1, 1^2$

	a_1 1	a_3 2	a_4 5
a_2 1			5
a_5 2	1	$1, 1^2$	5^2
a_3 5		2	$1^2, 4$

Tables $n=3p+1$.

	a_1 $2p-2$	a_p 3	a_{p+1} 2	a_{2p} 1
a_3 $2p-2$	1	3	2	1^2
a_2 3		2	2	1
a_1 2			1	1
a_{3p-2} 1	$2p-2$	3^{p-1}	2^p	1^{2p-2}

	a_1 2	a_2 3	a_3 $2p-2$	a_{3p-2} 1
a_{p+1} 2	1	3	$2p-2$	1^p
a_p 3		2	$2p-2$	1^{p-1}
a_1 $2p-2$			1	1
a_{2p} 1	2	3	$(2p-2)^2$	1^{2p-2}

	a_1 $2p-1$	a_p 5	a_{2p+1} 1
a_3 $2p-1$	1	5	1^2
a_1 5		1	1
a_{3p-2} 1	$2p-1$	5^{p-1}	1^{2p-1}

	a_1 5	a_3 $2p-1$	a_{3p-2} 1
a_p 5	1	$2p-1$	1^{p-1}
a_1 $2p-1$		1	1
a_{2p+1} 1	5	$(2p-1)^2$	1^{2p-1}

Tables $n=3p+2$.

	a_1 3	a_2 2	a_3 $2p-1$	a_{3p-1} 1
a_{p+1} 3	1	2	$2p-1$	1^p
a_p 2		1	$2p-1$	1^{p-1}
a_1 $2p-1$			1	1
a_{2p+1} 1	3	2	$(2p-1)^2$	1^{2p-1}

	a_1 $2p-1$	a_p 2	a_{p+1} 3	a_{2p+1} 1
a_3 $2p-1$	1	2	3	1^2
a_2 2		1	3	1
a_1 3			1	1
a_{3p-1} 1	$2p-1$	2^{p-1}	3^p	1^{2p-1}

TABLES $n = 3p + 2$ (*continued*).

		a_2 5	a_3 $2p-2$	a_{3p-1} 1
a_{p+1}	5	4	$2p-2$	1^p
a_1	$2p-2$		1	1
a_{2p}	1	5	$(2p-2)^2$	1^{2p-2}

		a_1 $2p-2$	a_{p+1} 5	a_{2p} 1
a_3	$2p-2$	1	5	1^2
a_2	5		4	1
a_{3p-1}	1	$2p-2$	5^p	1^{2p-2}

TADLES $n = 4p$.

		a_1 1	a_2 3	a_3 2	a_4 $2p-3$	a_{4p-4} 1
a_{p+1}	1		3	2	$2p-3$	1^p
a_p	3		1	2	$2p-3$	1^{p-1}
a_{2p}	2	1	3	$1, 1^2$	$(2p-3)^2$	1^{2p-2}
a_1	$2p-3$				1	1
a_{2p-1}	1		3	2	$(2p-3)^2$	1^{2p-3}

		a_1 $2p-3$	a_p 3	a_{p+1} 1	a_{2p-1} 1	a_{2p} 2
a_4	$2p-3$	1	3	1	1^2	2^2
a_2	3		1	1	1	2
a_1	1					2
a_{4p-4}	1	$2p-3$	3^{p-1}	1^p	1^{2p-3}	2^{2p-2}
a_3	2		3	1	1	$1, 1^2$

		a_1 2	a_3 5	a_4 $2p-4$	a_{4p-4} 1
a_{p+1}	2	1	5	$2p-4$	1^p
a_p	5		4	$2p-4$	1^{p-1}
a_1	$2p-4$			1	1
a_{3p-1}	1	2	5^2	$(2p-4)^2$	1^{3p-4}

		a_1 $2p-4$	a_p 5	a_{p+1} 2	a_{3p-1} 1
a_4	$2p-4$	1	5	2	1^3
a_3	5		4	2	1^2
a_1	2			1	1
a_{2p-4}	1	$2p-4$	5^{p-1}	2^p	1^{3p-4}

		a_1 3	a_2 3	a_4 $2p-2$	a_{4p-4} 1
a_{2p}	3	2	3	$(2p-2)^2$	1^{2p-2}
a_p	3		2	$2p-2$	1^{p-1}
a_1	$2p-2$			1	1
a_{p-1}	1			$2p-2$	1^{p-2}

		a_1 $2p-2$	a_{p-1} 1	a_p 3	a_{2p} 3
a_4	$2p-2$	1	1	3	3^2
a_{4p-4}	1	$2p-2$	1^{p-2}	3^{p-1}	3^{2p-2}
a_2	3			2	3
a_1	3				2

TABLES $n=4p$ (*continued.*)

	a_1 6	a_3 1	a_4 $2p-2$	a_{4p-4} 1
a_p 6	1	1	$2p-2$	1^{p-1}
a_{p-1} 1			$2p-2$	1^{p-2}
a_1 $2p-2$			1	1
a_{3p} 1	6	1^2	$(2p-2)^2$	1^{3p-3}

	a_1 $2p-2$	a_{p-1} 1	a_p 6	a_{3p} 1
a_4 $2p-2$	1	1	6	1^3
a_3 1			6	1^2
a_1 6			1	1
a_{4p-4} 1	$2p-2$	1^{p-2}	6^{p-1}	1^{3p-3}

TABLES $n=4p+1$.

	a_2 3	a_3 3	a_4 $2p-3$	a_{4p-3} 1
a_{p+1} 3	2	3	$2p-3$	1^p
a_{2p} 3	3	$1^2, 2$	$(2p-3)^2$	1^{2p-2}
a_1 $2p-3$			1	1
a_p 1		3	$2p-3$	1^{p-1}

	a_1 $2p-3$	a_p 1	a_{p+1} 3	a_{2p} 3
a_4 $2p-3$	1	1	3	3^2
a_{4p-3} 1	$2p-3$	1^{p-1}	3^p	3^{2p-2}
a_2 3			2	3
a_3 3		1	3	$1^2, 2$

	a_1 2	a_2 2	a_3 1	a_4 $2p-2$	a_{4p-3} 1
a_{2p} 2	1	3	1	$(2p-2)^2$	1^{2p-2}
a_p 3		1	1	$2p-2$	1^{p-1}
a_{p+1} 1		3	1	$2p-2$	1^p
a_1 $2p-2$				1	1
a_{2p+1} 1	2	3	1^2	$(2p-2)^2$	1^{2p-1}

	a_1 $2p-2$	a_p 3	a_{p+1} 1	a_{2p} 2	a_{2p+1} 1
a_4 $2p-2$	1	3	1	2^2	1^9
a_2 3		1	1	2	1
a_3 1		3	1	2	1^2
a_1 2				1	1
a_{4p-3} 1	$2p-2$	3^{p-1}	1^p	2^{2p-2}	1^{2p-1}

TABLES $n=4p+1$ (*continued*).

	α_1 3	α_3 4	α_4 $2p-3$	α_{4p-3} 1
α_{p+1} 3	1	4	$2p-3$	1^p
α_p 4		3	$2p-3$	1^{p-1}
α_1 $2p-3$			1	1
α_{3p} 1	3	4^2	$(2p-3)^2$	1^{3p-3}

	α_1 $2p-3$	α_p 4	α_{p+1} 3	α_{3p} 1
α_4 $2p-3$	1	4	3	1^3
α_3 4		3	3	1^2
α_1 3			1	1
α_{4p-3} 1	$2p-3$	4^{p-1}	3^p	1^{3p-3}

	α_1 7	α_4 $2p-1$	α_{2p-3} 1
α_p 7	1	$2p-1$	1^{p-1}
α_1 $2p-1$		1	1
α_{3p+1} 1	7	$(2p-1)^2$	1^{3p-2}

	α_1 $2p-1$	α_p 7	α_{4p+1} 1
α_4 $2p-1$	1	7	1^3
α_1 7		1	1
α_{4p-3} 1	$2p-1$	7^{p-1}	1^{3p-2}

TABLES $n=4p+2$.

	α_3 7	α_4 $2p-4$	α_{4p-2} 1
α_{p+1} 7	6	$2p-4$	1^p
α_1 $2p-4$		1	1
α_{3p} 1	7^2	$(2p-4)^2$	1^{3p-3}

	α_1 $2p-4$	α_{p+1} 7	α_{3p} 1
α_4 $2p-4$	1	7	1^3
α_3 7		6	1^2
α_{4p-2} 1	$2p-4$	7^p	1^{3p-3}

	α_1 1	α_2 3	α_3 2	α_4 $2p-2$	α_{4p-2} 1
α_p 1			2	$2p-2$	1^{p-1}
α_{p+1} 3		2	2	$2p-2$	1^p
α_{2p+1} 2	1	3	$1, 1^2$	$(2p-2)^2$	1^{2p-1}
α_1 $2p-2$				1	1
α_{2p} 1		3	2	$(2p-2)^2$	1^{2p-2}

	α_1 $2p-2$	α_p 1	α_{p+1} 3	α_{2p} 1	α_{2p+1} 2
α_4 $2p-2$	1	1	3	1^2	2^2
α_1 1					2
α_2 3			2	1	2
α_{2p+2} 1	$2p-2$	1^{p-1}	3^p	1^{2p-2}	2^{2p-1}
α_3 2		1	3	1	$1, 1^2$

TABLES $n=4p+2$ (*continued*).

	a_1 3	a_2 3	a_4 $2p-1$	a_{4p-2} 1
a_{2p+1} 3	2	3	$(2p-1)^2$	1^{2p-1}
a_p 3		1	$2p-1$	1^{p-1}
a_1 $2p-1$			1	1
a_{p+1} 1		3	$2p-1$	1^p

	a_1 $2p-1$	a_p 3	a_{p+1} 1	a_{2p+1} 3
a_4 $2p-1$	1	3	1	3^2
a_2 3		1	1	3
a_{4p-2} 1	$2p-1$	3^{p-1}	1^p	3^{2p-1}
a_1 3				2

	a_1 4	a_3 3	a_4 $2p-2$	a_{4p-2} 1
a_{p+1} 4	1	3	$2p-2$	1^p
a_p 3		2	$2p-2$	1^{p-1}
a_1 $2p-2$			1	1
a_{3p+1} 1	4	3^2	$(2p-2)^2$	1^{3p-2}

	a_1 $2p-2$	a_p 3	a_{p+1} 4	a_{3p+1} 1
a_4 $2p-2$	1	3	4	1^3
a_3 3		2	4	1^2
a_1 4			1	1
a_{4p-2} 1	$2p-2$	3^{p-1}	4^p	1^{3p-2}

TABLES $n=4p+3$.

	a_2 3	a_3 3	a_4 $2p-2$	a_{4p-1} 1
a_{p+1} 3	1	3	$2p-2$	1^p
a_{2p+1} 3	3	$1^2, 2$	$(2p-2)^2$	1^{2p-1}
a_1 $2p-2$			1	1
a_{p+2} 1	3	3	$2p-2$	1^{p+1}

	a_1 $2p-2$	a_{p+1} 3	a_{p+2} 1	a_{2p+1} 3
a_4 $2p-2$	1	3	1	3^2
a_2 3		1	1	3
a_{4p-1} 1	$2p-2$	3^p	1^{p+1}	3^{2p-1}
a_3 3		3	1	$1^2, 2$

	a_1 1	a_3 6	a_4 $2p-3$	a_{4p-1} 1
a_{p+2} 1	1	6	$2p-3$	1^{p+1}
a_{p+1} 6		5	$2p-3$	1^p
a_1 $2p-3$			1	1
a_{3p+1} 1	1	6^2	$(2p-3)^2$	1^{3p-2}

	a_1 $2p-3$	a_{p+1} 6	a_{p+2} 1	a_{3p+1} 1
a_4 $2p-3$	1	6	1	1^3
a_3 6		5	1	1^2
a_1 1			1	1
a_{4p-1} 1	$2p-3$	6^p	1^{p+1}	1^{3p-2}

TABLES $n = 4p+3$ (*continued*).

	a_1 2	a_2 3	a_3 1	a_4 $2p-1$	a_{4p-1} 1
a_{2p+1} 2	1	3	1	$(2p-1)^2$	1^{2p-1}
a_{p+1} 3		2	1	$2p-1$	1^p
a_p 1			1	$2p-1$	1^{p-1}
a_1 $2p-1$				1	1
a_{2p+2} 1	2	3	1^2	$(2p-1)^2$	1^{2p}

	a_1 $2p-1$	a_p 1	a_{p+1} 3	a_{2p+1} 2	a_{2p+2} 1
a_4 $2p-1$	1	1	3	2^2	1^2
a_3 1		1	3	2	1^3
a_2 3			2	2	1
a_1 2				1	1
a_{4p-1} 1	$2p-1$	1^{p-1}	3^p	2^{2p-1}	1^{2p}

	a_1 5	a_3 2	a_4 $2p-1$	a_{4p-1} 1
a_{p+1} 5	1	2	$2p-1$	1^p
a_p 2		1	$2p-1$	1^{p-1}
a_1 $2p-1$			1	1
a_{3p+2} 1	5	2^2	$(2p-1)^2$	1^{3p-1}

	a_1 $2p-1$	a_p 2	a_{p+1} 5	a_{3p+2} 1
a_4 $2p-1$	1	2	5	1^3
a_3 2		1	5	1^2
a_1 5			1	1
a_{4p-1} 1	$2p-1$	2^{p-1}	5^p	1^{3p-1}

On an Algebraical Form, and the Geometry of its Dual Connexion with a Polygon, plane or spherical. *By* T. COTTERILL, M.A.

[*Read February 8th,* 1872.]

The following is a *résumé* of some of the results and the method employed in a paper on this subject, read before the Society, Feb. 8, 1872. A portion of the MS. having been mislaid, and some improvements having suggested themselves, the whole paper will be given hereafter.

A polygon, plane or spherical, can be denoted in two ways—either by its angular points, or its lines equal in number (say m), taken in a