

VI.—CONSIDERATIONS ON THE FLOTATION OF ICEBERGS.

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IN all our text-books of Geology, the action of floating Ice is referred to as an agent of great power in producing physical changes. Its two chief forms are those of Coast Ice and Icebergs. Much has been written about the latter of these, but about the former very little. In the GEOLOGICAL MAGAZINE, July, 1876, in an article on Ice and Ice-work in Newfoundland, I endeavoured to show that the greater agent of the two was Coast Ice, a view which has been subsequently strengthened by observations on the Coast of Finland. In this paper I had occasion to refer to the laxity with which the conditions under which Icebergs float have been spoken about. Thus, in Jukes and Geikie's Text-Book of Geology, p. 416, we are told that because "about eight times more ice of an iceberg is below water than above," therefore "a mass which rises 300 feet above the waves has its bottom 2400 below them."

As no regard is paid to what the relative shape of ice above water is to that below, might it not be well to add, in order to render the harmlessness of the doctrine more evident, that the mere fact of planting a Union Jack upon the summit of the berg would cause an addition to its depth equal to eight times the height of the pole?

If this were only done, Icebergs might be talked about as grounding in very deep water, where they could "tear up the softer deposits of the sea-bed," and "rub down and groove the harder rocks" to an unlimited extent. This grounding in deep water I endeavoured to show to be, in the generality of cases, untenable, excepting, perhaps, in the case of bergs immediately in the vicinity of their origin, where they more or less approximate to parallelopipeds in their form. In doing this, I also showed that in consequence of the degrading action which takes place, more especially between wind and water, it would seem that bergs as they travel towards low latitudes must be looked upon as a form more like a peak which stands upon a sunken pedestal or foot, rather than as descending perpendicularly into the water. In such a case it is evident that no great depth could be obtained.

However, to take as favourable a view as possible of ice reaching down to abyssal depths, I will again assume a case which I took before (GEOLOGICAL MAGAZINE, July, 1876, p. 307), where we must imagine the portion of the berg beneath the water to be a general continuation of that above.

Such a figure I showed might be regarded as approximately equal to a cone or many-sided pyramid. In such a case I have shown mathematically that *the depth of ice below water is approximately equal to the height which is exposed above*, the slight difference which may exist depending on the ratio we take as existing between the specific gravity of ice and sea-water,—a conclusion from which I do not see the slightest reason to alter.

This being the case, it consequently follows that if it is accepted

that bergs exist at all approximating to that of a pinnacle standing upon a base, *the depth to which they may extend below the surface of the water is less than the height we see above*, and therefore in many cases, when we see a berg 300 feet above the water, we may with much reason assume that its depth beneath the surface of the water is less than 300 feet.

The case which I have considered is one which appears to be applicable to many icebergs, and, I think, to the generality of them.

It now remains to see how far such views may be carried, and also, for the sake of illustration, to consider the possible conditions under which some other forms of ice may be regarded as existing.

In the paper where the conclusion just referred to was arrived at, a cone approximating to a berg of ice was drawn as floating with its base downwards. The Rev. O. Fisher (*GEOL. MAG.*, 1876, p. 379) has, however, raised the question of the stable equilibrium of such a cone, which he thinks would not remain in the position as figured, but must turn over. Whether this would or would not be the case with the cone in question, I am not prepared to answer. The figure is only drawn to illustrate the calculation to which it is appended. As a practical illustration, to strengthen these views and to show that the cone of ice which I have taken will not float with its base downwards, Mr. Fisher takes a tetrahedron out of a set of models of crystals, and placing it in water finds that it floats with one of its angles downwards.

This I consider to be an unfair comparison, which no doubt has led many casual readers to the belief that a cone will also float with its apex downwards, and perhaps, in consequence, that my conclusions, being founded on false assumption, must also of necessity be false. Lest readers should be led into misconceptions of this sort, it may be well to consider how cones of ice would float.

First, if we take a slab of ice and place it upon water, we know that it will float horizontally. On the middle of this slab we might raise a small pinnacle of ice, and the mass would still keep horizontal. We might next increase this pinnacle round its sides without increasing its height until we reached the edges of our slab, and still we may imagine the block we have built up keeping its horizontal position. We should here have a figure approximating to the probable shape of an iceberg which has travelled into latitudes like those of Newfoundland,—a pinnacle supported on a foot or pedestal. Such a form approximates to a cone, and such a cone I believe would float, and does float with its base downwards, or in other words, from a consideration of this sort, it is evident to us that there are certain obtuse cones which would float with their apex upwards. Secondly, on the other hand, if I make a very acute or tall cone, it would never for a moment be expected to float vertically with its base downwards more than a tall stick of ice would be expected to retain such a position. Such a cone would, according to ordinary expectation and according to all probability, fall on its side and float more or less horizontally. It is also equally certain that such a cone would not float with its apex downwards, as Mr. Fisher's experiment might lead one to think.

Being thus convinced, from our own sense of reason, that there are cones of ice which can float with their base downwards, and also that there are others which can float with their base upwards, the question then is to define these cones.

1. Adopting from Thomson and Tait, Natural Philosophy, § 767, that where V is the volume of a body immersed in a fluid, A the area of its plane of flotation, k the radius of gyration of that plane, and h the height or distance between the centre of gravity of the floating body and that of the displaced fluid, for stable equilibrium we must have

$$A k^2 > V h$$

We shall find for a cone of ice to float with its vertex downwards in sea-water, the radius of the base of the cone must be greater than 196 times its height,—or, roughly, the diameter of the base cannot be less than two-fifths of the height.

2. Again adopting the same method for a cone of ice floating in sea-water with its base downwards and horizontal, we shall find that the radius of the cone must be greater than 1.05 times its height,—or roughly the diameter of the base cannot be less than twice the height.

NOTE.—Case I.¹ Moment of Inertia of a circular lamina about a diameter = $\frac{\pi R^2}{4}$, but this = $A k^2$

$$\therefore A k^2 = \frac{\pi R^2}{4}$$

$$\text{or } \pi R^2 k^2 = \frac{\pi R^4}{4} \therefore k^2 = \frac{R^2}{4} \therefore k = \frac{R}{2}$$

Let r be the radius of the base of the cone and a its height. Also let the density of the floating cone compared with the liquid be ρ , then—

$$AC : BC = 1 : \rho^{\frac{1}{3}}$$

$$\therefore \text{radius of plane of flotation is } r \rho^{\frac{1}{3}}$$

$$\therefore \text{radius of gyration } k = \frac{r \rho^{\frac{1}{3}}}{2} \quad (1)$$

The Area of the plane of flotation

$$A = \pi r^2 \rho^{\frac{2}{3}} \quad (2)$$

Let G be the centre of Gravity of the Cone and E that of the displaced water,

$$GC = \frac{3a}{4} \text{ and } EC = \frac{3}{4} a \rho^{\frac{1}{3}}$$

$$\therefore GE \text{ or } h = \frac{3a}{4} (1 - \rho^{\frac{1}{3}}) \quad (3)$$

V the immersed volume = the volume of the Ice Cone multiplied by

$$\rho \text{ or } \frac{\pi r^2 a}{3} \rho \quad (4)$$

Now substituting in $A k^2 > V h$

$$\pi r^2 \rho^{\frac{2}{3}} \cdot \frac{r^2 \rho^{\frac{2}{3}}}{4} > \frac{\pi r^2 a}{3} \rho \cdot \frac{3a}{4} (1 - \rho^{\frac{1}{3}})$$

$$\text{or } r > a \sqrt{\frac{1}{\rho^{\frac{1}{3}}} - 1}$$

¹ See Woodcut, Fig. 1, p. 69.

Now if the specific gravity of ice = 1.028 and that of sea-water .918, then ρ for sea-water and ice = .893

$$\text{whence } r > a \cdot 196$$

CASE II.¹—As in Case I. let r = the radius of the base of the cone, a = the height of the cone, and ρ as before.

$$\text{Volume of Cone A C M : vol. of B C N :: } 1 : 1 - \rho \therefore \text{AC : BC} = 1 : \sqrt[3]{1 - \rho}$$

$$\therefore \text{radius of plane of flotation} = r (1 - \rho)^{\frac{1}{3}} \text{ and } k \text{ the radius of gyration of plane of flotation} = \frac{r}{2} (1 - \rho)^{\frac{1}{3}} \quad (1)$$

$$\text{Area A of plane of flotation} = \pi r^2 (1 - \rho)^{\frac{2}{3}} \quad (2)$$

$$\text{The distance between G and E which are as before, or } h = \left(\frac{1}{\rho} - 1\right)^{\frac{3}{2}}$$

$$a (1 - \sqrt[3]{1 - \rho}) \quad (3)$$

$$\text{V the volume of displaced water} = \frac{\pi r^2 a \rho}{3} \quad (4)$$

Now substituting in $A k^2 > V h$

$$\pi r^2 (1 - \rho)^{\frac{2}{3}} \cdot \frac{r^2}{4} (1 - \rho)^{\frac{2}{3}} > \pi r^2 \rho \frac{a}{3} \cdot \left(\frac{1}{\rho} - 1\right)^{\frac{3}{2}} \cdot a (1 - \sqrt[3]{1 - \rho})$$

$$\text{or } r > a \sqrt{\frac{1}{(1 - \rho)^{\frac{1}{3}}} - 1}$$

$$\text{or } r > a \cdot 1.05$$

Approximations to these two limiting cones are represented in the woodcuts given on page 69. Fig. 1 represents a cone of ice floating with its apex downward, which is unstable, and in sea-water might fall on its side, whilst one less acute might float in this position. Any cone, when thus floating, has about $\frac{1}{25}$ of its whole depth above water. If such cones existed in nature, it is evident that they must be much more obtuse in form in order to withstand in such a position the shocks of waves and winds to which they would be subjected.

Fig. 2 represents a cone of ice floating with its apex upwards, and its base horizontal. Any cone which is more obtuse than this, when floating in sea-water, is stable. In this case .47, or, as before stated, nearly one-half of the height of the cone, is above water.

To test these results I had several small cones made out of Japanese boxwood (S.G. about .839), which was the most suitable wood for the purpose which I could obtain. The diameter of the base of these cones was in all cases 2 in., whilst their height, which was variable, was made above and below the limits as given by calculation where the specific gravity of the wood I was using took the place of the specific gravity of ice. These cones, when placed in water, behaved in a manner similar to the way I have stated that cones of ice will act.

¹ See Woodcut, Fig. 2, on opposite page (p. 69).

We have here the solution to two theoretical cases of supposed bodies of ice floating in sea-water, which I think will considerably aid us in forming some idea as to the depth to which icebergs extend beneath the surface of the water,—the practical solution of which problem is surrounded with so much difficulty. The results are obtained from two regular solids, but yet it is evident that they can be roughly applied to any solids which approximate to such forms. Now from Case I., where cones floating apex downwards are considered, it is evidently possible for floating ice to have a depth below the surface of the water in comparison to that which is

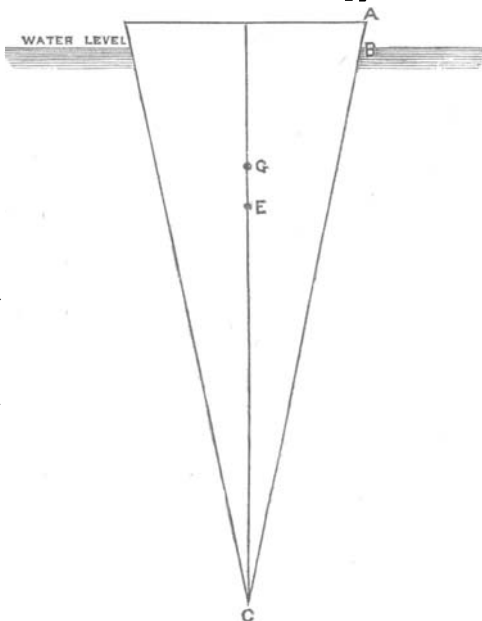


FIG. 1.

above immensely greater than has generally been believed. But the question now is, have forms approximating to such inverted cones any existence in nature? All that I can say to the contrary is by appealing to the results of observation and to the consequences of degradation upon a block of ice after leaving its parent the glacier,—both of which, as pointed out before (*GEOL. MAG.* 1876, p. 306), appear unfavourable to such views.

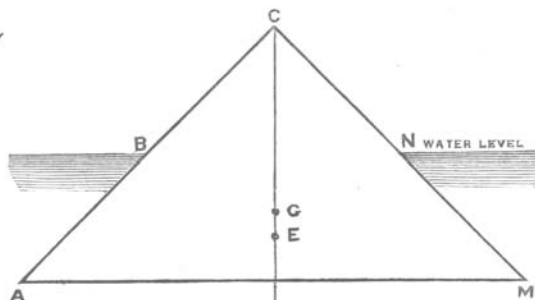


FIG. 2.

I might also add, as another argument against the probability of ice extending to abyssal depths, that pressure tends to liquefy ice, or, in other words, to lower the freezing-point of water, and ice at great depths is under great pressure. For example, ice at the

depth of 2400 feet would be under a pressure of about 73 atmospheres. Although this lowering of temperature, which can be easily calculated, is very small, it must nevertheless have some influence in the destruction of masses of ice should they extend to considerable depths, more especially so when we consider that the action is not merely a surface one, but one that extends throughout the mass.

The more probable form in which the generality of icebergs exist are those which have their limit represented by Case II., where we have a series of stable forms, more or less conical in their shape. Here the depth below the surface of the water never exceeds the height which is above, but is probably always less.

Of course many other forms of ice also approximating to regular solids might be supposed, in which the ratio of the depth of ice below water to that which is above would be greater than that of the inverted cone, and which would be less than that of the upright cone. Thus, for instance, such a solid as would be described by an equilateral hyperbola, revolving round one of its asymptotes, might be taken as pointing downwards or upwards. In the former case the ratio of the depth below the surface of the water to the height which is above might be infinitely greater than in the case of the inverted cone of ice, and in the latter case or pointing upwards the ratio of the depth below the surface of the water to the height which is above infinitely less than in the case of the upright cone.

To obtain the greatest height of ice above the surface of the water relatively to that which is below we must imagine a sheet of ice, from the upper surface of which a needle or pencil extends vertically upwards. The same figure reversed would give us the greatest depth to which ice could descend below the surface of the water. Such a case is however purely theoretical. In cubes which are in stable equilibrium with a face upwards, and in parallelepipeds which are in stable equilibrium with one of their largest faces upwards, the depth of ice below the surface of the water would be about eight times the height which is exposed above.

Combinations of regular solids might also be considered. Thus two cones might be supposed placed base to base, and floating one with its apex upwards and the other with its apex downwards.

First—let the volume of the lower cone V , whose height is H , be eight times the volume of the small cone v , whose height is h .

In this case we have

$$\frac{V}{v} = \frac{H}{h} = \frac{8}{1} \therefore H = 8h$$

or the depth below the surface of the water is eight times the height which is above.

Secondly—let the upper or smaller cone be less than $\frac{1}{8}$ the volume of the lower one, then the depth below water will be greater than eight times the height above.

Thirdly—let the upper or smaller cone be greater than $\frac{1}{8}$ the volume of the lower one, then the depth below water will be less than eight times the height which is above.

Continuing in this way, a number of cases might be considered which would show us that in some cases the depth of ice below the surface of the water is eight times that which is above, in some cases less than eight times, and in other cases greater than eight times.

What now remains to be done is to take the case which seems to be, for the generality of icebergs, the most probable one. This I believe to be the case where we have a pinnacle standing on a foot or pedestal, or in the limiting case a cone floating with its apex upwards. In this case *the height above the surface of the water will be generally greater than the depth which is below.* Until future observations show this view to be a wrong one, I think I shall be justified in keeping to the above result.

VII.—NOTES ON CORAL REEFS.

By STAFF-COMMANDER HENRY HOSKEN, R.N., H.M.S. *Pearl*.

[Communicated by R. H. Scott, F.R.S., Director of the Meteorological Office.]

I HAVE to thank you for your kind letter of the 24th of December, 1875; it affords me great satisfaction to hear that my remarks about the New Hebrides¹ have interested you.

Thinking that my observations on the soundings that I obtained off the east end of Vanikoro Island, Santa Cruz Group, might be useful, I propose to forward a copy of them, together with tracings, with the next Log that I shall complete and send into office; the originals were forwarded to the Hydrographical Department on the 31st December, 1875. A new edition of the Chart No. 986 with corrections has since been issued.

The chief interest of this discovery is not so much in its hydrographical importance, as in its connexion with the probable alteration of the geological formation of these reefs, or else, allowing that they existed at the time of the Survey, it shows that this "Barrier Reef" is not so different from the general rule as was at first supposed, when the Chart showed a gap of eight miles in the reef.

Before our arrival at Vanikoro Island, Commodore Goodenough and I had been remarking on the peculiarity of the apparent cessation of the "Barrier Reef" on the weather side of that island.

The *Pearl* was taken into the anchorage of Ocili Harbour, Tevai Bay, under sail, a strong Trade was blowing; when nearly abreast Dillon Head, and steering in on the course recommended, a shoal spot, upon which the sea occasionally broke heavily, was seen; this was only cleared by about a ship's length; particulars are given in copy of remarks. The patch appeared to consist of live coral.

Whilst the *Pearl* was steaming out, several soundings were obtained, and much discoloured water seen over shoal-looking ground, the description of which is given in Remarks, and the position shown in the tracings already alluded to.

It had been Commodore Goodenough's intention to have taken the *Pearl* inside the "Barrier Reef," but eventually it was considered

¹ See GEOL. MAG. 1876, Decade II. Vol. III. p. 82.