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BRIDGES AS ILLUSTRATIVE MATERIAL ON THE PARALLELOGRAM OF FORCES.¹

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The parallelogram of forces is far from being neglected in modern courses in elementary physics, but the students who have a vivid idea of its meaning and practical importance seem to be few and far between. Many times, in giving laboratory examinations in physics, I have asked some student who had just finished satisfactorily the familiar three spring balance experiment, what it was good for, and why any one might conceivably be glad to know the fact that he had just proved. I have still to find a boy who instinctively thought of looking ten feet over his head at the trussed roof of the laboratory. It is, therefore, my hope to emphasize once more the familiar fact that there is a wealth of practical applications of the parallelogram law that can be used to show students what this law is really good for. Among the more obvious of these applications are derricks, where the parallelogram law can be applied at the tip of the boom to determine the necessary strength of the boom hoist. There are also steam shovels, where the scoop is moved in any desired direction, against the resistance of the material that is being handled, by a push along a beam and a pull along a chain. The student should be led to see how the resultant of these two forces can be made to point in the desired direction and to be of the necessary magnitude. There are also kites, sailboats, and aeroplanes, where the pressure of the air against a surface has a component in such a direction as to propel the sailboat or to support the kite or aeroplane. There are hanging street lamps, hanging signs, guy-ropes on tents, and often guy-wires on telegraph poles where the wires have to turn a corner, and dozens of other applications that you will find if you look for them. But the commonest and perhaps the most interesting of all are to be found in the various roof and bridge

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trusses that abound in every neighborhood. It is chiefly of these that I wish to speak. Many of the details that I shall mention are perhaps too technical for class room use, but others are, I believe, entirely suitable for even the most elementary classes, and you will know better than I where the dividing line should come.

It is best to begin with what is called a "pinned" truss (see figure 1) rather than a "riveted" truss (see figure 2). In a pinned truss the members are provided with holes at each end, and are joined by pins, from three inches to two feet in diameter, which

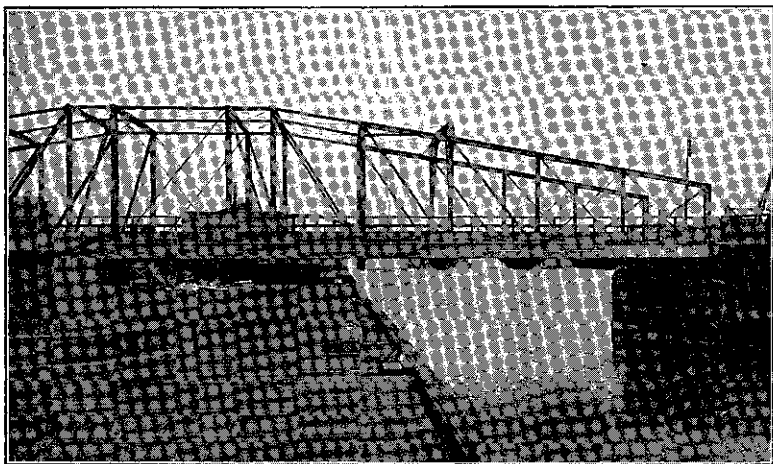


FIG. 1. A pinned truss, a cantilever Pratt.

are thrust through all the holes at a joint, like the pin of a door hinge. No twisting or bending can be transmitted through such a joint; the members are able only to push or to pull on the pin. In a riveted truss there are gusset plates at each joint, that look like the webs between a duck's toes, to which the members are fastened. The distinction between riveted and pinned trusses is not fundamental, in that all riveted trusses are designed as if they were pinned, the gusset plates being counted on merely to increase the stiffness of the truss.

It is also best to begin with a "simple truss," that is, one in which all the members are so arranged as to form triangles. Such a truss depends for its stiffness on the familiar fact that a triangle is determined when the lengths of its three sides are fixed, while a quadrilateral or other polygon can be deformed without changing the lengths of its sides. The experimental truss in figure 3 is a simple truss in this sense.

This experimental truss has proved to be very useful both for lecture demonstrations and for experimental tests of computed stresses, but many of the points that will here be demonstrated with its help, could be taught equally well by referring one's students to various real bridges in the neighborhood. The model is made of $1\frac{1}{4}$ inch 22 gauge aluminum tubing, with cast aluminum end pieces at the joints. These castings are of two kinds, "central" and "offset," and are interchangeable and reversible, so that three members can be brought to a single joint without throwing the tubes out of alignment. This is important because the truss is built double (see figure 5) so as to be self-supporting, and it is desirable to be able to use one width of cross brace throughout. The truss should not be thought of as two trusses with a roadway between, but rather as what is called a "pony truss," which can stand alone without cross bracing above the road bed. The pins are 14-inch lengths of thin tubing $\frac{7}{16}$ of an inch in diameter and fit somewhat loosely in the $\frac{1}{2}$

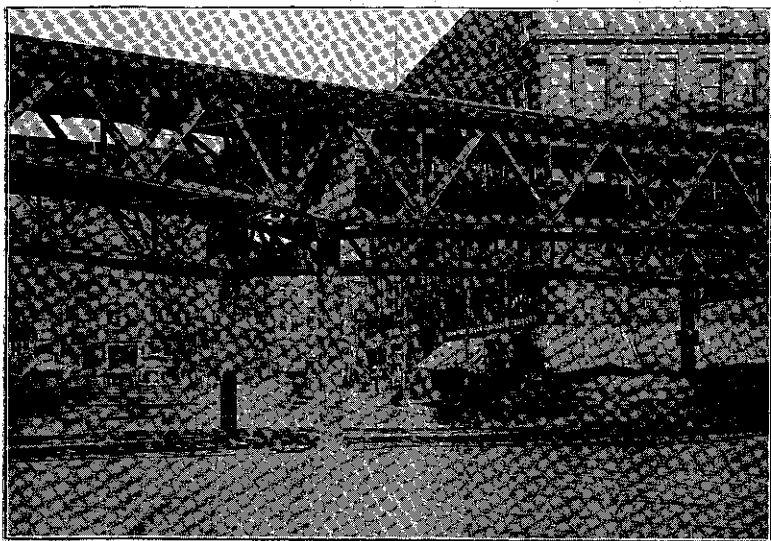


FIG. 2. A riveted truss, a Warren truss with divided panels.

inch holes. The standard members are 3, 4 or 5 feet long, so as to form right triangles and at the same time make the arithmetic easy. The whole truss is made as light as possible (32 pounds for the twelve foot span shown in figure 3; and 49 pounds for a similar fifteen foot span with double diagonals, in the middle panel) so that the effect of such concentrated loads as are easy to handle may not be masked by the effect of the weight of the truss itself.

Now let us consider how the laws of statics apply to such a truss as that in figure 3. Two treatments are possible. In the first we neglect the weight of the truss itself and work only with the concentrated loads (in this case two fifty-pound weights), and later eliminate the weight of the truss in making the corroborative measurements. In the second we take the weight of the

truss into account as we go along. The second may seem more reasonable at first, but it is harder to do. It is also less practical, for engineers, in designing a large truss, always neglect the weight of the truss as such, concentrating at the pins what experience has taught them will be an equivalent set of loads in addition to the expected pin loads due to other causes, such as the weight of the road bed, and of any wagons, crowds of pedestrians, or trains that may use the bridge. Then they compute the stresses that the members will have to stand, and design them to suit. And finally they check up the weights of these members and make sure that they are not in excess of the concentrated loads used to represent them. Let us use the first method and neglect the weight of the truss itself.

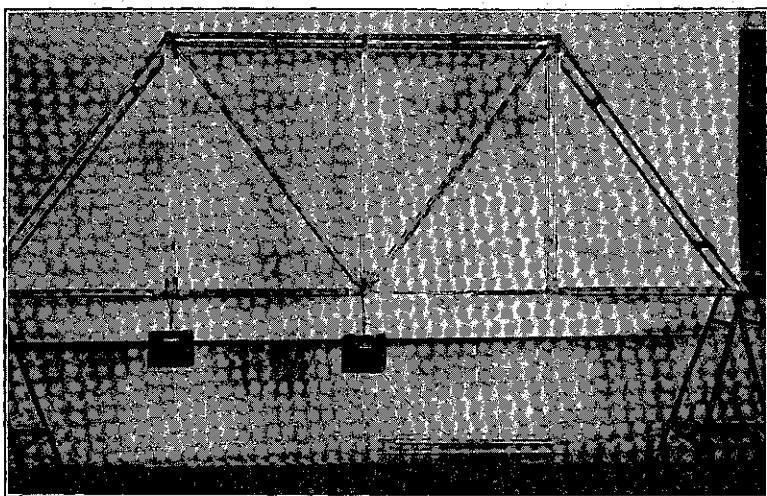


FIG. 3. An experimental model—testing a tension member.

We begin by computing the vertical reactions exerted *on* the truss in figure 3 by the supports; evidently the right hand pier must push up with a force of $\frac{1}{2} 50 + \frac{1}{4} 50 = 37.5$ lbs., and the left hand one with $\frac{1}{2} 50 + \frac{3}{4} 50 = 62.5$ lbs. As a check, we notice that $37.5 + 62.5 = 100 = 50 + 50$ lbs.

Next we “isolate” the right hand lower pin. By this I mean that we draw an auxiliary diagram of this pin by itself, magnified so that we see clearly just what we are talking about, and we show by arrows on this drawing all the forces exerted *on* the pin by the pier and the two members, and no other forces (see figure 4a). We have one force of 62.5 that is completely known,

and two others, P and T , that are known in direction but not in magnitude. The parallelogram of forces (see figure 4b) gives us the magnitude of P and T . Since either triangle in figure 4b is similar to one of the 3-4-5 triangles of the truss itself, T is $\frac{3}{4} \times 62.5 = 46.9$ lbs., and P is $\frac{5}{4} \times 62.5 = 78.1$ lbs.

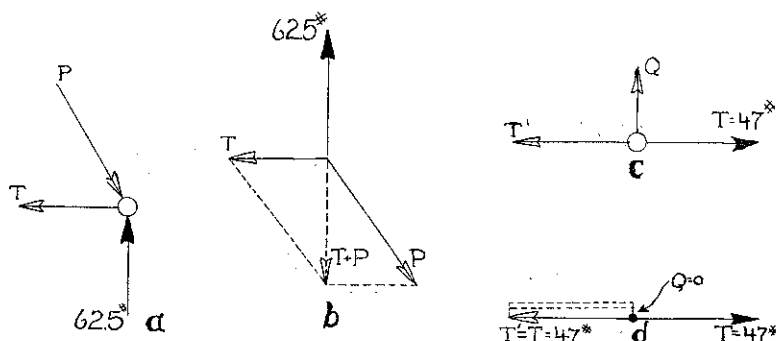


FIG. 4. Insolation sketches [(a) and (c)] and the corresponding parallelogram diagrams [(b) and (d)] for the first two pieces to be treated in the truss of figure 3.

This brings us to an aspect of real bridges that often interests students more than any other. Evidently the horizontal and inclined members that meet at the right hand pier have very different parts to play in the structure. The horizontal one has to pull on the pins at its two ends, and is said to be "in tension," while the inclined member has to push against the pins at its two ends, and is said to be "in compression."² In a large bridge this difference in function would involve a great difference in appearance between these members. The horizontal tension member would be composed of one or more steel straps or rods, enlarged at the ends to give room for the holes, as in the foreground of figure 5, while the inclined compression members would be much stouter and more complicated, as in the background of figure 5, so as not to buckle under compression. I have found that every one who has mastered this distinction has thereafter watched bridges with a new interest, noticing at once which members were built for compression and which for tension, and trying to find out why.

Having finished with the first pin we may proceed to the pin next to it in the bottom chord of the truss. Here (see figure 4c), the known force is $T = 47$ lbs. found above, and there are

² Notice that no member can possibly push at one end and pull at the other.

two unknown forces. The parallelogram of forces (figure 4d) turns out to have shrunk to a line, one side, Q , being zero. So $T^1 = T = 47$ lbs., and the second member in the bottom chord is in tension, while the first vertical is not stressed at all in this loading of the truss. If there were a concentrated load at this pin as well as at the other two, this vertical would, however, have its part to play like the rest.

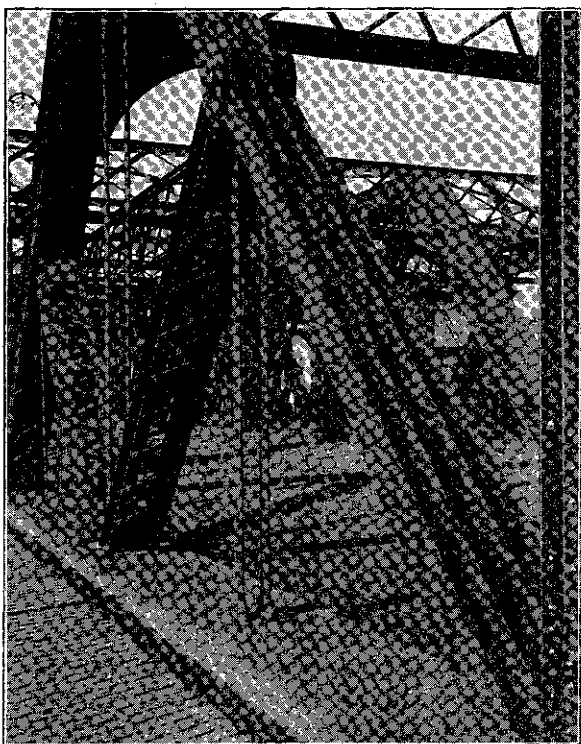


FIG. 5. Typical tension and compression members.

We could proceed next to the pin directly above this one, where we would find two known and two unknown forces. We would have to use the parallelogram of forces twice here, once to compound the two knowns into a single resultant, and again to determine the unknowns from this resultant as before.³ We would thus find the top chord member in compression and the diagonal in tension. And so we could go on through the whole truss, pin by pin, determining the nature and magnitude of the

³ Unless we use the equivalent process called the polygon of forces.

stress in every member, although three or four pins are usually enough to illustrate the point at issue.

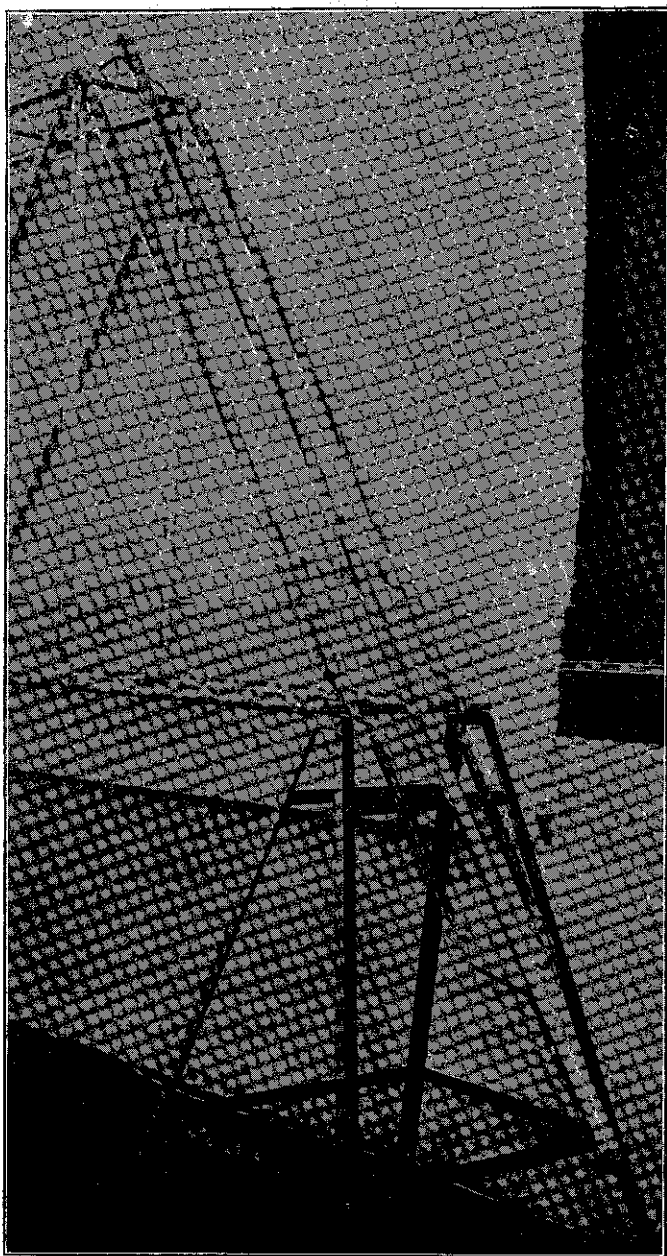


FIG. 6. Testing a compression member.

Computations of this sort can be tested on real bridges by means of a sensitive instrument called an extensometer, which measures the stretch in a tension member, or the shrinkage in a compression member, when a concentrated load, such as a locomotive, is put at various points on the bridge. These measurements often agree with the computed values to something like a tenth of one per cent. Similar tests on the model can be made

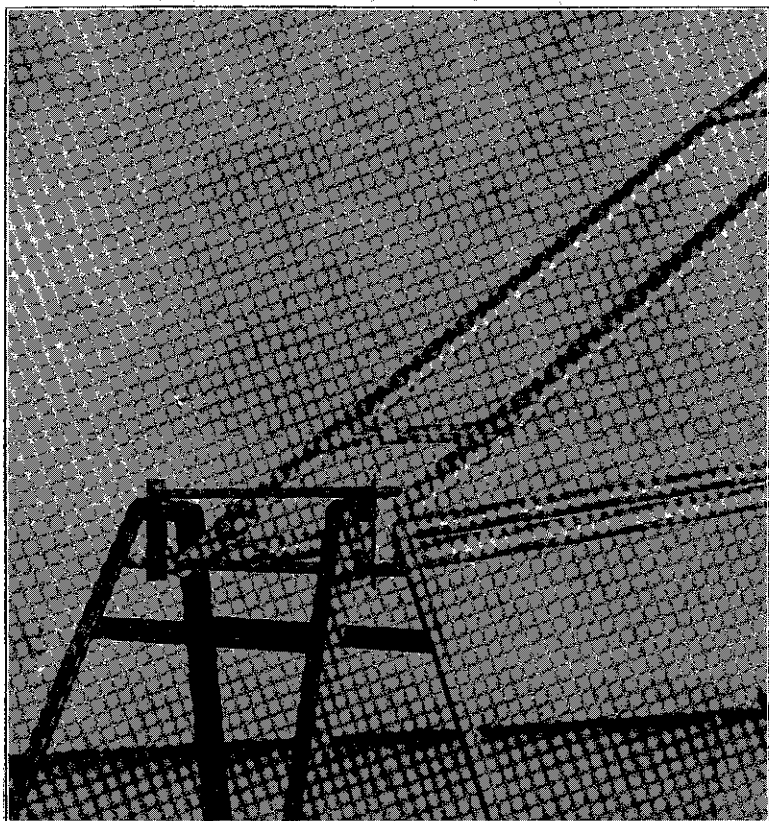


FIG. 7. Method of supporting one end of the model to avoid friction.

by the devices shown in figures 3 and 6. In figure 3 a pair of spring balances have been stretched between the two pins at the ends of a tension member, and a turn-buckle tightened until the pins are just pulled loose from contact with the tension member in question. Just the right tightness is secured by watching when the tension member begins to rattle when shaken.

When the balances are adjusted, the tension member can be removed altogether, as in figure 3, to make the demonstration more striking. In figure 6 one of the end diagonals is being tested for compression. An auxiliary piece of aluminum tubing supports the balances in such a position as to pull the two pins apart, and the same turn-buckle is used to tighten the balances until the member rattles when shaken. In both cases readings of the balances should be taken both before and after the concentrated loads are put on. The difference between the two sets of readings gives the stress due to the concentrated loads by themselves, the effect of the weight of the truss being eliminated. Such tests will usually come out within two per cent of the calculated values.

In testing the model truss it is necessary to take one precaution. We have assumed that the reactions at the supports are vertical. But if the truss rests on solid supports and gives a little under a concentrated load, there will be frictional components at the ends large enough to make a noticeable difference in the tests. In real bridges there is a corresponding difficulty, not only from the truss's stretching under load, but also from thermal expansion. To avoid this, one end of a real truss is usually supported on rollers. In testing the model the equivalent arrangement shown in figure 7 is more convenient.

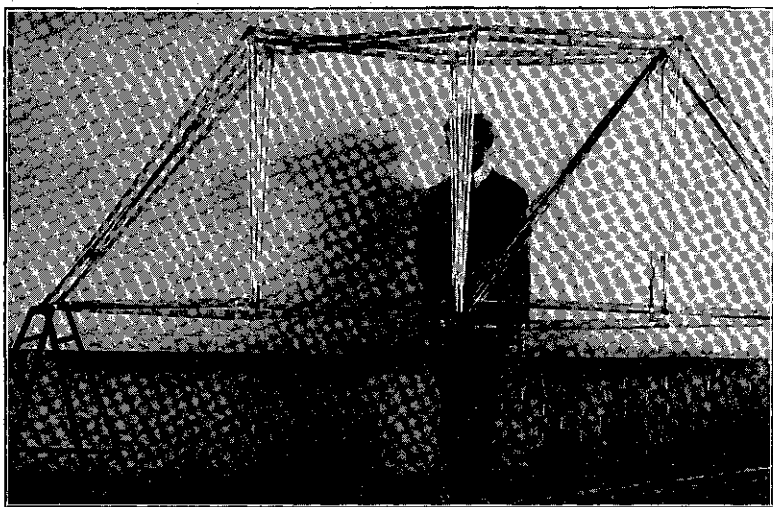


FIG. 8. Double exposure showing how truss deforms when falling.

There are two other interesting points that can be demonstrated with the model, "pin-packing" and "the shear in a panel." By pin-packing is meant the way in which the various members that come to a joint should be arranged on the pin there. If the two members that bring the greatest forces to a pin are widely

separated on the pin, they evidently have a considerable tendency to bend the part of the pin that lies between them, since they are like a couple with a large lever arm. If they lie close together, as they should, the lever arm is much smaller. In other words, in a well packed joint, the forces exerted by the members should balance each other as far as may be, not only for the joint as a whole, but for all the various groups of adjacent members considered separately. This matter is hard to explain on paper, but on the model, where one can see things in three dimensions, it is very simple.

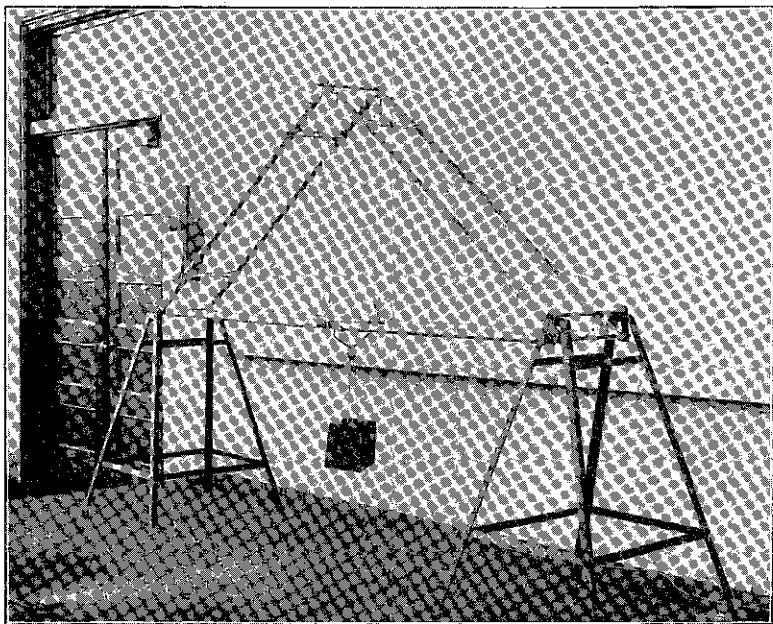


FIG. 9. King post bridge truss.

To demonstrate the "shear in a panel," one removes one of the internal diagonals of the truss, say the left hand one in figure 3.⁴ The truss will then collapse, unless supported by the demonstrator. This can be done in this particular case either by pulling *up* on the middle pin, or by pushing *down* on the pin on the other side of the panel. The reason is evident from figure 8, which is a double exposure. It shows that when the middle pin drops, the side pin rises, and so we can support the truss by preventing either of these tendencies. If we want to make the truss rigid, we can put in an additional member along either di-

⁴ The truss will be easier to handle if all concentrated loads are removed first.

agonal of the panel; but if it springs from the middle lower pin, it will be in tension, while if it springs from the side lower pin, it will be in compression. Another way of looking at this is to notice that when the rectangular panel deforms in figure 8, one diagonal lengthens and the other shortens. A tension member along the first diagonal, or a compression member along the second, will keep the truss in position.

If a truss of this sort has all its diagonals so set as to be in tension, it is called a Pratt truss; if all the diagonals are in compression, it is called a Howe truss. When it was common to make the compression members of a truss of wood, and when the iron rods for the tension members were expensive in comparison, Howe trusses with long diagonal compression members and short vertical tension members, were common. Now that steel mem-

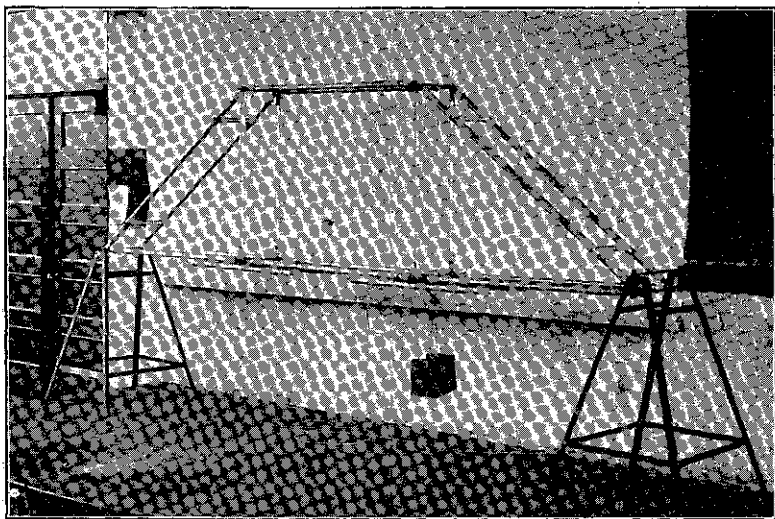


FIG. 10. Stiffened Queen post bridge truss.

bers are commonly used, compression members are much more expensive than tension members (see figure 5) and most trusses are Pratt trusses.

This brings us to the question of "counters" or crossed diagonals, such as are found in the middle panels of most bridges. A little consideration, and particularly a little experimenting with the model, will convince one that a concentrated load on any pin except the middle pin of a Pratt truss, will *tend* to put every diagonal between the load and the middle of the truss in compression. If the concentrated load is large enough in compari-

son with the weight of the truss, the stresses in some of these diagonals will actually "reverse," and they may buckle, unless designed as compression members. Now it is usually cheaper to build two tension members than one compression member, and so the middle panels, where reversal will come first, have crossed diagonals, one of which slackens off, while the other takes up the whole shear in the panel.

It should be noticed that the likelihood of reversing the shear in a given panel depends, not on the absolute size of the largest expected concentrated load, but on its size *relatively* to the weight of the truss itself. In real bridges conditions are usually such as to make counters necessary only in two or three panels near the middle. But if models of these bridges are made for laboratory experiments, their weight will be smaller in comparison with the loads they can carry, and counters may have to be provided in all the panels, unless the diagonals can stand some compression, as in the model here illustrated. This is an interesting example of the fact, well known to engineers, that the weights of very large spans increase faster than their strengths, so that there is a pretty definite limiting length of span, beyond which no truss could carry even its own weight, to say nothing of any road-bed or traffic.

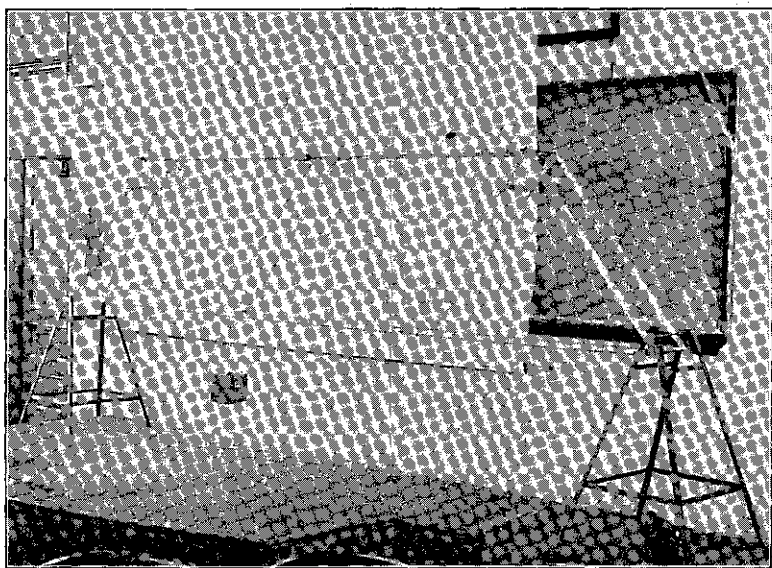


FIG. 11. Warren truss.

All this can be demonstrated on one set-up of the model. If additional members are available, or time can be taken to dismantle the model and set it up in a new form,⁵ many other interesting trusses can be demonstrated. Thus figure 9 shows a King post truss that is common in all small country bridges. The

⁵ This takes from one to two hours for complicated forms which require rear-

inclined members are in compression, and are ordinarily wooden timbers. The vertical is in tension, and is ordinarily an iron rod. Figure 10 shows a Queen post truss. It is not a pure truss, in that it would collapse under nonsymmetrical loads if the lower chord were hinged at the two pins. The truss is used to stiffen a beam which otherwise might not be strong enough to carry its load. How effectively it does this is shown in the figure, where a load of fifty pounds is carried on a twelve foot span by a lower chord formed by tying on a couple of extra members with hemp string. Queen post trusses are common in country bridges, and inverted Queen post trusses are especially common under freight cars.

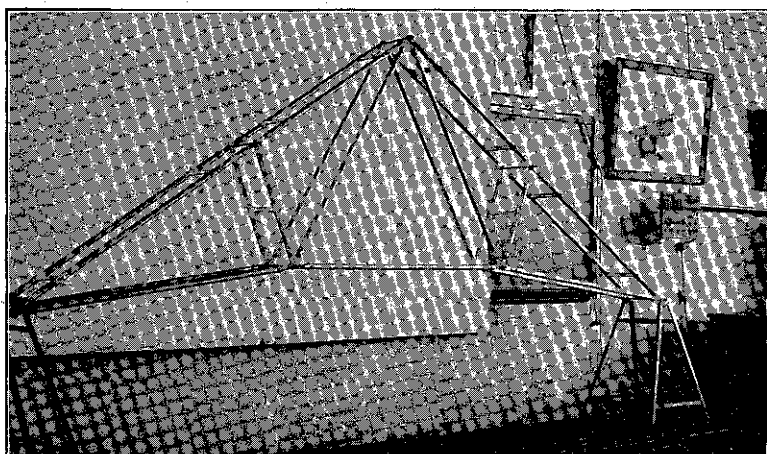


FIG. 12. Belgian or Fink roof truss.

Figure 11 shows a form of bridge truss called a Warren truss. Its disadvantage is that all the inclined members have to be built as compression members, since reversals of stress in them are frequent. Multiple Warren trusses, in which several independent sets of isosceles triangles are superposed, are common, both in steel trusses and in the old fashioned wooden "covered bridges" of New England. The truss in figure 2 would probably be called a Warren truss with divided panels. And finally, figure 12 shows a common form of roof truss. Often each of the roof slopes is divided into four or even six panels, instead of two, each panel having a tension diagonal. This truss is convenient, in that the two halves of a rather large span can be built separately in the factory and shipped on flat cars, so that but little work of erection is left to be done in the field.