The Bordered Antilogarithm Table
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The Committee, though fully recognising the desirability of variety of treatment in the teaching of Mathematics as a general rule, is of opinion that uniformity of method in certain fundamental processes in Arithmetic would be a great convenience, especially in multiplication and division of decimal fractions.

If boys remained at the same school during the whole of their Arithmetic course, each school would be independent in this matter and might reasonably object to any attempt to influence its actions. But such a large proportion of boys at some time change their school (principally in passing from a Preparatory School to a Public School), that the work in certain parts of the Public Schools is handicapped by the want of uniformity of method.

The Committee is consequently endeavouring to find out whether such uniformity is generally considered desirable as far as boys entering the Public Schools are concerned, and, if so, whether anything can be done to secure, if not complete uniformity, at any rate some approximation to it.

At a preliminary discussion the Committee examined the different methods in common use. It appeared that, though much might be said in favour of various methods, the only one at all likely to secure general support is the one described in the accompanying circular.

If the replies to these questions should lead to any result, it is proposed to inform the Preparatory Schools, and it is hoped that a considerable number of them will adopt the methods agreed upon.—Yours faithfully.

G. W. PALMER, *Hon. Sec.*

THE BORDERED ANTILOGARITHM TABLE.

BY PROF. G. H. BRYAN, F.R.S., AND T. G. CREAK, M.A.

WHILE in recent years the market has been flooded with small books of mathematical tables, each of which reproduces most of the defects of the others, there are few tables, if any, which meet all requirements when it is necessary to use logarithms for operations involving both multiplication and division. In such cases logarithms of reciprocals are constantly needed. Few tables contain these, a notable exception being the tables at the end of Carson and Smith's *Algebra*. On the other hand, tables of reciprocals are often given, which are rarely of use except in geometrical optics, and only confusion is occasioned by the separation of the tables for logarithms and antilogarithms, and of those for sines and cosines. Either the table of antilogarithms or that of logarithms may be banished with advantage.

If we retain the table of logarithms, it is impossible to obtain logarithms of reciprocals without performing an additional operation, which increases the work and the risk of error. Further, in the lowest parts of the scale the differences are large, and it is difficult to pick out the number which represents the logarithm *correctly* to four or five places of decimals as the case may be.

Now a table of *antilogarithms* may be used to find logarithms of reciprocals in just the same way that sines and cosines can be taken from the same table. The left-hand column and top line contain the actual logarithms of the numbers tabulated in the area of the present table, while the right-hand column and bottom line give the complementary logarithms or logarithms of the reciprocals.

Moreover, in order to secure greater uniformity in the degree of accuracy of working, it will be seen that on the first page the results are tabulated to five significant figures, and on the second to four. The reason of this is that, in order to find the logarithm of a number correct to four places, the number must be given to five figures in the lower parts of the scale, whereas in the higher parts it need only be given to four figures, and, furthermore, a number in the higher parts of the scale can only be found correct to four figures from a table of four-figure logarithms. It will be seen that by this method the mean differences are neither too small nor too large. The important point to be noticed is that, in order to find a logarithm correct to four places, the mean differences must not be the same in any two columns, and, to ensure this, numbers less than 4 must be known correctly to five significant figures. Two examples will make this clear.

Ex. 1. Multiply 1·058 by 8·926, using four-figure logarithms, the data being approximate.

Here 1·058 means some number between 1·0575 and 1·0584$\dot{9}$. With the present table we find, therefore,

log 1·058	between	·0243	and	·0247
log 8·926	=	·9506	=	·9506
log product	between	·9749	and	·9753
product	between	9·439	and	9·448

Ex. 2. Find the number whose four-figure logarithm is ·9457.

Even if the result has *not* been arrived at by the addition of several logarithms, it represents a logarithm lying between ·94565 and ·94574$\dot{9}$. If we use a table of five-figure antilogarithms, or a seven-figure table, we shall find that the number lies between 8·8237 and 8·8257. Thus, even if we take the result as 8·825 to four significant figures, there is a possible error of ± 1 in the last place, and the fifth significant figure is of no value whatever.

More generally, since a four-figure logarithm represents an approximate value of a logarithm, the true value of which may differ from it by $\pm 0{\cdot}00005$, and since $0{\cdot}00005 = \log 1{\cdot}000115$, it follows that the number, when determined from a four-figure logarithm, may differ from its correct value by $\pm 0{\cdot}000115$ of the whole, or 0·0115 per cent. This does not take into account the cumulative error introduced when logarithms are added together or multiplied by some factor in the calculation of powers.

We may look at the matter in another way by regarding four-figure logarithms as a collection of 10,000 numbers from 0000 to 9999. Accuracy in working depends on choosing the correct one of these numbers, and this is facilitated by tabulating each one separately, as is done here, and giving the required data to the degree of approximation necessary to make the correct choice in every instance.

As the second half of the table does not occupy a whole page, the space is filled up by tables of the logarithms and logarithms of reciprocals of numbers from 10 to 99, as well as the logs and log reciprocals of a few of the most important constants. As there is room on the page, these are given to five places, a plan which should secure greater accuracy when powers and products are found, owing to figures carrying from the fifth to the fourth place.

Personally I have always held that logarithms should be taught before indices, and if this is done it may be necessary to defer the introduction of this table till the pupils have got fairly advanced in their work. As, however, the opposite practice commonly prevails, the use of these tables should, in most cases, facilitate matters. For this reason I have headed the tables " Antilogarithms or Powers of 10." It may be found even better to drop the somewhat clumsy name " Antilogarithm " altogether, and to call the table merely one of " Powers of 10." Whether this change is desirable is probably a point on which differences of opinion exist.

ANTILOGARITHMS.

Index or Log.	0	1	2	3	4	5	6	7	8	9	10	Differences. 1	2	3	4	5	6	7	8	9	
·00	10000	023	046	069	093	116	139	162	186	209	10233	2	5	7	9	12	14	16	19	21	·99
·01	10233	257	280	304	328	351	375	399	423	447	10471	2	5	7	10	12	14	17	19	21	·98
·02	10471	495	520	544	568	593	617	641	666	691	10715	2	5	7	10	12	15	17	20	22	·97
·03	10715	740	765	789	814	839	864	889	914	940	10965	3	5	8	10	13	15	18	20	23	·96
·04	10965	990	$\overline{015}$	$\overline{041}$	$\overline{066}$	$\overline{092}$	$\overline{117}$	$\overline{143}$	$\overline{169}$	$\overline{194}$	$\overline{11220}$	3	5	8	10	13	15	18	20	23	·95
·05	11220	246	272	298	324	350	376	402	429	455	11482	3	5	8	11	13	16	18	21	24	·94
·06	11482	508	535	561	588	614	641	668	695	722	11749	3	5	8	11	13	16	19	21	24	·93
·07	11749	776	803	830	858	885	912	940	967	995	12023	3	5	8	11	14	16	19	22	25	·92
·08	12023	050	078	106	134	162	190	218	246	274	12303	3	6	8	11	14	17	20	22	25	·91
·09	12303	331	359	388	417	445	474	503	531	560	12589	3	6	9	11	14	17	20	23	26	·90
·10	12589	618	647	677	706	735	764	794	823	853	12882	3	6	9	12	15	18	21	24	26	·89
·11	12882	912	942	972	$\overline{002}$	$\overline{032}$	$\overline{062}$	$\overline{092}$	$\overline{122}$	$\overline{152}$	$\overline{13183}$	3	6	9	12	15	18	21	24	27	·88
·12	13183	213	243	274	305	335	366	397	428	459	13490	3	6	9	12	15	18	21	25	28	·87
·13	13490	521	552	583	614	646	677	709	740	772	13804	3	6	9	13	16	19	22	25	28	·86
·14	13804	836	868	900	932	964	996	$\overline{028}$	$\overline{060}$	$\overline{093}$	$\overline{14125}$	3	6	10	13	16	19	22	26	29	·85
·15	14125	158	191	223	256	289	322	355	388	421	14454	3	7	10	13	16	20	23	26	30	·84
·16	14454	488	521	555	588	622	655	689	723	757	14791	3	7	10	13	17	20	24	27	30	·83
·17	14791	825	859	894	928	962	997	$\overline{031}$	$\overline{066}$	$\overline{101}$	$\overline{15136}$	3	7	10	14	17	21	24	28	31	·82
·18	15136	171	205	241	276	311	346	382	417	453	15488	4	7	11	14	18	21	25	28	32	·81
·19	15488	524	560	596	631	668	704	740	776	812	15849	4	7	11	14	18	22	25	29	32	·80
·20	15849	885	922	959	996	$\overline{032}$	$\overline{069}$	$\overline{106}$	$\overline{144}$	$\overline{181}$	$\overline{16218}$	4	7	11	15	18	22	26	30	33	·79
·21	16218	255	293	331	368	406	444	482	520	558	16596	4	8	11	15	19	23	26	30	34	·78
·22	16596	634	672	711	749	788	827	866	904	943	16982	4	8	12	15	19	23	27	31	35	·77
·23	16982	$\overline{022}$	$\overline{061}$	$\overline{100}$	$\overline{140}$	$\overline{179}$	$\overline{219}$	$\overline{258}$	$\overline{298}$	$\overline{338}$	$\overline{17378}$	4	8	12	16	20	24	28	32	36	·76
·24	17378	418	458	498	539	579	620	660	701	742	17783	4	8	12	16	20	24	28	32	36	·75
·25	17783	824	865	906	947	989	$\overline{030}$	$\overline{072}$	$\overline{113}$	$\overline{155}$	$\overline{18197}$	4	8	12	17	21	25	29	33	37	·74
·26	18197	239	281	323	365	408	450	493	535	578	18621	4	8	13	17	21	25	30	34	38	·73
·27	18621	664	707	750	793	836	880	923	967	$\overline{011}$	$\overline{19055}$	4	9	13	17	22	26	30	35	39	·72
·28	19055	099	143	187	231	275	320	364	409	454	19498	4	9	13	18	22	26	31	35	40	·71
·29	19498	543	588	634	679	724	770	815	861	907	19953	5	9	14	18	23	27	32	36	41	·70
·30	19953	999	$\overline{045}$	$\overline{091}$	$\overline{137}$	$\overline{184}$	$\overline{230}$	$\overline{277}$	$\overline{324}$	$\overline{370}$	$\overline{20417}$	5	9	14	19	23	28	32	37	42	·69
·31	20417	464	512	559	606	654	701	749	797	845	20893	5	10	14	19	24	29	33	38	43	·68
·32	20893	941	989	$\overline{038}$	$\overline{086}$	$\overline{135}$	$\overline{184}$	$\overline{232}$	$\overline{281}$	$\overline{330}$	$\overline{21380}$	5	10	15	19	24	29	34	39	44	·67
·33	21380	429	478	528	577	627	677	727	777	827	21878	5	10	15	20	25	30	35	40	45	·66
·34	21878	928	979	$\overline{029}$	$\overline{080}$	$\overline{131}$	$\overline{182}$	$\overline{233}$	$\overline{284}$	$\overline{336}$	$\overline{22387}$	5	10	15	29	25	31	36	41	46	·65
·35	22387	439	491	542	594	646	699	751	803	856	22909	5	10	16	21	26	31	37	42	47	·64
·36	22909	961	$\overline{014}$	$\overline{067}$	$\overline{121}$	$\overline{174}$	$\overline{227}$	$\overline{281}$	$\overline{336}$	$\overline{388}$	$\overline{23442}$	5	11	16	21	27	32	37	43	48	·63
·37	23442	496	550	605	659	714	768	823	878	933	23988	5	11	16	22	27	33	38	44	49	·62
·38	23988	$\overline{044}$	$\overline{099}$	$\overline{155}$	$\overline{210}$	$\overline{266}$	$\overline{322}$	$\overline{378}$	$\overline{434}$	$\overline{491}$	$\overline{24547}$	6	11	17	22	28	34	39	45	50	·61
·39	24547	604	660	717	774	831	889	946	$\overline{003}$	$\overline{061}$	$\overline{25119}$	6	11	17	23	29	34	40	46	51	·60
·40	25119	177	236	293	351	410	468	527	586	645	25704	6	12	18	23	29	35	41	47	53	·59
·41	25704	763	823	882	942	$\overline{002}$	$\overline{062}$	$\overline{122}$	$\overline{182}$	$\overline{242}$	$\overline{26303}$	6	12	18	24	30	36	42	48	54	·58
·42	26303	363	424	485	546	607	669	730	792	853	26915	6	12	18	24	31	37	43	49	55	·57
·43	26915	977	$\overline{040}$	$\overline{102}$	$\overline{164}$	$\overline{227}$	$\overline{290}$	$\overline{353}$	$\overline{416}$	$\overline{479}$	$\overline{27542}$	6	13	19	25	31	38	44	50	56	·56
·44	27542	606	669	733	797	861	925	990	$\overline{054}$	$\overline{119}$	$\overline{28184}$	6	13	19	26	32	39	45	51	58	·55
·45	28184	249	314	379	445	510	576	642	708	774	28840	7	13	20	26	33	39	46	52	59	·54
·46	28840	907	973	$\overline{040}$	$\overline{107}$	$\overline{174}$	$\overline{242}$	$\overline{309}$	$\overline{376}$	$\overline{444}$	$\overline{29512}$	7	13	20	27	34	40	47	54	60	·53
·47	29512	580	648	717	785	854	923	992	$\overline{061}$	$\overline{130}$	$\overline{30200}$	7	14	21	28	34	41	48	55	62	·52
·48	30200	259	339	409	479	549	620	690	761	832	30903	7	14	21	28	35	42	49	56	63	·51
·49	30903	974	$\overline{046}$	$\overline{117}$	$\overline{189}$	$\overline{261}$	$\overline{333}$	$\overline{405}$	$\overline{477}$	$\overline{550}$	$\overline{31623}$	7	14	22	29	36	43	50	58	65	·50
·50	31623	696	769	842	916	989	$\overline{063}$	$\overline{137}$	$\overline{211}$	$\overline{285}$	$\overline{32359}$	7	15	22	29	37	44	52	59	66	·49
·51	32359	434	509	584	659	735	809	885	961	$\overline{037}$	$\overline{33113}$	8	15	23	30	38	45	53	60	68	·48
·52	33113	189	266	343	420	497	574	651	729	806	33884	8	15	23	31	39	46	54	62	69	·47
·53	33884	963	$\overline{041}$	$\overline{119}$	$\overline{198}$	$\overline{277}$	$\overline{356}$	$\overline{435}$	$\overline{514}$	$\overline{594}$	$\overline{34674}$	8	16	24	32	40	47	55	63	71	·46
·54	34674	754	834	914	995	$\overline{075}$	$\overline{156}$	$\overline{237}$	$\overline{318}$	$\overline{400}$	$\overline{35481}$	8	16	24	32	40	48	56	65	73	·45
·55	35481	563	645	727	810	892	975	$\overline{058}$	$\overline{141}$	$\overline{224}$	$\overline{36308}$	8	16	25	33	41	50	58	66	74	·44
·56	36308	392	475	559	644	728	813	898	983	$\overline{068}$	$\overline{37154}$	8	17	25	34	42	51	59	68	76	·43
·57	37154	239	325	411	497	584	670	757	844	931	$\overline{38019}$	9	17	26	35	43	52	61	69	78	·42
·58	38019	107	194	282	371	459	548	637	726	815	38905	9	18	27	35	44	53	62	71	80	·41
·59	38905	994	$\overline{084}$	$\overline{174}$	$\overline{264}$	$\overline{355}$	$\overline{446}$	$\overline{537}$	$\overline{628}$	$\overline{719}$	$\overline{39811}$	9	18	27	36	45	54	63	72	82	·40
·60	39811	902	994	$\overline{087}$	$\overline{179}$	$\overline{272}$	$\overline{365}$	$\overline{458}$	$\overline{551}$	$\overline{644}$	$\overline{40738}$	9	19	28	37	46	56	65	74	83	·39
	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	Colog
												Differences.									

ANTILOGARITHMIC RECIPROCALS.

ANTILOGARITHMIC RECIPROCALS.

ANTILOGARITHMS OR POWERS OF 10.

ANTILOGARITHMS.

Index or Log.	0	1	2	3	4	5	6	7	8	9	10	Diff. 1	2	3	4	5	6	7	8	9	
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	4074	1	2	3	4	5	6	7	7	8	·39
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	4169	1	2	3	4	5	6	7	8	9	·38
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	4266	1	2	3	4	5	6	7	8	9	·37
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	4365	1	2	3	4	5	6	7	8	9	·36
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	4467	1	2	3	4	5	6	7	8	9	·35
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	4571	1	2	3	4	5	6	7	8	9	·34
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	4677	1	2	3	4	5	6	7	9	10	·33
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	4786	1	2	3	4	5	7	8	9	10	·32
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	4898	1	2	3	4	6	7	8	9	10	·31
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	5012	1	2	3	5	6	7	8	9	10	·30
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	5129	1	2	4	5	6	7	8	9	11	·29
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	5248	1	2	4	5	6	7	8	10	11	·28
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	5370	1	2	4	5	6	7	9	10	11	·27
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	5495	1	3	4	5	6	8	9	10	11	·26
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	5623	1	3	4	5	6	8	9	10	12	·25
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	5754	1	3	4	5	7	8	9	10	12	·24
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	5888	1	3	4	5	7	8	9	11	12	·23
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	6026	1	3	4	5	7	8	10	11	12	·22
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	6166	1	3	4	6	7	8	10	11	13	·21
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	6310	1	3	4	6	7	9	10	12	13	·20
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	6457	1	3	4	6	7	9	10	12	13	·19
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	6607	2	3	5	6	8	9	11	12	14	·18
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	6761	2	3	5	6	8	9	11	12	14	·17
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	6918	2	3	5	6	8	10	11	13	14	·16
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	7079	2	3	5	6	8	10	11	13	15	·15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	7244	2	3	5	7	8	19	12	13	15	·14
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	7413	2	3	5	7	9	10	12	14	15	·13
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	7586	2	3	5	7	9	10	12	14	16	·12
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	7762	2	4	5	7	9	11	12	14	16	·11
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	7943	2	4	5	7	9	11	13	15	16	·10
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	8128	2	4	6	7	9	11	13	15	17	·09
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	8318	2	4	6	8	10	11	13	15	17	·08
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	8511	2	4	6	8	10	12	14	15	17	·07
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	8710	2	4	6	8	10	12	14	16	18	·06
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	8913	2	4	6	8	10	12	14	16	18	·05
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	9120	2	4	6	8	10	12	15	17	19	·04
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	9333	2	4	6	8	11	13	15	17	19	·03
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	9550	2	4	7	9	11	13	15	17	20	·02
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	9772	2	4	7	9	11	13	16	18	20	·01
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	10000	2	5	7	9	11	14	16	18	20	·00
	10	9	8	7	6	5	4	3	2	1	0	−1	−2	−3	−4	−5	−6	−7	−8	−9	Colog
												Differences.									

ANTILOGARITHMIC RECIPROCALS.

ANTILOGARITHMIC RECIPROCALS.

LOGARITHMS 10 TO 99.

	0	1	2	3	4	5	6	7	8	9
1	00000	04139	07918	11394	14613	17609	20412	23045	25527	27875
2	30103	32222	34242	36173	38021	39794	41497	43136	44716	46240
3	47712	49136	50515	51851	53148	54407	55630	56820	57978	59106
4	60206	61278	62325	63347	64345	65321	66276	67210	68124	69020
5	69897	70757	71600	72428	73239	74036	74819	75587	76343	77085
6	77815	78533	79239	79934	80618	81291	81954	82607	83251	83885
7	84510	85126	85733	86332	86923	87506	88081	88649	89209	89763
8	90309	90848	91381	91908	92428	92942	93450	93952	94448	94939
9	95424	95904	96379	96848	97313	97772	98227	98677	99123	99564

N	Log N
π	0·49715
$180/\pi$	1·75812
e	0·43429
$\text{Log}_{10}e$	$\bar{1}$·63778
Water d. 62·3	1·79449
$g=32$ 2	1·50786
$g=981$	2·99167
Speed 88/60	0·16633
Ton 2240	3·35025
Mile 5280	3·72263

LOGARITHMS OF RECIPROCALS.

	0	1	2	3	4	5	6	7	8	9
·1	100000	95861	92082	88606	85387	82391	79588	76955	74473	72125
·2	69897	67778	65758	63827	61979	60206	58503	56864	55284	53760
·3	52288	50864	49485	48149	46852	45593	44370	43180	42022	40894
·4	39794	38722	37675	36653	35655	34679	33724	32790	31876	30980
·5	30103	29243	28400	27572	26761	25964	25181	24413	23657	22915
·6	22185	21467	20761	20066	19382	18709	18045	17393	16749	16115
·7	15490	14874	14267	13668	13077	12494	11919	11351	10791	10237
·8	09691	09152	08619	08092	07572	07058	06550	06048	05552	05061
·9	04576	04096	03621	03152	02687	02228	01773	01323	00877	00436

N	Log 1/N
π	$\bar{1}$·50285
$180/\pi$	$\bar{2}$·24188
e	$\bar{1}$·56571
$\text{Log}_{10}e$	0·36222
Water d. 62·3	$\bar{2}$·20551
$g=32·2$	$\bar{2}$·49214
$g=981$	$\bar{3}$·00833
Speed 88/60	$\bar{1}$·83367
Ton 2240	$\bar{4}$·64975
Mile 5280	$\bar{4}$·27737