

A COMMUNICATION CONCERNING THE TYPE $px^2 + qx + r$.

Essex High School, Essex, Ontario, May 27, '07.

Editors of SCHOOL SCIENCE AND MATHEMATICS,

440 Kenwood Terrace, Chicago.

Dear Sirs:

The question of precedence of the method of factoring the type $px^2 + qx + r$, as given in the April number of SCHOOL SCIENCE AND MATHEMATICS, has been raised by Prof. Smith in the June number. Whether or not this method was first published in the United States in 1900 as stated, it certainly was published some 14 years earlier in America. In "The Elements of Algebra," by J. A. McLellan, and published in Toronto, 1886, will be found on page 110, article 106, the identical method so well demonstrated by Mr. Toan in the April number.

The second method illustrated by Prof. Smith, or one very similar, is, I believe, quite generally taught to 3rd and 4th year High School students throughout Ontario.

The solution of the quadratic equation is taught about as follows:

$$\begin{aligned} px^2 + qx + r &= 0 \\ \text{or } x^2 + \frac{q}{p}x + \frac{r}{p} &= 0 \\ \text{or } \left(x + \frac{q}{2p}\right)^2 - \left(\frac{q^2 - 4pr}{4p^2}\right) &= 0 \\ \text{or } \left(x + \frac{q}{2p} + \frac{\sqrt{q^2 - 4pr}}{2p}\right)\left(x + \frac{q}{2p} - \frac{\sqrt{q^2 - 4pr}}{2p}\right) &= 0 \\ \text{or } x = -\frac{q}{2p} \pm \frac{\sqrt{q^2 - 4pr}}{2p} \end{aligned}$$

In the process of this solution the factors of the quadratic expression have been obtained, and when a formula becomes desirable the one will answer a dual purpose.

Thus to factor $6x^2 + 11x - 10$ the student writes first the roots of $6x^2 + 11x - 10 = 0$

$$\text{i. e. } x = -\frac{11}{12} + \frac{\sqrt{121 + 240}}{12} \text{ and } \therefore x + \frac{11}{12} - \frac{19}{12} = 0$$

$$\text{or } x = -\frac{11}{12} - \frac{\sqrt{121 + 240}}{12} \text{ and } \therefore x + \frac{11}{12} + \frac{19}{12} = 0$$

Hence factors required must be

$$6\left(x + \frac{11-19}{12}\right)\left(x + \frac{11+19}{12}\right) = 6\left(x - \frac{2}{3}\right)\left(x + \frac{5}{2}\right) = (3x - 2)(2x + 5)$$

Of course, multiplication by $4p$ may be used instead of division by p ; but, having used both, I prefer the latter. Again referring to Mr. Toan's method of factoring $12x^2 - 11x - 5$ compare this:

$$\begin{array}{ccc} 12 & 6 & (4) \\ 1 & 2 & (3) \end{array} \qquad \begin{array}{cc} (5) & 1 \\ (1) & 5 \end{array}$$

$$-15 + 4 = -11$$

$$\therefore 12x^2 - 11x - 5 = 4x - 5 (3x + 1)$$

The great majority of such examples are solved readily by the average student by trial. The above work indicates the method by trial reduced to a science. Write in order all the possible pairs of factors of the end coefficients repeating one set reversed i. e. $\frac{5}{4}$ and $\frac{1}{2}$. Starting with $\frac{1}{2}$ try cross products first with $\frac{5}{4}$ then with $\frac{1}{4}$ looking for a difference of 11. This failing $\frac{1}{2}$ may be crossed out. Proceed in this manner until the coefficients of the factors are found. If every pair fails then the expression is not factorable.

Once more experience in teaching both Mr. Toan's method and this makes me use the latter almost exclusively.

Yours truly,

R. W. ANGLIN, Prin.

A COMMUNICATION.

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Prof. Ira M. DeLong, University of Colorado, Boulder, Colo.

My Dear Sir:—I was very much interested in the solution of the Frustum Theorem given by Dr. Blakslee in the December, 1906, number of *SCHOOL SCIENCE AND MATHEMATICS*, especially since I have for some time used a similar proof with my classes in Solid Geometry. I enclose my solution.

Yours very respectfully,

NELSON L. RORAY.

Lemma:—In a series of equal ratios the sum of the mean proportionals of each antecedent to its consequent is equal to the mean proportional of the sum of the antecedents to the sum of the consequents.

That is, if

$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'} = \text{etc.}$$

then does

$$\sqrt{aa'} + \sqrt{bb'} + \sqrt{cc'} + \dots = \sqrt{(a+b+c+\dots)(a'+b'+c'+\dots)}$$

Proof:

$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'} = \dots$$

$$\frac{aa'}{a'^2} = \frac{bb'}{b'^2} = \frac{cc'}{c'^2} = \dots$$

$$\frac{\sqrt{aa'}}{a'} = \frac{\sqrt{bb'}}{b'} = \frac{\sqrt{cc'}}{c'} = \dots$$

$$\frac{\sqrt{aa'} + \sqrt{bb'} + \sqrt{cc'} + \dots}{a' + b' + c' + \dots} = \frac{\sqrt{aa'}}{a'}$$