5. On the Centroid of a Trapezoid
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By symmetry the remaining group of three follows.

The properties of the polar triangle form a corollary to the above equations. G. HEPPEL.

4. Geometrical determination of K in the trilinear relation $\alpha\beta = K\gamma^2$.

TQ, TQ′ (Fig. 11) are two tangents to a conic, P any point on the curve. Through P and the centre O are drawn lines PG, PH, EPF, and Oa, Ob, Oc parallel to TQ, TQ′, QQ′ respectively. PL, PM, PN are perpendicular to TQ, TQ′, QQ′.

Then, since the diameter OT bisects QQ′, PP′, and EF, EP′ = PF.

$$\therefore\ Oc^2 : Oa^2 = EP.EP' : EQ^2,$$
$$= EP.PF : EQ^2.$$

Since the right-angled triangles PEL, PGN are similar,

$$\therefore\ PE : PG(=EQ) = PL : PN.$$

Similarly, $$PF : PH(=FQ') = PM : PN,$$
$$\therefore\ PE.PF : EQ.FQ' = PL.PM : PN^2,$$

and $$EQ : FQ' = PG : PH = TQ : TQ' = Oa : Ob,$$

$$\therefore\ \frac{PL.PM}{PN^2} = \frac{PE.PF}{EQ.FQ'} = \frac{PE.PF}{EQ^2}\cdot\frac{EQ}{FQ'} = \frac{Oc^2}{Oa^2}\cdot\frac{Oa}{Ob},$$
$$= \frac{Oc^2}{Oa.Ob}.$$

It can easily be deduced from the above that in the trilinear relation—

$$\alpha\beta = K\gamma\delta,$$

if Oa, Ob, Oc, Od are the semi-diameters parallel to the four lines to which α, β, γ, δ are respectively perpendicular,

$$K = \frac{Oc.Od}{Oa.Ob}.$$ J. J. MILNE.

5. On the centroid of a trapezoid.

Let ABCD (Fig. 12) be a trapezoid whose parallel sides AB, DC contain p and q units of length respectively. Bisect AB, DC at E, F, and let BD, EF cut at H.

By VI. 1 triangle ABD : triangle BDC : : AB : DC,

$$: : p : q,$$
$$: : 3p : 3q.$$

The centroid of triangle ABD is that of masses $2p$ at E and p at D. The centroid of triangle BDC is that of masses $2q$ at F and q at B.

Now $$EH : HF : : BH : HD,$$
$$: : BE : DF,$$
$$: : p : q.$$

Hence masses p at D and q at B are equivalent to a mass $p+q$ at H, and therefore to masses p at F and q at E.

Hence the system has the centroid of masses $2p+q$ at E and $p+2q$ at F.

$\therefore$ it is at G on EF where $(2p+q)EG = (p+2q)FG$.

If we denote the centroids of triangles ABD, BDC by L and M, and adopt the notation of Möbius, the latter part of the demonstration would run thus—

$$(3p+3q)G = 3pL + 3qM,$$
$$= 2pE + pD + 2qF + qB,$$
$$= 2pE + 2qF + (p+q)H,$$
$$= 2pE + 2qF + qE + pF,$$
$$= (2p+q)E + (2q+p)F.$$

E. M. LANGLEY.

6. On a proof of XI. 4.

Euclid's own demonstration, being rather long, is now usually superseded either by the demonstration which Legendre gives in his *Elements of Geometry*, Bk. V. prop. 4, or by a third demonstration in which the perpendicular is produced to the other side of the plane. In Wilson's *Solid Geometry* this third method of proof is ascribed (erroneously) to Legendre, and in my own edition of Euclid to A. L. Crelle. The latter certainly gives it, without any hint of its authorship, in an article (dated 1834) in his *Journal*, vol. xlv. pp. 35, 36 (1853). He had however published it, along with the proofs of Euclid and Legendre, in his *Lehrbuch der Elemente der Geometrie*, vol. ii. p. 532 (1827), and added the remark: "This proof is by Cauchy."

The correctness of this ascription to Cauchy is confirmed by Lacroix, who gives the proof in his *Elements of Geometry*, § 196 (12th edition, 1822), with the note:

"This demonstration, of the same kind as that of Euclid but simpler, has been communicated to me by Mr. Cauchy, a very distinguished young geometer." J. S. MACKAY.

APPROXIMATIONS AND REDUCTIONS.

(Continued from No. 1, which contains fully-worked numerical examples of the application of the methods of "Practice" to such multipliers as those given below.)

"It is an abiding delusion of the opponent of decimals that he *will* suppose the decimalist to be under a contract never to use a common fraction."—*De Morgan.*

7. Degrees to radians ·01745329

$\frac{1}{60}(1+\frac{1}{20})(1-\frac{1}{400}-\frac{1}{6000}) = {}$·01745333.

8. Minutes to radians ·000290888

(i) ·00029$(1+\frac{3}{1000})$ = ·00029087,

(ii) ·0006$(\frac{1}{2}-\frac{1}{70}-\frac{1}{1100})$ = ·000290883.

9. Seconds to radians ·00000484814

·00001$(\frac{1}{2}-\frac{1}{70}-\frac{1}{1100})$ = ·00000484805.

10. Revolutions p. min. to radians p. sec. ·104719755

$\frac{1}{10}(1+\frac{1}{20})(1-\frac{1}{400}-\frac{1}{6000}) = {}$·10472.

11. Radians p. sec. to revolutions p. min. 9·5493

$10(1-\frac{3}{100}-\frac{3}{200}) = 10(1-\frac{1}{20}+\frac{1}{200}) = 9{\cdot}55$,

i.e. multiply by 10 and subtract $4\frac{1}{2}$ per cent.

12. Miles to kilometres 1·60933

$2(1-\frac{1}{5})(1+\frac{1}{200}+\frac{1}{1400})$ = 1·609$\frac{3}{7}$,

or $\frac{10}{6}(1-\frac{1}{30}-\frac{1}{1000})$ = 1·609$\frac{4}{9}$.

13. Kilometres to miles ·62138

(i) $(\frac{1}{2}+\frac{1}{8})(1-\frac{1}{200}-\frac{1}{1400})$ = ·6214$\frac{2}{7}$,

(ii) ·6 + $\frac{1}{70}+\frac{1}{140}$ = ·6214$\frac{2}{7}$.

Note that, approximately, 5 miles = 8 kilometres, or a mile and a quarter (*i.e.* 10 furlongs) = 2 kilometres, so that if our mile were increased by a quarter of its present length, the series *miles*, *furlongs*, *chains*, and *links* would