

During the winter of 1889-90, seventy cars were put in use and performed satisfactory service.

There are a number of constructive details of the system described which can undoubtedly be simplified, as more experience is gained from actual operation.

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ON THE FORCE OF IMPACT OF WAVES AND THE  
STABILITY OF THE SUPERSTRUCTURE OF  
BREAKWATERS.

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Great difficulty has always been experienced in determining the safe limits of construction for marine works subjected to the action of breaking waves, on account of the ignorance prevailing in regard to the magnitude of the force of impact of such waves. One of the greatest marine engineers, Smeaton, we are told, used to refer to such force as one being *subject to no calculation*; while, others, like Sir Samuel Brown, Captain Taylor and Minard, estimated it as being equivalent to a pressure from seventy pounds to 144 pounds per square foot. These estimates, however, were shown to be a mere bagatelle by the results obtained with marine dynamometers, but even these results fail to account for the destructive power of breaking waves, for no one would at present dare to proportion the dimensions of the superstructure of a breakwater to the observed pressure reported by the dynamometer.

From this brief sketch of the status of our knowledge regarding the force of impact of breaking waves, it may be readily appreciated how important it is to determine the highest limit to which such force, under favorable circumstances, may reach, by an exact method based upon a well-grounded scientific principle, and this is what has been aimed at in the following pages.

The amount of energy which is necessary to cause the propagation of a wave is equal to the energy which such

wave would require if it had to be moved *en masse* with its velocity of propagation upon a frictionless level-surface.

Denoting by  $s$  the full length of the wave; by  $k$  its height measured from crest to trough; by  $\gamma$  the weight of one cubic foot of sea-water; by  $g$  the acceleration of gravity, and by  $v$  the velocity of propagation, the amount of energy represented by one linear foot of such wave can be expressed approximately by

$$E = \frac{s k \gamma v^2}{4g}$$

This energy, when the wave breaks under favorable conditions, may be wholly discharged against every linear foot of the superstructure of a breakwater during the interval of time ( $s/v$ ) which the wave requires to propagate its own length. Therefore, if  $f$  represents the mean force of impact of the wave per linear foot of superstructure during such time, we can write the equation

$$E = f v \times \frac{s}{v};$$

which gives

$$E = \frac{\gamma k v^2}{4g}$$

Since we know that in the beginning the force of impact is zero, we may express the maximum force of impact by twice the mean; or,

$$F = \frac{\gamma k v^2}{2g} \quad (1)$$

This force may be considerably reduced when the impact is oblique, but the obliquity of the impact cannot be always relied upon. In fact, quoting from Harcourt, Harbours and Docks, we have the following opinion in regard to the obliquity of the impact:

"Though the angle of incidence as to exposure may be oblique, in practice this obliquity must always be more or less modified by the tendency for the waves to finally fall upon the shore on lines parallel with the coast line."

Therefore, it would not be safe to discount from the

value of  $F$  given by (1) in proportioning the dimensions of the cross-section of the superstructure of a breakwater, even when the impact may seem to be oblique.

Now, let  $H$  represent the height of the low-water line above the bottom plane of the superstructure;  $R$ , the range of tide;  $h$ , the elevation of the tops of the superstructure above the plane of high water, and  $x$ , the width of the cross-section, supposed rectangular. The height of the centre of impact above the bottom plane of the superstructure is a maximum at high water, and may be safely put equal to  $(H + R)$ . Denoting, then, by  $\gamma$ , the weight of one cubic foot of superstructure, the buoyancy being taken into account, we can form the equation of moments

$$\frac{1}{2} x^2 \gamma_1 (H + R + h) = \frac{1}{2} \frac{\gamma k v^2}{g} (H + R);$$

from which we derive

$$x = v \sqrt{\frac{\gamma k (H + R)}{\gamma_1 g (H + R + h)}} \quad (2)$$

This gives the width of the cross-section of the superstructure to resist overturning.

If we denote by  $\mu$  the coefficient of friction of the superstructure upon its bed, the condition to prevent sliding will be

$$x \gamma_1 (H + R + h) \mu = \frac{\gamma k v^2}{2g};$$

from which we derive another value of  $x$  which, to avoid confusion, will be indicated by

$$x_1 = \frac{\gamma k v^2}{2 \gamma_1 g \mu (H + R + h)} \quad (3)$$

From the experiments on friction of large stones or blocks made by Thomas Stevenson, F.R.S.E., F.G.S., the value of  $\mu$  varies from 0.65 to 0.79. But taking into account the tremor of the superstructure caused by wind and waves, it will be safer to assume  $\mu = 0.5$ , and write

$$x_1 = \frac{\gamma k v^2}{\gamma_1 g (H + R + h)} \quad (4)$$

Hence, we have the following simple relation

$$x_1 = \frac{x^2}{H + R} \quad (5)$$

in which  $x$  is the value offered by (2).

This relation shows that the same width of cross-section, which is sufficient to balance overturning, may not be sufficient to prevent sliding, or *vice-versa*. When we find  $x_1 < x$ , it is plain that the safe value is  $x$ ; and when we find  $x_1 > x$ , it is equally plain that the safe value is  $x_1$ . By pursuing this practice, we provide against sliding and against overturning at the same time.

Of course, a coefficient for safety ought to be applied in determining the values of  $x$  and  $x_1$ , but this we leave to the discretion of the engineer, and shall now proceed to a test of our formulæ.

In the year 1883, the superstructure of the breakwater built in the Oswego Harbor, N. Y., was set back by the force of the waves during a gale to the extent of three feet at one particular point. The superstructure consists of cribs whose net weight per linear foot is estimated to be about 65,000 pounds. The elevation of the superstructure above the level of the water is about twelve feet, which we have to use instead of  $h$ , since the waves were much higher than twelve feet. The velocity of propagation of the waves was observed to be between thirty and forty miles per hour, or between forty-four and fifty-nine feet per second. Taking the higher limit, our equation (1) furnishes

$$F = 40,500 \text{ pounds,}$$

and consequently the coefficient of friction is found to be  $\mu = 0.62$ , which is almost identical with the lowest value of  $\mu$  determined by Stevenson's experiments.

With such close agreement there cannot be any doubt left as to the soundness of the formulæ given above and of the principles involved.