On the Proportions of Ships of least Skin Resistance for a given Speed and Displacement. By Prof. RANKINE.

From the London Athenæum, Sept., 1863.

The author referred to a previous paper which he had read to the British Association in 1861, and in which he had stated the results of a theoretical investigation of the "skin-resistance" of ships, and verified those results by a comparison with those of experiments. In the course of that paper he had stated, that the theory gives, for the proportion of length to breadth which produces least skin-resistance with a given displacement and speed, that of *seven* to *one*, nearly. This is the case when the figures and proportions of the cross-sections are given, so that the draft of water bears a fixed proportion to the breadth. But, when the draft of water has a fixed absolute value, the theory gives a somewhat different result; for the proportion of length to breadth which produces the least skin-resistance is found to increase as the draft of water becomes shallower.

Proceedings British Association.

For the Journal of the Franklin Institute.

On the Integral of $\frac{dz^2}{dx^2}=\frac{8w}{R}z^{\frac{3}{2}}+C$.

By DE VOLSON WOOD. Prof. of Civ. Eng. Univ. of Mich.

In an article on "*Beams of Uniform Strength,*" published in Vol. xlii, third series, page 110 of this *Journal*, will be found the equation at the head of this article in which C is the constant of integration. I raised the query whether it could be integrated in finite terms. Upon a recent examination of it, I find that it may be integrated by means of the higher transcendentals, in the following way:

We have $$dx=\frac{dz}{\sqrt{C+\frac{8w}{R}z^{\frac{3}{2}}}}=\sqrt{\frac{R}{8w}}\ \frac{dz}{\sqrt{C_1^2+z^{\frac{3}{2}}}}\quad . \quad . \quad (1)$$

where $C_1^2=\frac{RC}{8w}$.

Let $z^{\frac{3}{2}}=-u^2$ and the equation becomes

$$dx=-\frac{2}{3C_1}\sqrt{\frac{R}{8w}}\ \frac{u^{\frac{1}{3}}\,du}{\sqrt{1-\frac{u^2}{C_1^2}}}\quad . \quad . \quad . \quad (2)$$

Now let $$\frac{u}{C_1}=\left(1-y^2\right)^{\frac{3}{2}}\quad \therefore\quad u^{\frac{1}{3}}=C_1^{\frac{1}{3}}\left(1-y^2\right)^{\frac{1}{2}}$$

$$du=\tfrac{3}{2}(1-y^2)^{\frac{1}{2}}dy$$

(This y is not the depth of the beam as used in the problem.)

These substituted in the preceding equation gives

$$dx = -C^{\frac{1}{4}} \frac{\sqrt{R}}{2w} \frac{1-y^2}{\sqrt{3-3y^2+y^4}} dy \quad . \quad . \quad (3)$$

As the solution of this, in finite form, involves the use of *elliptic functions*—a branch of mathematics extensively studied by comparatively few students—I shall only indicate the steps.

Equation (3) is in a convenient form to put in ordinary form for elliptic functions. See *Le Gendre's Elliptique Fonctions*, page 9, 6th paragraph. The roots of the denominator are all imaginary; hence it comes under the first case. This case in the form of Equation (3) is given on page 55 of Le Gendre.

The form is

$$x = \int \frac{(f+gy^2)\,dy}{\sqrt{a^2+2aby^2\cos.\theta+b^2y^4}} \quad . \quad . \quad . \quad . \quad (4)$$

The integral of which is given on page 56, and is

$$= \frac{fb+ga}{b\sqrt{ab}} \mathrm{F}_{(c,\varphi)} - \frac{2ga}{b\sqrt{ab}} \mathrm{E}_{(c,\varphi)} \quad . \quad . \quad . \quad (5)$$

Equation (3) compared with (5) gives

$$f=1 \qquad a^2=3 \qquad \cos.\theta = -\tfrac{1}{2}\sqrt{3}$$

$$g=-1 \qquad b^2=1 \qquad \therefore \qquad \theta=120°,$$

which substituted in (5) gives

$$x = -\left(\frac{RC}{8w}\right)^{\frac{1}{6}} \left(\frac{R}{2w}\right)^{\frac{1}{2}} \left[\frac{1-\sqrt{3}}{\sqrt[4]{3}} \mathrm{F}_{(c,\varphi)} + 2\sqrt[4]{3}\, \mathrm{E}_{(c,\varphi)}\right] \quad (6)$$

We also have $c = \sin.\tfrac{1}{2}\varphi$.

$$\cos.^2\varphi = \frac{-by^2 - a\cos.\theta + \sqrt{a^2+2aby^2\cos.\theta+b^2y^4}}{2a\sin.\frac{1}{2}\theta} \quad . \quad . \quad (7)$$

$$\therefore \quad \varphi = 0 \text{ for } y = 0$$

Equation (6) gives $x=0$ for $\varphi=0$ as it showed, since x and y are both zero at the origin.

To construct the curve we may assume y, and by the relations given above, find u and $z^{\frac{1}{3}}$, which last value will be the ordinate. To find the corresponding abscissa, substitute the assumed value of y in equation (7) and find φ, and then Table IX, Vol. II, Le Gendre, will give $\mathrm{F}_{(c,\varphi)}$ and $\mathrm{E}_{(c,\varphi)}$ which in equation (6) will give φ.

It is difficult to find C, the first constant of integration, because its value is contained in a transcendental function.

This solution induces me to make a remark upon the integration of expressions.

We know that the cases which will admit of integration in finite form are comparatively few. Any known function may be differentiated (or disintegrated), and the result is *typical* of the infinitessimal-elements which result from the function.

But it is not *generally* possible to pass from the differential to the integral, because we do not know the *form* of the function which will give the *form* of the given differential. It is generally necessary to have the function (and this is known first) and the typical infinitessimal-element before we can deduce rules for passing from the latter to the former. Thus we know how to integrate sin. $x\,dx$, because we know the form of the function from which it is derived, viz: — cos. x; but we cannot integrate log. $x\,dx$ (in finite form), because we do not know the form of the function from which it is derived. To integrate it we must have some transcendental function of higher order, which, in the course of time, may possibly become known.

But all transcendental functions do not require transcendentals of a higher order. Thus the integral of the trigonometrical functions, sin. $x\,dx$, tan. $x\,dx$, sec. $x\,dx$, &c., are expressed in other trigonometrical functions. Still it is more generally the case that integration raises a function to a higher transcendental, and the integral of this higher one demands still higher.

Thus $\int \frac{dx}{\sqrt{1-x^2}} = \sin.^{-1} x$; but $\sin.^{-1} x\,dx$ is not integrable.

$\int \frac{\cos. x\,dx}{\sin. x} = \log._e \sin. x$, but $\log._e \sin. x\,dx$ is not integrable.

$\int \frac{2\,dx}{x^3} = \frac{-1}{x^2}$, and $\int \frac{-dx}{x^2} = \frac{1}{x}$, and $\int \frac{dx}{x} = \log. x$, but log. $x\,dx$ is not integrable.

It is sometimes the case that algebraic expressions may be transformed so that their integral can be expressed in known transcedentals. Such is the case with the expression which forms the subject of this article. All differential expressions may be regarded as being derived from some function.

Hence, from this brief view of the subject, we infer, that if we knew all possible forms of transcendentals, we might integrate all differential expressions.

On a Mercurial Air Pump. By Mr. J Swan.

From the London Athenæum, Sept., 1863.

In general arrangement and appearance this instrument resembles a barometer, with very large lower reservoir, having an inlet and outlet pipe at the top of this, each provided with a stop-cock; and with the upper part of the barometer tube very greatly enlarged—in fact, a reservoir at the top, and a reservoir at the bottom. The upper reservoir, termed a vacuum-chamber, is surmounted by a ball-valve opening outward, and has also a tube with a stop-cock communicating with the vessel to be exhausted. The vacuum-chamber, tube, and a portion of the lower reservoir are, in the normal condition of the apparatus, to to be occupied by mercury. The remaining space within the lower