

Shuffling a Deck of Cards – A Process Theoretically Reversible

Marek Berezowski¹⁾

Politechnika Krakowska, Kraków, Poland

Abstract

Shuffling a standard 52-card deck may seem trivial, but from the perspective of mathematics and physics, it is a fascinating example of a dynamical system with an enormous number of possible states. This analysis demonstrates that the shuffling process is theoretically reversible, and the probability of a spontaneous decrease in entropy is not zero, although it is extremely small. In this paper, we discuss deck permutations, system entropy, and the probability of returning to an ordered state, using principles from nonlinear dynamics [1–4].

Keywords

deck of cards, entropy

1. Introduction

Consider a standard 52-card deck. Shuffling can be treated as an **isolated system** – the deck does not exchange cards with the environment. The total number of possible arrangements is:

$$N = 52! \approx 8,0658 * 10^{67} \text{ (1).}$$

If we assume symmetry with respect to reversal (i.e., the left-to-right arrangement is considered identical to its mirror image), the number of unique permutations reduces to:

$$N_{sym} = \frac{52!}{2} \approx 4.0329 * 10^{67} \text{ (2).}$$

Thus, shuffling the deck is an example of a system with an enormous number of states, where a decrease in entropy – returning to an ordered arrangement – is theoretically possible,

¹⁾ Corresponding author: Marek Berezowski, e-mail: marek.berezowski@pk.edu.pl

but practically extremely unlikely [1,3].

2. System Entropy

The information entropy of a perfectly shuffled deck can be expressed as [2]:

$$S = \lg_2(N_{sym})=226.6 \text{ bitów (3).}$$

For comparison, initially, when the deck is arranged sequentially (e.g., by suit and rank), entropy is $S=0$. This indicates that entropy increases from an ordered state to a maximum level of disorder as shuffling progresses.

3. Shuffling as Nonlinear Dynamics

Each shuffle can be considered a reversible **permutation transformation** – mathematically deterministic, but practically random [4].

3.1 Dynamics Theory

Shuffling a deck meets the conditions of dynamics:

- finite state space (N_{sym}) [1],
- deterministic process in a mathematical model [4],
- practical randomness [3].

Returning the deck to an ordered state is theoretically possible, but the probability is extremely low:

$$p_{return} = \frac{1}{52!} \approx \frac{1}{8} * 10^{-67} \text{ (4),}$$

4. Analogy to Thermodynamics

Similar to a gas in a box (Penrose, 1979) [1], as the deck is shuffled, entropy rises and the system becomes increasingly disordered. Each subsequent shuffle maintains entropy at a maximum level – or in very rare cases may slightly decrease it.

4.1 Expected Time to Return to Ordered State

The expected time for the deck to return to its initial state can be estimated from the distribution:

$$N_{sym} = \frac{52!}{2} \approx 4.033 * 10^{67} = t_{approx} \text{ (5)},$$

where t_{approx} is the average return time for random shuffling:

1) Human

Assuming one shuffle per second:

$$t_{human} \approx 4.033 * 10^{67} \text{ seconds} = 1.28 * 10^{60} \text{ years (6)}$$

2) Classical Supercomputer

- Assuming 10^{12} shuffles per second:

$$t_{computer} = 1.28 * 10^{48} \text{ years (7)},$$

3) Quantum Computer

- Hypothetical quantum computer processing 10^{30} arrangements per second:

$$t_{qcomputer} = 1.28 * 10^{30} \text{ years (8)}.$$

For comparison, the age of the Universe is estimated at $\sim 1.4 * 10^{10}$ years. Even with extreme acceleration, returning to the initial state is practically impossible.

5. Conclusions

- Shuffling a 52-card deck is a simple yet ideal example of a dynamics system, where entropy rises to a maximum [4].
 - The deck's entropy remains relatively small compared to the number of possible configurations.
 - The example shows that a decrease in entropy is theoretically possible, though practically extremely unlikely [3].
 - This analysis allows understanding of the principles of statistics, thermodynamics, and dynamics in systems [1,4].
 - The theoretical return to the initial state exists, but in practice it is unrealistic, illustrating an analogy to thermodynamic laws in the Universe [1,3]. And that's why, for example, a broken glass won't put itself back together, because it's unimaginably unlikely [5].
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Data Availability Statement

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

6. References

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