

THURSDAY, FEBRUARY 9, 1893.

THE MILKY WAY.

The Milky Way from the North Pole to 10° of South Declination, drawn at the Earl of Rosse's Observatory at Birr Castle. By Otto Boeddicker. (London: Longmans, Green, and Co., 1892.)

DR. OTTO BOEDDICKER devoted the clear moonless nights for five years, from October 1884 to October 1889, to delineating the Milky Way as it appeared to his unaided eyes at Parsonstown, Ireland. His drawings were deposited in the library of the Royal Astronomical Society, and a note accompanying them was read at the meeting of the Society in November 1889. The work now before us consists of four excellent lithographic reproductions of these drawings, a brief introductory preface being added by Dr. Boeddicker.

The working maps for the drawings were taken from Argelander's "*Uranometria Nova*," the Milky Way being inserted by means of stump and lead pencil. This medium was found very unsuitable for photographic reproduction, and in preparing the lithographic stones for these charts photography was used for the stars, and the Milky Way was introduced by hand work. Mr. W. H. Wesley, the Assistant Secretary of the Royal Astronomical Society, is responsible for this latter portion of the work, and the results are a splendid testimony to his care and skill. Dr. Boeddicker is to be congratulated upon having secured the services of so excellent an artist.

Plate I. is a detailed drawing of the Section Cygnus-Scutum, Plate II. of the Section Cassiopeiæ, and Plate III. of the Section Aurigæ-Gemini-Monocerotis. In these plates an attempt has been made to represent accurately the appearance of the galaxy, all the differences of luminosity being represented as they actually appeared to Dr. Boeddicker. In Plate IV. a general view on a smaller scale of the whole Milky Way from the North Pole to 10° south declination is given, the contrast being deliberately exaggerated in order to bring out clearly all the details.

The area of the Milky Way indicated on these drawings is very much greater than that on any previously published representations, while for delicate details and faithful reproduction of contrast the plates are unapproached. In many respects Dr. Boeddicker's drawings are a new revelation, branches, wisps, and feelers being shown extending from the main body so as to include stars, clusters and nebulae, and even whole constellations not previously recognized as connected with or forming part of the Milky Way. Polaris, γ Arietis, Præsepe, the Pleiades, the Hyades, the great nebula in Andromeda, and nearly the whole of the constellation Orion, are thus joined to the galactic circle. Numerous bright patches, channels, rifts, and interlacing lines of luminous matter hitherto unsuspected are revealed by Dr. Boeddicker's long and patient work, and exponents of disc, spiral, and other theories as to the construction of the Milky Way will find considerable difficulty in accounting for the details shown.

It is very difficult to compare drawings of the Milky Way made by different observers without optical aid.

There are such wide variations in unaided vision, so many peculiarities introduced by long and short sight, by astigmatism, by irradiation in the retina, and by other physical and physiological imperfections, that it may safely be asserted that no two persons get exactly the same naked-eye impression of such a vague object as the Milky Way. As no details are given about Dr. Boeddicker's eyes we are probably justified in inferring that they are practically normal, but we doubt whether any other observer, even with special training, could check or correct these charts with reasonable prospect of convincing the original artist of error in the representation of the Milky Way as it appeared to him. Individual peculiarities of sight are minimized by the use of slight optical aid, and two equally experienced observers would be more likely to agree in their delineations of the Milky Way if they used similar telescopes, of say 1-inch aperture, or even ordinary opera glasses. Dr. Boeddicker's appeal to other observers to "verify and correct" his work will probably bring him plenty of correspondence, but can scarcely lead to any important correction in his magnificent drawings.

Dr. Boeddicker considers that "the first step necessary towards the knowledge of the sidereal universe is a thorough acquaintance with the Milky Way as it appears to the naked eye," and hopes that by comparison and the superposition of naked-eye drawings on photographs "some knowledge of the structure of the Milky Way *in the line of sight* may be obtained." This idea is founded on the theory that there is a direct connection between the magnitudes of stars and their distances. Littrow's analysis of Argelander's catalogue of stars certainly seemed to justify belief in this connection, but recent work has entirely disproved the hypothesis. Measurements of the parallax of stars indubitably prove that some faint stars are near, while some of the brightest are at such distances as to have no appreciable parallax. Thus α Orionis, α Virginis, α Leonis, and α Cygni have no parallax, while the 5th magnitude star 61 Cygni has a parallax of between $0''.4$ and $0''.5$. Photographs of the Pleiades show that we have in that cluster stars differing by as much as 13 magnitudes at approximately the same distance from us. Russell's photograph of α Crucis plainly indicates a direct physical connection between that star and many stars of the 14th and 15th magnitudes which should, according to the theory, be nearly 1000 times more distant. Streaks of nebulae connect α Cygni and γ Cygni with long lines and stars of about the 16th magnitude in Dr. Max Wolf's photographs of the Milky Way. From considerations of parallax observations of stars and from examination of photographs we are forced to conclude that there is no real connection between magnitude and distance, and that the differences of magnitude of stars are due to differences of size and physical condition. Stars differ enormously in light-giving power, and the actual light emitted by α Cygni must be nearly a million times greater than that from the faint stars directly connected with it and at practically the same distance from us. There is therefore very little chance of adding to our knowledge of the Milky Way "in the line of sight" by superposition of naked-eye drawings on photographs.

In his preface Dr. Boeddicker frequently speaks of

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"nebulousness," "nebulous light," and "nebulous matter," when he means luminosity and luminous matter. In ante-spectroscopic days the terms nebula and cluster were used almost indiscriminately, a nebula being looked upon as simply an irresolvable cluster, and this error still survives in many astronomical text-books and compilations, but Dr. Boeddicker should have avoided it. When we consider that the majority of the stars in the cluster which we call the Milky Way are of the Sirian type, we see how misleading is the use of the terms nebulous light and nebulous matter. A. T.

THE THEORY OF SUBSTITUTIONS AND ITS APPLICATIONS TO ALGEBRA.

The Theory of Substitutions and its Applications to Algebra. By Dr. Eugen Netto. Translated by F. N. Cole, Ph.D. (Mich.: Ann Arbor, 1892.)

THE theory of substitutions abstractly considered is concerned with the enumeration and classification of the permutations of a set of n different letters x_1, x_2, \dots, x_n . It is scarcely apparent at first sight that a far-reaching mathematical theory could be built on a basis so simple, still less that there should be any connection between this and the complicated question of the solution of algebraical equations by means of radicals. It may be worth while, in order to excite the interest of mathematical readers in the work before us, to mention one or two points in the Theory of Substitutions which will give an inkling of the nature of its connection with the interesting problem just mentioned.

The operation of replacing—say in any function $\phi(x_1, x_2, x_3)$ —any permutation of the letters, say x_1, x_2, x_3 , by any other, say x_1, x_3, x_2 , is called a *substitution*. This operation is denoted explicitly by $\begin{pmatrix} x_1, x_2, x_3 \\ x_1, x_3, x_2 \end{pmatrix}$, or shortly by a single letter s . Thus $s\phi(x_1, x_2, x_3) = \phi(x_1, x_3, x_2)$; and again: If t denote the substitution $\begin{pmatrix} x_1, x_2, x_3 \\ x_3, x_2, x_1 \end{pmatrix}$, $t\phi(x_1, x_3, x_2) = \phi(x_3, x_1, x_2)$. We may indicate the successive application of the two substitutions s and t by multiplying the symbols st in the order of application: thus $st\phi(x_1, x_2, x_3) = \phi(x_3, x_1, x_2)$ and $ts\phi(x_1, x_2, x_3) = \phi(x_2, x_3, x_1)$. In particular, the repetition of the same substitution may be represented by powers of the symbol; thus $s^2\phi(x_1, x_2, x_3) = \phi(x_1, x_2, x_3)$. The identical substitution $\begin{pmatrix} x_1, x_2, x_3 \\ x_1, x_2, x_3 \end{pmatrix}$ is represented by unity. The total number of different substitutions of n letters is obviously $n!$; consequently, if we form the consecutive powers of any substitution we shall ultimately arrive at a power s^m which will be the identical substitution, m being some positive integer not exceeding $n!$: m is called the *order* and n the *degree* of the substitution.

If among the substitutions of any given degree we can select a set which have the property that the product of any two furnishes another substitution belonging to the set, we obtain what is called a *group of substitutions*. The whole of the $n!$ substitutions of n letters obviously form a group, and the identical substitution by itself forms a group. It is easy, however, to see that in general there are other groups among the substitutions of a given

degree. Consider, for example, any rational function $\phi(x_1, x_2, \dots, x_n)$ which is not wholly asymmetric: there must exist a set of substitutions each of which leaves the value of ϕ unaltered. A substitution which is the product of any number of these must also leave ϕ unaltered: hence the set in question forms a group. We have here a fundamental point in the theory of substitutions, viz., the existence of a group of substitutions and the correlation therewith of rational functions which are unaltered by all the substitutions of the group. The group is said to belong to all the functions which it leaves unaltered; and these functions are said to form a family which is characterized by the group. Thus the group of a wholly asymmetric function is the identical group consisting of the substitution 1 ; the group of the wholly symmetric functions consists of the whole of the $n!$ substitutions of the n^{th} degree; the group of the alternating functions consists of all those substitutions which are equivalent to an even number of transpositions, and so on. It is obvious that every rational function determines a group of substitutions, and it may be shown that, conversely, for every group of substitutions we may construct an infinity of rational functions which are unaltered by the substitutions of the group. The significance of this correlation between a group and a family of functions depends on the following important theorem, which is due in substance to Lagrange. If ψ be a rational function which is unaltered by all the substitutions of the group of ϕ (in other words, if the group of ψ contain the group of ϕ) then ψ can be expressed as a rational function of ϕ , and the n elementary symmetric functions

$$C_1 = \Sigma x_i, C_2 = \Sigma x_i x_j, \dots, C_n = x_1 x_2 \dots x_n.$$

A particular case of this is the theorem that if the groups of ψ and ϕ be identical, then each can be expressed as a rational function of the other, and of the elementary symmetric functions. A limiting case of this theorem is the familiar result that every rational symmetric function can be expressed as a rational function of the elementary symmetric functions. As a special example consider the two wholly asymmetric functions $\psi = ax_1 + bx_2$, $\phi = a/x_1 + b/x_2$: these both belong to the identical group, since they are changed by every substitution of the letters x_1, x_2 . Hence ψ can be rationally expressed as a function of ϕ, C_1, C_2 . The actual expression is in fact

$$\psi = \{2(a-b)^2 C_2 - (a^2 + b^2) C_1^2 + (a+b) C_1 C_2 \phi\} / \{(a+b) C_1 + 2 C_2 \phi\} / (a-b)^2 (C_1^2 - 4 C_2).$$

The application of the theory of substitutions is limited in the first instance to rational functions. Its use in the theory of the solution of algebraical equations by means of radicals is based on the following important result in the theory of irrational functions. Any root of a solvable equation $f(x) = 0$ can be expressed as a rational integral function of certain elements V_1, V_2, \dots, V_n , the coefficients of which are rational functions of the coefficients of $f(x)$ and of primitive roots of unity. The quantities V_1, V_2, \dots, V_n are on the one hand rational integral functions of the roots of $f(x) = 0$ and of primitive roots of unity, and on the other hand are determined by a series of equations

$$V_a p_a = F_a(V_{a-1}, V_{a-2}, \dots, V_n),$$

where p_a is a prime number and F is a rational function of the V 's. For example, in the case of the cubic