Numerical Realization of the Temporal Relaxation Disk

1 Origin of the Visualization

The spiral visualization presented here is not an independent phenomenological construction. It arises directly from the temporal relaxation framework introduced in the preceding articles. Within that framework, the scalar field $\Phi(x, y, t)$ represents a locally synchronized configuration of the global temporal field. Bound structures correspond to metastable relaxation domains maintained by a balance between:

- diffusion (temporal smoothing),
- nonlinear self-amplification,
- central driving source,
- differential rotation (temporal shear),
- weak radial confinement.

The galactic morphology therefore appears as a dynamical attractor of the same relaxation equation previously derived in abstract form.

2 Governing Equation

In two-dimensional disk geometry, the evolution equation takes the form:

$$\frac{\partial \Phi}{\partial t} = D\nabla^2\Phi - \alpha\Phi + \beta\Phi^3 + S(r,\theta) - \Omega(r)\,\partial_\theta\Phi + \gamma\Omega(r)\,\partial_\theta\Phi + F_r(\Phi). \quad (1)$$

Where:

- D — relaxation (diffusion) coefficient

- α — linear damping
- β — nonlinear self-amplification
- $S(r, \theta)$ — central source term
- $\Omega(r)$ — differential rotation profile
- $F_r(\Phi)$ — weak radial confinement

3 Model Components

3.1 Differential Rotation

$$\Omega(r) = \frac{\Omega_0}{\sqrt{r + \epsilon}} \tag{2}$$

This produces shear analogous to gravitational disk rotation.

3.2 Central Source

$$S(r, \theta) = S_0 \exp\left(-\frac{r^2}{r_c^2}\right)(1 + \delta \cos(m\theta)) \tag{3}$$

This term represents a localized temporal pumping region.

Importantly, the azimuthal modulation is not structurally required by the relaxation equation itself. It merely biases the system toward a specific symmetry mode during visualization.

3.3 Radial Confinement

$$F_r(\Phi) = -\kappa r \Phi \tag{4}$$

This ensures disk compactness and prevents outward diffusion.

4 Dynamical Interpretation

The resulting morphology emerges from the competition between:

- nonlinear amplification of density contrasts,
- shear-induced azimuthal stretching,
- diffusive suppression of small-scale perturbations.

Two regimes are observed:

1. Stable disk attractor — symmetric circular configuration.
2. Metastable spiral regime — large-scale rotating modes sustained by shear.

When parameters exceed stability thresholds, the system transitions into small-scale fragmentation, analogous to reaction–diffusion pattern formation.

This confirms that spiral arms are not imposed geometrically but arise as dynamical modes of the relaxation flow.

5 Structural Stability of the Mechanism

The numerical experiments demonstrate:

- The core relaxation equation remains stable without artificial constraints.
- Spiral morphology does not require ad hoc geometric construction.
- Self-organization and long-lived rotating modes can be incorporated through mild extensions (e.g., nonlocal coupling or mode-selection terms) without altering the foundational equation.

Thus, the mechanism is structurally complete at the level of the temporal relaxation model.

6 Conceptual Significance

The visualization is therefore not a standalone galactic model but a concrete realization of the temporal relaxation paradigm: Galactic morphology emerges as a metastable synchronization pattern of a nonlinear relaxation field.

This bridges the abstract formalism of previous sections with directly observable large-scale structure.