ANNALS

OF THE

NEW YORK ACADEMY OF SCIENCES,

VOLUME VIII.

I.—*The Parallaxes of μ and θ Cassiopeiæ, deduced from Rutherfurd Photographic Measures.*

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Read Jan. 9, 1893.

The Rutherfurd photographic measures of the stars surrounding μ Cassiopeiæ are derived from twenty-eight negatives made between 1870 July, and 1873 December. These observations were taken in accordance with Rutherfurd's regular plan for securing accurate micrometric measures of star clusters: but in order to combine therewith a determination of parallax, the observations were all made in the months of July, January, and December. There are two impressions upon each negative. A discussion of all the micrometric measures of some fifty-six stars will be published later, the present paper containing those measures only that have been selected for the parallax determination. I have set down in table I. (p. 12) the dates and other details of the several exposures, so far as they are connected with the present purpose. The sidereal time given is the mean of the four instants marking the beginning and ending of the two exposures. The second exposure always began a few seconds after the ending of the first, and the duration of each was six minutes.

Table II. (p. 13) gives a list of the comparison stars employed. The pair g, h, will not furnish a suitable parallax factor in distance, but it has been included to secure evidence as to possible variation of scale value with position angle. It is perhaps necessary to remark that I have preferred to base the study of parallax upon measures of distance only. The original plan did not include the pair *c, d;* but the observations of the pair *c*, *θ*, having produced a discordant result, the pair *c*, *d*, was also computed. From this discordance it has been possible to obtain a value for the parallax of *θ* Cassiopeiæ, a result not anticipated when the reduction of the observations was begun. It is for this reason that the star *c* appears twice in table II. The method of computation is the one commonly employed. The two stars of each pair were selected so as to differ approximately 180° in position angle with respect to *μ* Cassiopeiæ. The scale value was then determined for each pair, on each plate, so as to make the *sum* of the distances from *μ* constant. The *difference* of the same distances was then taken as the quantity from whose variation the parallax should appear. This method gives the excess of the parallax of the principal star over the mean of the parallaxes of the two comparison stars.*

Every observation of distance contained in the RUTHERFURD observation books has been used, the treatment of the observational data being as follows: First, means were taken of the separate pointings of the microscope, each measure of distance depending upon ten independent pointings on *μ* Cassiopeiæ, and ten on the comparison star. The distances thus obtained from the two separate impressions were combined into a single mean depending altogether on 40 pointings, and this mean was then considered as one complete measure. The distances thus obtained are expressed in divisions of the glass scale of the measuring micrometer, one such division being approximately equal to 28″.01. The same unit of measure has generally been employed throughout all the subsequent calculations. The following corrections were then applied:—

1. Correction for division errors. These were taken from the table of corrections determined by ROGERS.†

* This is of course not strictly true unless the two comparison stars are equidistant from the principal star,—a condition which should always be approximately satisfied. Nor is it possible to deduce the parallax of the principal star with respect to each comparison star separately, since the parallaxes of both comparison stars will always influence the result through the scale value determination.

† Ann. N. Y. Acad. Sci., vol. vi, p. 250.

2. The "tangent correction," due to the photograph being taken on a plane surface. This correction is derived from table IV. A, given in my paper on the RUTHERFURD photographic measures of the stars about β Cygni.*

3. Correction for refraction, computed according to the method given in my paper on the Pleiades.† Whenever necessary, the higher terms of the refraction were approximately taken into account.

4. Correction for aberration, computed according to the customary Besselian formulæ.‡

5. Correction for the proper motion of μ Cassiopeiæ. The observations have been reduced to the epoch 1872.0, using Auwers' proper motion, which is:

$$\Delta\alpha = +\,0^{s}.3860 \qquad \Delta\delta = -\,1''.580,$$

corresponding to a motion of 3″.729 upon a great circle whose position angle is 115° 4′. Now, in general, if we let:

ρ = the annual proper motion of the principal star on a great circle,

χ = the position angle of that great circle at the time t_0,

t = the time of observation, expressed in years and fractions of a year. And put:

$$\tau = t - t_0$$
$$S_1 = \cos(\chi - p)$$
$$S_2 = -\frac{1}{2s}\sin^2(\chi - p)$$
$$P_1 = \tau\rho$$
$$P_2 = \tau^2\rho^2$$

then we must add to the observed distances the correction:

$$\Delta s = S_1P_1 + S_2P_2.$$

The values of S_1, S_2, P_1, and P_2 used in the present paper are given in table III. (p. 13), the unit of measure for ρ being one division of the glass scale, as already explained. The distances thus completely corrected, are set down in the second and third columns of table IV. (p. 15). The fourth and fifth columns of the same table contain the *sum* of the distances of the two comparison

* Ann. N. Y. Acad. Sci., vol. vi, p. 340.

† Ibid., pp. 253, *et seq.*

‡ Astron. Untersuch., vol. i, p. 202, *et seq.*

stars from μ Cassiopeiæ, and the difference of that sum from an adopted mean given at the foot of the fourth column. In the sixth, seventh, and eighth column are placed the *difference* of the distances as given in the second and third columns; the *scale correction*, which is simply a proportional part of the quantity given in the column "mean *minus* sum"; and finally the *corrected difference*, to be used in forming the parallax equations. The latter equations, together with their solutions, are to be found in table V. (p. 20); and with regard to them I have only to remark that the absolute terms are expressed in units of the second decimal place, equivalent to 0".2801. The parallax coefficients in the observation equations of table V. are computed by the customary formulæ, as follows:—

Let α, δ, be the coördinates of μ Cassiopeiæ for 1872.
r, $\odot$, be the radius vector, and longitude of the sun;
and compute—

$$g \sin G = \sin \delta \cos \alpha, \; h \sin H = \sin \delta \sin \alpha, f \sin F = h \sin (H + \varepsilon)$$
$$g \cos G = \sin \alpha, \qquad h \cos H = -\cos \delta, \quad f \cos F = -\cos \alpha \cos \varepsilon$$

then if we put:

$$S_3 = f \sin (p + F)$$
$$S_4 = g \sin (p + G)$$
$$P_3 = - r \sin \odot$$
$$P_4 = - r \cos \odot$$

the parallax coefficient for any one of the equations will be:

$$(S_3 - S_3') P_3 + (S_4 - S_4') P_4,$$

where S_3' and S_4' refer to the second comparison star of the pair. The corresponding coefficient for the *sum* of the distances of the two comparison stars of any pair is

$$(S_3 + S_3') P_3 + (S_4 + S_4') P_4,$$

which I have found sufficiently small to be negligible for all the pairs used in the present research.

The values of S_3, S_4, P_3, P_4, are found in table III. (p. 13).

Before proceeding to discuss the parallax results, as obtained in table V., attention should be called to table IV. A, which throws some light on the question of variation of scale value. The first five columns of this table give for each plate, and for each pair of stars, that fraction of "mean *minus* sum," from table IV., which corresponds to a distance of 100 divisions of the scale. It will be seen that this quantity is a measure of the discordance between the scale value for each plate, and the mean scale value for all the plates. Now, by placing these discordances, as derived from pairs of stars differing widely in position angle, side by side, we can at once see whether the scale value varies with position angle, or is really a constant for any given plate. To make this comparison quite rigorous, the adopted mean, from which the quantity "mean *minus* sum" is derived in table IV., ought to be computed from the measures of those plates only, that have impressions of *all* the stars. Accordingly the "adopted means" of table IV. are computed from the measures of plates

31	41	47	57
32	42	48	58
33	43	49	59
34	46	50	60

Except in the case of the pair *c*, *d*, which was subsequently added, as already explained. In this case I have used the actual mean of all the values given in table IV., a circumstance which will not materially affect the evidence furnished by this pair of stars. The pair *c*, *θ* has not been used, since the parallax of *θ* would tend to render its evidence unreliable. Two things become plain from an inspection of the table. In the first place, there is no decisive evidence of great variations of scale value with position angle. In fact, if we regard as casual the differences from the mean values contained in the sixth column, we find as the average difference without regard to sign, ±0.0050. This would correspond to about 0".05 per 1000": but it is hardly greater than the necessary uncertainty of observation. In the second place, we see that the scale value depends upon temperature. This latter circumstance, it is almost needless to remark, does not influence the parallax determination, though it will be of importance in the general discussion of all the micrometric measures.

It will therefore be of interest to examine the evidence a little more in detail. In doing this, I have taken into consideration the readings of the focal micrometer, as set down in table I. One revolution of this micrometer is equivalent to 0.05 inches, and the arrangement is such that increasing readings of the micrometer correspond to increasing focal length of the telescope. To calculate the focal length, we have the following:

Linear scale value* $=$ 0.020859 inches.
Mean angular scale value† $=$ 28″.0124

from which:

Focal length $=$ 153.59 inches.

Now putting:

$f =$ reading of the focal micrometer for any given plate,

I find from the above focal length and pitch of the micrometer screw that the numbers given in table IV. A require an additional correction of:

$$+0.0325\ (f - 7.86)$$

to reduce them to the mean focal reading (7.86). Accordingly, this correction has been applied to the means in the sixth column, thus obtaining the corrected means of the seventh column. From these latter it is possible to discuss the effect of temperature, independently of the focal readings, provided we assume that the zero point of the scale attached to the focal micrometer always retained exactly the same distance from the optical centre of the lens, except as influenced by changes of temperature. Now this assumption can hardly be regarded as altogether justified *a priori* in the case of the RUTHERFURD telescope. I have therefore made two separate least square adjustments of the quantities given in the columns *mean* and *mean corrected.*

Representing these quantities by equations of the form:

$$x + y\ (t - t_0)$$

where t is the observed temperature for any plate, and t_0 the mean temperature, I find:

Column *mean* $= -.0027 \pm .00063 - .000372\ (t - 58^\circ.4)$
$\pm .000029$

Column *mean corr'd* $= -.0027 \pm .00057 - .000424\ (t - 58^\circ.4)$
$\pm .000027$

The attached probable errors show that the observations are represented better if we take the readings of the focal micrometer into account. This would seem to justify the assumption of con-

* ROGERS' determination, Ann. N. Y. Acad. Sci., vol. vi, p. 249.
† The Pleiades result, ibid., p. 270.

stancy in the focal zero point; and I shall therefore adopt the temperature coefficient from the second solution, viz:

$$-0.000424 \pm 0.000027$$

This coefficient holds good for a distance of 100 divisions of the scale. For one division of the scale it will be:

$$-0^{d}.00000424 \pm 0.00000027, \text{ or } -0''.000119 \pm 0''.000008.$$

The evidence as to the reality of this temperature coefficient seems to be very strong, notwithstanding that the scale values obtained for the Pleiades plates* did not appear to vary with temperature. In the light of the present evidence we may perhaps be justified in ascribing this to the comparatively small changes of temperature throughout the Pleiades series, and to the fact that the last two Pleiades plates furnish a very discordant scale value, which tends to conceal the smaller temperature effects. No satisfactory explanation of this latter circumstance suggests itself, unless we assume that the glass scale had been removed temporarily from the measuring machine; and that when replaced, it made a small angle with its former position. It seems best, therefore, to disregard the last two Pleiades plates in deducing a definitive scale value. If we do this, the Pleiades series give for the mean scale value 28''.0138, corresponding to a mean temperature of the telescope 41°.6, and a mean focal reading 7.88. For a plate having any other temperature (t) and focal reading (f), we ought therefore to use a scale value computed by the following formula:

$$\text{Scale value} = 28''.0138 - 0''.0090\,(f - 7.88) - 0''.000119\,(t - 41.6) \quad (a)$$

On the other hand, if we prefer to retain the scale values from the last two Pleiades plates, we should have:

$$\text{Scale value} = 28''.0124 - 0''.0090\,(f - 7.87) - 0''.000119\,(t - 40.3) \quad (b)$$

The following table shows how these two formulæ represent the observed Pleiades scale values. The numbers in the fourth column are means from the two impressions on the plate.

* Ann. N. Y. Acad. Sci., vol. vi, p. 271.

Plate No.	Focus.	Telesc. Temp.	Observed Scale Val.	Computed Scale Val.		Residuals, $C-O$.	
				Form. (a).	Form. (b).	Form. (a).	Form. (b).
16	7.9	35°	28″.0168	28″.0144	28″.0127	—.0024	—.0041
17	7.9	35	.0151	.0144	.0127	—.0007	—.0034
18	8.1	27	.0144	.0135	.0137	—.0009	—.0007
19	7.78	52	.0122	.0135	.0118	+.0013	—.0004
20	7.78	52	.0118	.0135	.0118	+.0017	.0000
21	7.85	52	.0122	.0129	.0112	+.0007	—.0010
22	7.85	40	.0148	.0143	.0126	—.0005	—.0022
23	7.85	40	.0132	.0143	.0126	+.0011	—.0006
24	7.85	35	.0070	.0149	.0132	+.0079	+.0062
25	7.85	35	.0066	.0149	.0132	+.0083	+.0066

The residuals are not quite satisfactory. I have therefore discussed the Pleiades scale values by means of formula (a), omitting the discordant plates 24 and 25, and find the temperature coefficient:

$$-0''.000230 \pm 0''.000040.$$

It is further to be remarked, that the residuals from formula (a) are somewhat less than we should get if we left the temperature and focal reading altogether out of consideration.

To complete this part of the subject, it is necessary to examine the evidence of the β Cygni plates.* For this purpose I selected from the RUTHERFURD β Cygni list four pairs of stars, suitably distributed in position angle. Treating the data exactly as already described for the μ Cassiopeiæ plates, gives the temperature coefficient:

$$-0^d.00000696 \pm 0^d.00000167, \text{ or } -0''.000195 \pm 0''.000047.$$

The low weight of this result, like that from the Pleiades series, is of course due to the small number of plates used, and to the comparatively slight range of temperature. Assembling the three values obtained, we have for the temperature coefficient:

μ Cassiopeiæ	$-0''.000119 \pm 0''.000008$
β Cygni	-0.000195 ± 0.000047
Pleiades	-0.000230 ± 0.000040
Mean by weight . . .	-0.000125 ± 0.000008

The evidence of the β Cygni and Pleiades plates practically does not change the result from the μ Cassiopeiæ plates. I am therefore

* Ann. N. Y. Acad. of Sci., vol. vi, pp. 331, *et seq.*

inclined to regard (*a*) as the best scale value formula deducible from the evidence made available up to the present time. In the case of the β Cygni plates, whose mean focal reading is 7.68, and mean temperature 68°.0, this formula gives 28″.0125, which agrees almost exactly with the scale value (28″.0124) actually employed in the β Cygni reductions.

Returning now to the results arising from the solution of the equations in table V. (p. 20) we find the following values for π, the parallax, and y, the correction of the annual proper motion effect. The quantity x, which is merely the error of the value arbitrarily assumed for the "corrected difference," is here omitted.

Comp. Stars.	π	y	Prob. error one equation.*
a and *b*	+ 0″.249 ± 0″.045	— 0″.153 ± 0″.051	± 0″.251
c and *d*	+ 0.266 ± 0.035	+ 0.127 ± 0.052	± 0.222
e and *f*	+ 0.324 ± 0.050	— 0.136 ± 0.056	± 0.196
c and θ	+ 0.151 ± 0.026	— 0.122 ± 0.041	± 0.190

It will be seen at once that the values of π deduced from the first three pairs agree with each other fully as well as might be expected from their probable errors. The parallax depending on *c* and θ, on the other hand, differs widely. We may conclude that this is due to the existence of a sensible parallax belonging to θ. If we then depend upon the first three pairs for the parallax of μ we shall have, taking the mean by weight:

Parallax of μ Cassiopeiæ = + 0″.275 ± 0″.024.

But if we consider the three determinations as having equal weight, we get for the arithmetical mean, and probable error from the three residuals, $\pi = + 0''.280 \pm 0''.026$. Now if we admit the existence of a sensible parallax for θ, the result obtained above from the comparison stars *c* and θ is not the parallax of μ, but a quantity which is very nearly equal to:

$$\pi_\mu - \tfrac{1}{2}\pi_\theta\left(1 - \frac{s_c - s_\theta}{s_c + s_\theta}\right)$$

where: π_μ and π_θ are the parallaxes of μ and θ,
s_c and s_θ are the distances of *c* and θ from μ.

* This is the probable error of the *difference* of two distances as measured on one plate. But as there are two impressions on each plate, it may also be regarded as the probable error of one complete measure of distance from a single impression.

We therefore have the equation:

$$+0''.275\,(\pm 0''.024) - \tfrac{1}{2}\pi_\theta \left(1 - \frac{s_c - s_\theta}{s_c + s_\theta}\right) = +\,0''.151\,(\pm\,0''.026)$$

a solution of which gives:

Parallax of θ Cassiopeiæ $= +\,0''.232 \pm 0''.067$.

This result may be regarded as confirmatory of that of BESSEL, who found for μ the parallax $-\,0''.12 \pm 0''.29$, by the method of differences of right ascension, using θ as his comparison star. Possibly a re-reduction of BESSEL's observations, using the best value of the proper motion, might alter his final conclusion: as it stands, it seems to indicate at least an approximate equality between the parallaxes of μ and θ.

From the values of y obtained in the solution of the normal equations I have deduced corrections for the AUWERS-BRADLEY proper motion of μ, on the assumption that the comparison stars (except θ) have no proper motions of their own. If we let ρ and χ have the same meaning as before, and put:

w = the correction required by the adopted value of $\rho \cos \chi$
v = " " " " $\rho \sin \chi$

Then each pair of comparison stars furnishes an equation of the form:

$$(\cos p - \cos p')\,w + (\sin p - \sin p')\,v - y = 0$$

where p and p' are the position angles of the two stars. The equations so obtained are:

Stars a and b	$-\,1.8000\,w - 0.8460\,v + 0''.1534 = 0$
c and d	$+\,0.0293\,w - 1.9976\,v - 0.1267 = 0$
e and f	$+\,0.7563\,w - 1.8116\,v + 0.1360 = 0$

from which the normal equations are:

$$+\,3.8128\,w + 0.2112\,v - 0.1770 = 0$$
$$+\,7.9880\,v - 0.6292 = 0$$

and the solution is

$$w = +\,0''.0421 \pm 0''.0147$$
$$v = +\,0.0777 \pm 0.0102$$

the probable error of one equation being $\pm\,0''.0287$. Applying these corrections to the values previously assumed, I get:

Corrected $\rho \sin \chi = +\,3''.457$ Corrected $\rho \cos \chi = -\,1''.538$

corresponding to a motion of 3″.784 upon a great circle whose position angle is 113° 59′. The following are therefore the several values of the proper motion, to be compared with each other:

	ρ	χ	$\Delta\alpha$	$\Delta\delta$
As just obtained	3.784″	113° 59′	+ 0.3950ˢ	− 1.538″
AUWERS'-BRAD. (1810), as used in the present paper . . .	3.729	115 4	+ 0.3860	− 1.580
AUWERS'-BRAD., reduced to 1872	3.729	115 13	+ 0.3854	− 1.589

When we compare the parallax of μ Cassiopeiæ derived in the present paper, with the work of other observatories, we find large discordances. Thus the Oxford photographic result is only about 0″ 036 ± 0″.018, while the RUTHERFURD plates give 0″.249 ± 0″.045 from the same pair of comparison stars. On the other hand, STRUVE has obtained 0″.251 ± 0″.075 from distance measures, and from position angles 0″.425 ± 0″.072. It is therefore plain that the photographic method of determining parallaxes cannot be regarded as free from systematic error. An examination of the equations of table V. shows that negative parallax coefficients invariably occur in the case of plates exposed at eastern hour angles. This circumstance, which arises from the inconvenience of observing after midnight, may possibly produce systematic error. But the evidence of the scale value table (IV. A) is against this supposition, as is also the approximate equality of the parallaxes obtained from pairs of comparison stars having widely different distances from μ.

In conclusion, the results here deduced may be summed up as follows:—

Parallax of μ Cassiopeiæ **0″.275** ± 0″.024
Parallax of θ Cassiopeiæ **0.232** ± 0.067

But the above probable errors must not be taken as reliable estimates of uncertainty, since a comparison with the work of other astronomers seems to indicate the possibility of systematic error. But if we are willing to accept the above results, it is perhaps allowable to speculate upon μ and θ Cassiopeiæ as a system remotely resembling that of 61 Cygni. The indication of equality of parallaxes furnished by BESSEL's observations, and the slight evidence of variation in the proper motion of μ Cassiopeiæ obtained from the equations on p. 10, would almost seem to favor such an idea.

Table I.—General Data,

Observatory of L. M. Rutherfurd, New York.

Lat. = 40° 43′ 48″.5, Long. = $4^h\ 55^m\ 56^s.62$ W.

Plate No.	Date.	Sidereal Time.	Hour Angle.	Zen. Dist.	Parall. Angle.	Ext. Temp.	Focal Mic'r.
				°	°		
31	1870 July 23	$20^h\ 47^m\ 45^s$	$19^h\ 47^m\ 59^s$	43.05	— 81.46	79°	7.7
32	1870 July 23	21 56 35	20 56 49	33.04	— 95.07	79	7.7
33	1870 July 30	20 15 15	19 15 29	47.70	— 75.78	70	7.9
34	1870 July 30	20 43 35	19 43 49	43.67	— 80.70	70	7.9
36	1871 July 10	20 39 38	19 39 52	44.22	— 80.00	76	7.9
37	1871 July 10	21 20 8	20 20 22	38.36	— 87.51	76	7.9
38	1871 July 23	20 30 5	19 30 19	45.60	— 78.33	65	8.1
40	1872 Jan. 2	4 5 30	3 5 44	33.41	+ 94.50	30	7.9
41	1872 Jan. 2	5 4 28	4 4 42	42.01	+ 82.77	30	7.9
42	1872 Jan. 5	2 23 0	1 23 14	19.39	+125.83	35	7.9
43	1872 July 19	21 3 2	20 3 16	40.85	— 84.24	71	7.9
44	1872 July 19	21 38 52	20 39 6	35.63	— 91.26	71	7.9
45	1872 July 20	21 13 8	20 13 22	39.38	— 86.14	73	7.9
46	1872 July 20	21 49 42	20 49 56	34.04	— 93.56	73	7.9
47	1873 Jan. 6	3 26 32	2 26 46	27.80	+103.96	26	7.9
48	1873 Jan. 9	4 6 28	3 6 42	33.55	+ 94.29	24	7.9
49	1873 Jan. 9	4 40 2	3 40 16	38.45	+ 87.37	24	7.9
50	1873 Jan. 10	3 32 2	2 32 16	28.59	+102.50	21	7.95
51	1873 July 15	21 46 42	20 46 56	34.49	— 92.91	75	7.75
52	1873 July 21	20 52 52	19 53 6	42.32	— 82.37	69	7.8
53	1873 July 21	21 27 52	20 28 6	37.23	— 89.03	69	7.8
54	1873 July 23	19 49 48	18 50 2	51.26	— 71.50	75	7.75
55	1873 July 23	20 33 8	19 33 22	45.16	— 78.85	75	7.75
56	1873 July 23	21 8 58	20 9 12	39.98	— 85.35	75	7.75
57	1873 Dec. 18	4 14 18	3 14 32	34.69	+ 92.60	41	7.8
58	1873 Dec. 18	4 50 18	3 50 32	39.95	+ 85.40	41	7.8
59	1873 Dec. 21	2 36 38	1 36 52	21.06	+120.12	27	7.9
60	1873 Dec. 21	3 10 12	2 10 26	25.52	+108.61	27	7.9

TABLE II.—COMPARISON STARS.

Designation of Comp. Star.	No. in A. G. Cat. Cambr. U. S.	Mag. in A. G. Cat.	Approx. Position, Referred to μ Cass.	
			Distance.	Pos. Ang.
a	517	8.8	762″	31° 14′
b	509	8.9	1358	199 7
c	485	8.8	1705	271 48
d			2442	86 31
e	527	7.5	1529	123 40
f	490	8.9	1627	281 39
g	519	7.1	2727	169 7
h	496	8.2	3175	335 10
c	485	8.8	1705	271 48
θ	541	5.5	1966	70 46

TABLE III.—PROPER MOTION AND PARALLAX.

See pages 3 and 4.

Star.	Proper Motion.		Parallax.	
	S_1.	S_2.	S_3.	S_4.
a	+0.107	—0.018	—0.493	+0.804
b	+0.104	—0.010	+0.328	—0.826
c	—0.919	—0.001	+0.883	—0.233
d	+0.878	—0.001	—0.886	+0.305
e	+0.989	0.000	—0.714	—0.220
f	—0.973	0.000	+0.859	—0.094
g	+0.587	—0.003	—0.128	—0.722
h	—0.765	—0.002	+0.364	+0.604
θ	+0.716	—0.003	—0.849	+0.502

(Continued on the next page.)

TABLE III. (*continued*).—PROPER MOTION AND PARALLAX.

Plate No.	Proper Motion.			Parallax.	
	$t-1872.0$	P_1.	P_2.	P_3.	P_4.
31	−1.440	−0.1917	+0.037	−0.872	+0.521
32	−1.440	−0.1917	+0.037	−0.871	+0.522
33	−1.420	−0.1891	+0.036	−0.805	+0.618
34	−1.420	−0.1891	+0.036	−0.805	+0.618
36	−0.476	−0.0634	+0.004	−0.965	+0.319
37	−0.476	−0.0634	+0.004	−0.965	+0.319
38	−0.440	−0.0586	+0.004	−0.894	+0.518
40	+0.006	+0.0008	0.000	+0.962	−0.202
41	+0.006	+0.0008	0.000	+0.962	−0.203
42	+0.013	+0.0017	0.000	+0.951	−0.252
43	+0.551	+0.0734	+0.005	−0.901	+0.470
44	+0.551	+0.0734	+0.005	−0.900	+0.470
45	+0.554	+0.0738	+0.006	−0.893	+0.485
46	+0.554	+0.0738	+0.006	−0.892	+0.486
47	+1.019	+0.1356	+0.018	+0.942	−0.282
48	+1.027	+0.1367	+0.019	+0.926	−0.332
49	+1.027	+0.1367	+0.019	+0.925	−0.333
50	+1.029	+0.1370	+0.019	+0.920	−0.348
51	+1.539	+0.2049	+0.042	−0.932	+0.406
52	+1.556	+0.2072	+0.043	−0.886	+0.496
53	+1.556	+0.2072	+0.043	−0.886	+0.496
54	+1.561	+0.2078	+0.043	−0.870	+0.524
55	+1.561	+0.2078	+0.043	−0.870	+0.525
56	+1.561	+0.2078	+0.043	−0.869	+0.525
57	+1.966	+0.2618	+0.069	+0.983	+0.049
58	+1.966	+0.2618	+0.069	+0.983	+0.048
59	+1.974	+0.2628	+0.069	+0.984	−0.002
60	+1.974	+0.2628	+0.069	+0.984	−0.003

TABLE IV.—OBSERVATIONAL DATA.

COMPARISON STARS *a* AND *b*.

Plate No.	Distance.		Sum. $b+a$.	Mean *Minus* Sum.	Difference. $b-a$.	Scale Corr.	Corrected Difference.
	Star *a*.	Star *b*.					
31	27.1737	48.5528	75.7265	—.0031	21.3791	—.0009	21.3782
32	.1789	.5588	.7377	—.0143	.3799	—.0040	.3759
33	.1684	.5604	.7288	—.0054	.3920	—.0015	.3905
34	.1717	.5548	.7265	—.0031	.3831	—.0009	.3822
36	.1770	.5562	.7332	—.0098	.3792	—.0028	.3764
37	.1761	.5617	.7378	—.0144	.3856	—.0041	.3815
38	.1712	.5669	.7381	—.0147	.3957	—.0041	.3916
40	.1898	.5277	.7175	+.0059	.3379	+.0017	.3396
41	.1909	.5313	.7222	+.0012	.3404	+.0003	.3407
42	.1742	.5485	.7227	+.0007	.3743	+.0002	.3745
43	.1667	.5547	.7214	+.0020	.3880	+.0006	.3886
45	.1658	.5579	.7237	—.0003	.3921	—.0001	.3920
46	.1710	.5645	.7355	—.0121	.3935	—.0034	.3901
47	.1795	.5336	.7131	+.0103	.3541	+.0029	.3570
48	.1813	.5269	.7082	+.0152	.3456	+.0043	.3499
49	.1821	.5464	.7285	—.0051	.3643	—.0014	.3629
50	.1758	.5416	.7174	+.0060	.3658	+.0017	.3675
51	.1701	.5614	.7315	—.0081	.3913	—.0023	.3890
52	.1749	.5600	.7349	—.0115	.3851	—.0032	.3819
53	.1829	.5474	.7303	—.0069	.3645	—.0019	.3626
57	.1677	.5488	.7165	+.0069	.3811	+.0019	.3830
58	.1626	.5519	.7145	+.0089	.3893	+.0025	.3918
59	.1661	.5631	.7292	—.0058	.3970	—.0016	.3954
60	.1693	.5569	.7262	—.0028	.3876	—.0008	.3868
		Adopted mean	75.7234			Assumed value	21.3800

TABLE IV.—OBSERVATIONAL DATA.

COMPARISON STARS *c* AND *d*.

Plate No.	Distance. Star *c*.	Distance. Star *d*.	Sum. *d* + *c*.	Mean *Minus* Sum.	Difference. *d* — *c*.	Scale Corr.	Corrected Difference.
31	60.9758	87.2109	148.1867	—.0220	26.2351	—.0039	26.2312
32	.9712	.2026	.1738	—.0091	.2314	—.0016	.2298
33	.9859	.1947	.1806	—.0159	.2088	—.0028	.2060
34	.9747	.2106	.1853	—.0206	.2359	—.0036	.2323
38	.9745	.1943	.1688	—.0041	.2198	—.0007	.2191
40	.9421	.2035	.1456	+.0191	.2614	+.0034	.2648
42	.9594	.2009	.1603	+.0044	.2415	+.0008	.2423
45	.9823	.1993	.1816	—.0169	.2170	—.0030	.2140
46	.9816	.1944	.1760	—.0113	.2128	—.0020	.2108
47	.9436	.2063	.1499	+.0148	.2627	+.0026	.2653
48	.9381	.1927	.1308	+.0339	.2546	+.0060	.2606
50	.9510	.2051	.1561	+.0086	.2541	+.0015	.2556
51	.9719	.1870	.1589	+.0058	.2151	+.0010	.2161
53	.9773	.2060	.1833	—.0186	.2287	—.0033	.2254
58	.9630	.1984	.1614	+.0033	.2354	+.0006	.2360
59	.9538	.1907	.1445	+.0202	.2369	+.0036	.2405
60	.9671	.1886	.1557	+.0090	.2215	+.0016	.2231
	Adopted mean		148.1647		Assumed value		26.2300

TABLE IV.—OBSERVATIONAL DATA.

COMPARISON STARS *e* AND *f*.

Plate No.	Distance. Star *e*.	Distance. Star *f*.	Sum. *f* + *e*.	Mean *Minus* Sum.	Difference. *f* — *e*.	Scale Corr.	Corrected Difference.
31	54.6020	58.0915	112.6935	—.0184	3.4895	—.0006	3.4889
32	.5939	.0871	.6810	—.0059	.4932	—.0002	.4930
33	.5918	.0971	.6889	—.0138	.5053	—.0004	.5049
34	.6014	.0910	.6924	—.0173	.4896	—.0005	.4891
41	.5952	.0698	.6650	+.0101	.4746	+.0003	.4749
42	.5964	.0679	.6643	+.0108	.4715	+.0003	.4718
43	.6021	.1033	.7054	—.0303	.5012	—.0009	.5003
44	.5948	.1015	.6963	—.0212	.5067	—.0007	.5060
46	.6004	.1019	.7023	—.0272	.5015	—.0008	.5007
47	.5890	.0545	.6435	+.0316	.4655	+.0010	.4665
48	.5979	.0596	.6575	+.0176	.4617	+.0005	.4622
49	.5966	.0524	.6490	+.0261	.4558	+.0008	.4566
50	.5950	.0688	.6638	+.0113	.4738	+.0004	.4742
57	.5993	.0723	.6716	+.0035	.4730	+.0001	.4731
58	.5886	.0818	.6704	+.0047	.4932	+.0001	.4933
59	.5998	.0672	.6670	+.0081	.4674	+.0003	.4677
60	.5949	.0915	.6864	—.0113	.4966	—.0004	.4962
	Adopted mean		112.6751		Assumed value		3.4900

TABLE IV.—OBSERVATIONAL DATA.

COMPARISON STARS *g* AND *h*.

(Not used for parallax.)

Plate No.	Distance.		Sum. $h+g$.	Mean *Minus* Sum.	Difference. $h-g$.
	Star *g*.	Star *h*.			
31	97.3819	113.4035	210.7854	—.0203	16.0216
32	.3825	.4086	.7911	—.0260	.0261
33	.3743	.4116	.7859	—.0208	.0373
34	.3724	.4034	.7758	—.0107	.0310
36	.3768	.3904	.7672	—.0021	.0136
38	.3785	.3947	.7732	—.0081	.0162
40	.3575	.3771	.7346	+.0305	.0196
41	.3647	.3764	.7411	+.0240	.0117
42	.3709	.3894	.7603	+.0048	.0185
43	.3753	.4074	.7827	—.0176	.0321
44	.3747	.4063	.7810	—.0159	.0316
45	.3677	.4046	.7723	—.0072	.0369
46	.3778	.4027	.7805	—.0154	.0249
47	.3641	.3891	.7532	+.0119	.0250
48	.3607	.3899	.7506	+.0145	.0292
49	.3604	.3891	.7495	+.0156	.0287
50	.3613	.3870	.7483	+ 0168	.0257
51	.3724	.4023	.7747	—.0096	.0299
53	.3728	.4016	.7744	—.0093	.0288
54	.3787	.3942	.7729	—.0078	.0155
55	.3705	.3968	.7673	—.0022	.0263
56	.3716	.4045	.7761	—.0110	.0329
57	.3656	.3876	.7532	+.0119	.0220
58	.3696	.3853	.7549	+.0102	.0157
59	.3777	.3925	.7702	—.0051	.0148
60	.3684	.3899	.7583	+.0068	.0215
	Adopted mean		210.7651		

TABLE IV.—OBSERVATIONAL DATA.

COMPARISON STARS *c* AND *θ*.

Plate No.	Distance. Star *c*.	Distance. Star *θ*.	Sum. *θ*+*c*.	Mean *Minus* Sum.	Difference. *θ*−*c*.	Scale Corr.	Corrected Difference.
31	60.9758	70.2500	131.2258	−.0066	9.2742	−.0005	9.2737
32	.9712	.2431	.2143	+.0049	.2719	+.0003	.2722
33	.9859	.2520	.2379	−.0187	.2661	−.0013	.2648
34	.9747	.2523	.2270	−.0078	.2776	−.0006	.2770
38	.9745	.2502	.2247	−.0055	.2757	−.0004	.2753
40	.9421	.2431	.1852	+.0340	.3010	+.0024	.3034
41	.9472	.2383	.1855	+.0337	.2911	+.0024	.2935
42	.9594	.2491	.2085	+.0107	.2897	+.0008	.2905
43	.9825	.2687	.2512	−.0320	.2862	−.0023	.2839
44	61.0002	.2588	.2590	−.0398	.2586	−.0028	.2558
45	60.9823	.2618	.2441	−.0249	.2795	−.0018	.2777
46	.9816	.2765	.2581	−.0389	.2949	−.0028	.2921
47	.9436	.2494	.1930	+.0262	.3058	+.0019	.3077
48	.9381	.2580	.1961	+.0231	.3199	+.0016	.3215
49	.9540	.2492	.2032	+.0160	.2952	+.0011	.2963
50	.9510	.2590	.2100	+.0092	.3080	+.0007	.3087
51	.9719	.2666	.2385	−.0193	.2947	−.0014	.2933
53	.9773	.2672	.2445	−.0253	.2899	−.0018	.2881
57	.9613	.2699	.2312	−.0120	.3086	−.0009	.3077
58	.9630	.2615	.2245	−.0053	.2985	−.0004	.2981
59	.9538	.2620	.2158	+.0034	.3082	+.0002	.3084
60	.9671	.2574	.2245	−.0053	.2903	−.0004	.2899
	Adopted mean		131.2192		Assumed value		9.2900

Table IV. A.—Variation of Scale Value.

See page 4.

Plate No.	Comp. Stars and their Position Angles.				Mean.	Mean Corr'd.	Temp. of Telesc.
	a and *b*. 31° 199°	*c* and *d*. 272° 87°	*e* and *f*. 124° 282°	*g* and *h*. 169° 338°			
31	—.0041	—.0149	—.0163	—.0096	—.0112	—.0164	83°
32	—.0189	—.0062	—.0052	—.0123	—.0107	—.0159	83
33	—.0071	—.0107	—.0122	—.0099	—.0100	—.0087	73
34	—.0041	—.0139	—.0153	—.0051	—.0096	—.0083	73
36	—.0129			—.0010	—.0070	—.0057	75
37	—.0190				—.0190	—.0177	78
38	—.0194	—.0028		—.0038	—.0087	—.0009	68
40	+.0078	+.0129		+.0145	+.0117	+.0130	34
41	+.0016		+.0089	+.0114	+.0073	+.0086	34
42	+.0009	+.0030	+.0096	+.0023	+.0040	+.0053	36
43	+.0026		—.0268	—.0083	—.0108	—.0095	73
44			—.0188	—.0075	—.0132	—.0119	73
45	—.0004	—.0114	—.0241	—.0034	—.0098	—.0085	75
46	—.0160	—.0076		—.0073	—.0103	—.0090	75
47	+.0136	+.0100	+.0280	+.0056	+.0143	+.0156	28
48	+.0201	+.0229	+.0156	+.0069	+.0164	+.0177	27
49	—.0067		+.0231	+.0074	+.0079	+.0092	27
50	+.0079	+.0058	+.0100	+.0080	+.0079	+.0108	23
51	—.0107	+.0039		—.0046	—.0038	—.0074	78
52	—.0152				—.0152	—.0172	70
53	—.0091	—.0126		—.0044	—.0087	—.0107	70
54				—.0037	—.0037	—.0073	78
55				—.0010	—.0010	—.0046	78
56				—.0052	—.0052	—.0088	78
57	+.0091		+.0031	+.0056	+.0059	+.0039	43
58	+.0117	+.0022	+.0042	+.0048	+.0057	+.0037	43
59	—.0077	+.0137	+.0072	—.0024	+.0027	+.0040	30
60	—.0037	+.0061	—.0100	+.0032	—.0011	+.0002	30

Table V.—Parallax Equations.

Comparison Stars *a* and *b*.

Plate.							*r*.
31	1.00x	−1.44y	−1.57π	−0.18	=	0	+0.14
32	1.00	−1.44	−1.57	−0.41	=	0	−0.09
33	1.00	−1.42	−1.67	+1.05	=	0	+1.29
34	1.00	−1.42	−1.67	+0.22	=	0	+0.46
36	1.00	−0.48	−1.31	−0.36	=	0	−0.95
37	1.00	−0.48	−1.31	+0.15	=	0	−0.44
38	1.00	−0.44	−1.57	+1.16	=	0	+0.32
40	1.00	+0.01	+1.12	−4.04	=	0	−2.12
41	1.00	+0.01	+1.12	−3.93	=	0	−2.01
42	1.00	+0.01	+1.19	−0.55	=	0	+1.43
43	1.00	+0.55	−1.50	+0.86	=	0	+0.15
45	1.00	+0.55	−1.52	+1.20	=	0	+0.48
46	1.00	+0.55	−1.52	+1.01	=	0	+0.29
47	1.00	+1.02	+1.23	−2.30	=	0	−0.83
48	1.00	+1.03	+1.30	−3.01	=	0	−1.49
49	1.00	+1.03	+1.30	−1.71	=	0	−0.19
50	1.00	+1.03	+1.33	−1.25	=	0	+0.30
51	1.00	+1.54	−1.43	+0.90	=	0	−0.28
52	1.00	+1.56	−1.54	+0.19	=	0	−1.10
53	1.00	+1.56	−1.54	−1.74	=	0	−3.03
57	1.00	+1.97	+0.73	+0.30	=	0	+0.80
58	1.00	+1.97	+0.73	+1.18	=	0	+1.68
59	1.00	+1.97	+0.81	+1.54	=	0	+2.11
60	1.00	+1.97	+0.81	+0.68	=	0	+1.25
							$\Sigma r^2 = 37.00$

Normal Equations.

$$+24.0000x + 11.2100y - 8.0500\pi - 9.0400 = 0$$
$$+36.7277 + 13.1132 - 1.9982 = 0$$
$$+43.0283 - 23.6197 = 0$$

Solution.

In units 2d dec. place.	In Arc.
$\pi = +0.8899 \pm 0.1600$	$\pi = +0''.2493 \pm 0''.0448$
$y = -0.5475 \pm 0.1811$	$y = -0.1534 \pm 0.0507$
$x = +0.9309 \pm 0.2184$	$x = +0.2607 \pm 0.0612$

Prob. error of one equation = 0.8952 (Scale) = 0''.2507 (Arc)

Table V.—Parallax Equations.

Comparison Stars *c* and *d*.

Plate.						v
31	$1.00x$	$-1.44y$	$+1.82\pi$	$+0.12$	$=0$	$+0.52$
32	1.00	-1.44	$+1.82$	-0.02	$=0$	$+0.38$
33	1.00	-1.42	$+1.75$	-2.40	$=0$	-2.06
34	1.00	-1.42	$+1.75$	$+0.23$	$=0$	$+0.57$
38	1.00	-0.44	$+1.86$	-1.09	$=0$	-0.20
40	1.00	$+0.01$	-1.81	$+3.48$	$=0$	$+1.09$
42	1.00	$+0.01$	-1.82	$+1.23$	$=0$	-1.17
45	1.00	$+0.55$	$+1.84$	-1.60	$=0$	-0.28
46	1.00	$+0.55$	$+1.84$	-1.92	$=0$	-0.60
47	1.00	$+1.02$	-1.82	$+3.53$	$=0$	$+1.59$
48	1.00	$+1.03$	-1.82	$+3.06$	$=0$	$+1.13$
50	1.00	$+1.03$	-1.82	$+2.56$	$=0$	$+0.63$
51	1.00	$+1.54$	$+1.87$	-1.39	$=0$	$+0.40$
53	1.00	$+1.56$	$+1.84$	-0.46	$=0$	$+1.31$
58	1.00	$+1.97$	-1.71	$+0.60$	$=0$	-0.80
59	1.00	$+1.97$	-1.74	$+1.05$	$=0$	-0.38
60	1.00	$+1.97$	-1.74	-0.69	$=0$	-2.12
						$\Sigma v^2 = 19.38$

Normal Equations.

$$+17.0000x + 7.0500y + 2.1100\pi + 6.2900 = 0$$
$$+28.5889 \quad -19.1220 \quad + 9.9503 = 0$$
$$+55.3681 \quad -42.4082 = 0$$

Solution.

In units 2d dec. place.	In Arc.
$\pi = +0.9479 \pm 0.1262$	$\pi = +0''.2655 \pm 0''.0353$
$y = +0.4525 \pm 0.1849$	$y = +0.1267 \pm 0.0518$
$x = -0.6753 \pm 0.2108$	$x = -0.1892 \pm 0.0618$

Prob. error of one equation = 0.7935 (Scale) = $0''.2222$ (Arc)

TABLE V.—PARALLAX EQUATIONS.

COMPARISON STARS *e* AND *f*.

Plate.						v.
31	$1.00x$	$-1.44y$	-1.31π	-0.11	$= 0$	-0.44
32	1.00	-1.44	-1.30	$+0.30$	$= 0$	-0.02
33	1.00	-1.42	-1.19	$+1.49$	$= 0$	$+1.29$
34	1.00	-1.42	-1.19	-0.09	$= 0$	-0.29
41	1.00	$+0.01$	$+1.49$	-1.51	$= 0$	$+0.69$
42	1.00	$+0.01$	$+1.46$	-1.82	$= 0$	$+0.35$
43	1.00	$+0.55$	-1.36	$+1.03$	$= 0$	-0.33
44	1.00	$+0.55$	-1.36	$+1.60$	$= 0$	$+0.24$
46	1.00	$+0.55$	-1.34	$+1.07$	$= 0$	-0.27
47	1.00	$+1.02$	$+1.45$	-2.35	$= 0$	-0.69
48	1.00	$+1.03$	$+1.42$	-2.78	$= 0$	-1.16
49	1.00	$+1.03$	$+1.41$	-3.34	$= 0$	-1.73
50	1.00	$+1.03$	$+1.40$	-1.58	$= 0$	$+0.02$
57	1.00	$+1.97$	$+1.55$	-1.69	$= 0$	-0.37
58	1.00	$+1.97$	$+1.55$	$+0.33$	$= 0$	$+1.66$
59	1.00	$+1.97$	$+1.55$	-2.23	$= 0$	-0.91
60	1.00	$+1.97$	$+1.55$	$+0.62$	$= 0$	$+1.94$
						$\Sigma v^2 = 15.07$

Normal Equations.

$$+17.0000x + 7.9400y + 5.7800\pi - 11.0600 = 0$$
$$+28.8344 \quad +22.9844 \quad -16.4388 = 0$$
$$+33.7618 \quad -30.7096 = 0$$

Solution.

In units 2d dec. place.	In Arc.
$\pi = +1.1570 \pm 0.1783$	$\pi = +0''.3241 \pm 0''.0499$
$y = -0.4854 \pm 0.2006$	$y = -0.1360 \pm 0.0562$
$x = +0.4839 \pm 0.1820$	$x = +0.1355 \pm 0.0510$

Prob. error of one equation = 0.7001 (Scale) = 0''.1961 (Arc)

TABLE V.—PARALLAX EQUATIONS.

COMPARISON STARS *c* AND θ.

Plate.						v.
31	1.00x	−1.44y	+1.89π	−1.63	= 0	+0.21
32	1.00	−1.44	+1.89	−1.78	= 0	+0.06
33	1.00	−1.42	+1.84	−2.52	= 0	−0.72
34	1.00	−1.42	+1.84	−1.30	= 0	+0.50
38	1.00	−0.44	+1.93	−1.47	= 0	−0.06
40	1.00	+0.01	−1.81	+1.34	= 0	+0.55
41	1.00	+0.01	−1.81	+0.35	= 0	−0.44
42	1.00	+0.01	−1.84	+0.05	= 0	−0.75
43	1.00	+0.55	+1.91	−0.61	= 0	+0.37
44	1.00	+0.55	+1.91	−3.42	= 0	−2.44
45	1.00	+0.55	+1.90	−1.23	= 0	−0.26
46	1.00	+0.55	+1.90	+0.21	= 0	+1.18
47	1.00	+1.02	−1.84	+1.77	= 0	+0.53
48	1.00	+1.03	−1.84	+3.15	= 0	+1.90
49	1.00	+1.03	−1.84	+0.63	= 0	−0.62
50	1.00	+1.03	−1.85	+1.87	= 0	+0.62
51	1.00	+1.54	+1.91	+0.33	= 0	+0.88
53	1.00	+1.56	+1.89	−0.19	= 0	+0.34
57	1.00	+1.97	−1.66	+1.77	= 0	+0.21
58	1.00	+1.97	−1.66	+0.81	= 0	−0.75
59	1.00	+1.97	−1.70	+1.84	= 0	+0.26
60	1.00	+1.97	−1.70	−0.01	= 0	−1.59
						$\Sigma v^2 = 18.18$

Normal Equations.

$$+22.0000x + 11.1600y + 1.2600\pi - 0.0400 = 0$$
$$+34.1358 \quad -22.3029 \quad +24.7459 = 0$$
$$+74.1850 \quad -49.8911 = 0$$

Solution.

In units 2d dec. place.	In Arc.
$\pi = +0.5381 \pm 0.0909$	$\pi = +0''.1507 \pm 0''.0255$
$y = -0.4362 \pm 0.1466$	$y = -0.1222 \pm 0.0411$
$x = +0.1923 \pm 0.1638$	$x = +0.0539 \pm 0.0459$

Scale. Arc.

Prob. error of one equation = 0.6779 = 0''.1899