On Rocking Stones. *By* E. J. Routh, F.R.S.

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The following problem seems to me interesting from its generality, and the simplicity of the results to which we are led. I have not thought it necessary to do more than indicate the general course of the demonstration.

A heavy body oscillates in three dimensions, with one degree of freedom, on a fixed rough surface of any form in such a manner that there is no rotation about the common normal.

1. Let O be the point of contact, Oz the common normal, Oy a tangent to the arc of rolling determined by the geometrical conditions of the question, OI the instantaneous axis. Then OI, Oy are conjugate diameters in the relative indicatrix.

The relative indicatrix is a conic having its centre at O and lying in the common tangent plane at O, such that the difference of the curvatures of the normal sections through any radius vector OR varies as $\frac{1}{OR^2}$.

2. Let ρ, ρ' be the radii of the normal sections through Oy, taken positively when the curvatures are in opposite directions, and let

$$\frac{1}{s}=\frac{1}{\rho}+\frac{1}{\rho'}.$$

Measure a length $s \cdot \sin^2 yOI$ along the common normal Oz, and describe a cylinder on it as diameter, the axis being parallel to OI. If the centre of gravity of the body be inside, the equilibrium is stable; if outside and above, unstable. This cylinder may therefore be called the cylinder of stability.

3. Let G be the centre of gravity, and let OG produced cut the cylinder of stability in V; then if K be the radius of gyration about OI, the length L of the simple equivalent pendulum is given by

$$\frac{K^2}{L}=GV \cdot \sin^2 GOI.$$

This equation may also be written in the form

$$\frac{K^2}{L}=s\cos Goz \cdot \sin^2 yOI - OG \cdot \sin^2 GOI.$$

This result may be obtained by taking moments about the instantaneous axis.

4. The motion of the upper body is the same as if the fixed surface were plane and the curvature of the upper body at the point of contact altered so that the relative indicatrix remain the same as before. This supplies an easy method of finding the oscillations in any particular case.