magnetic stress $B^2/8\pi$, is there spoken of as if it were of the same nature as a simple longitudinal stress of compression, producing a contraction of the length in consequence of the elasticity of the metal....But I see no ground for treating this purely hypothetical strain as a 'correction' to be applied, either one way or the other, to the observed changes of length." Mr. L. R. Wilberforce concurred in this opinion. To answer these criticisms it was necessary to devise an experiment that would isolate this effect from all others. As I saw no way of doing this in the case of iron I turned to the analogous case of a dielectric in an electrostatic field; for since Quincke had stated that the variation in the length was proportional to $H^2/8\pi$, it seemed that most of the effect was due to this stress. My recent experiments compel me to think that my original correction was erroneous and that Professor Ewing was right.

XV. *On a Theorem analogous to the Virial Theorem.*

By Lord RAYLEIGH, *F.R.S.**

AS an example of the generality of the theorem of Clausius, Maxwell† mentions that "in any framed structure consisting of struts and ties, the sum of the products of the pressure in each strut into its length, exceeds the sum of the products of the tension of each tie into its length, by the product of the weight of the whole structure into the height of its centre of gravity above the foundations." It will be convenient to sketch first the proof of the purely statical theorem of which the above is an example, and afterwards of the corresponding statical applications of the analogue. The proof of the general dynamical theorem will then easily follow.

If X, Y, Z denote the components, parallel to the axes, of the various forces which act upon a particle at the point x, y, z, then since the system is in equilibrium,

$$\Sigma X=0, \quad \Sigma Y=0, \quad \Sigma Z=0.$$

If we multiply these equations by x, y, z respectively, and afterwards effect a summation over all the particles of the system, we obtain a result which may be written

$$\Sigma[x\,.\,\Sigma X+y\,.\,\Sigma Y+z\,.\,\Sigma Z]=0. \quad . \quad . \quad . \quad (1)$$

The utility of the equation depends upon an alteration in the manner of summation, and in particular upon a separation

* Communicated by the Author.

† 'Nature,' vol. x. p. 477, 1874; 'Scientific Papers,' vol. ii. p. 410.

of the forces R (considered positive when repellent) which act mutually between two particles along their line of junction ρ. If x, y, z and x', y', z' be the coordinates of the particles, we have so far as regards the above-mentioned forces,

$$X(x'-x)+Y(y'-y)+Z(z'-z)=R\rho\ ;$$

or with summation over every pair of particles $\Sigma R\rho$. The complete equation may now be written

$$\Sigma(Xx+Yy+Zz)+\Sigma R\rho=0, \quad . \quad . \quad . \quad (2)$$

where in the first summation X, Y, Z represent the components of the *external* forces operative at the point x, y, z. In Maxwell's example the only external forces are the weights of the various parts of the system (supposed to be concentrated at the junctions of the struts and ties), and the reactions at the foundations.

The analogous theorem, to which attention is now called, is derived in a similar manner from the equally evident equation

$$\Sigma[x\,.\,\Sigma Y+y\,.\,\Sigma X]=0. \quad . \quad . \quad . \quad . \quad (3)$$

We have to extract from the summation on the left the force R mutually operative between the particles at x, y, z and at x', y', z'; and we shall limit ourselves to the case of two dimensions. If X, Y be the components of force acting upon the latter particle, ρ the distance between the particles, and ϕ the inclination of ρ to the axis of x, we have

$$Y(x'-x)+X(y'-y)=R\rho\sin 2\phi\ ;$$

so that if now X, Y represent the total *external* force acting at x, y, (3) becomes

$$\Sigma\,[xY+yX]+\Sigma\,R\rho\sin 2\phi=0. \quad . \quad . \quad . \quad (4)$$

where the first summation extends to every particle and the second to every *pair* of particles.

If the external force at x, y be P and be inclined at an angle α, we have

$$X=P\cos\alpha, \quad Y=P\sin\alpha\ ;$$

so that, if $x=r\cos\theta$, $y=r\sin\theta$ as usual, (4) may be written

$$\Sigma\,Pr\sin(\theta+\alpha)+\Sigma\,R\rho\sin 2\phi=0. \quad . \quad . \quad (5)$$

As simple examples of these equations, consider the square framework with one diagonal represented in figs. 1 and 2, and take the coordinate axes parallel to the sides of the square. Since $\sin 2\phi=0$ for all four sides of the square, the only R that occurs is that which acts along the diagonal where $\sin 2\phi = -1$. In fig. 1 opposed forces P act at the middle points of the sides, but since in each case $\theta + \alpha = 0$, the terms containing P disappear. Hence $R = 0$.

Fig. 1.

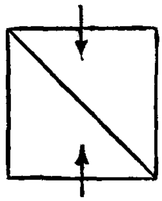
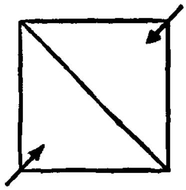


Fig. 2.



In fig. 2, where external forces P act diagonally at the unconnected corners, $\sin(\theta + \alpha) = -1$, and since $\rho = 2r$, $R = -P$, signifying that the diagonal piece acts as a tie under tension P. In neither case would the *weight* of the members disturb the conclusion.

The forces exercised by the containing vessel upon a liquid confined under hydrostatic pressure p contribute nothing to the left-hand member of (4). The normal force acting inwards upon the element of boundary ds is pds, so that

$$X = -p\,dy, \qquad Y = p\,dx,$$

and accordingly

$$\Sigma [xY + yX] = \tfrac{1}{2} p \int d(x^2 - y^2),$$

vanishing when the integration extends over the whole boundary.

Abandoning now the supposition that the particle at x, y is at rest, we have

$$\frac{d^2(xy)}{dt^2} = 2\frac{dx}{dt}\frac{dy}{dt} + x\frac{d^2y}{dt^2} + y\frac{d^2x}{dt^2},$$

so that if m be the mass of the particle, X, Y the components of force acting upon it,

$$2m\frac{dx}{dt}\frac{dy}{dt} = m\frac{d^2(xy)}{dt^2} + xY + yX; \quad . \quad . \quad . \quad (6)$$

or with summation over all the particles of the system,

$$2\Sigma m\frac{dx}{dt}\frac{dy}{dt} = \frac{d^2}{dt^2}\Sigma(mxy) + \Sigma(xY + yX). \quad . \quad . \quad (7)$$

We now take the mean values with respect to time of the various terms in (7). If the system be such that

$$\frac{d}{dt}\Sigma(mxy)$$

does not continually increase, we obtain, as in the case of the virial theorem,

$$2\Sigma m\frac{dx}{dt}\frac{dy}{dt}=\Sigma(x\mathrm{Y}+y\mathrm{X}). \quad . \quad . \quad . \quad . \quad (8)$$

It would seem that this equation has application to the molecular theory of the viscosity of gases, analogous to that of the virial as applied to hydrostatic pressure.

XVI. *The Classes of Progressive Long Waves.* *By* R. F. GWYTHER, *M.A.**

ADOPTING the method employed by Lord Rayleigh in his paper "On Waves" †, write

$$\phi+i\psi=\mathrm{F}(x+iy)$$

where F is a real function, and therefore

$$\psi=\mathrm{F}'y-\mathrm{F}'''\frac{y^3}{3!}+\text{ etc.}, \quad . \quad . \quad . \quad . \quad (1)$$

while the condition expressing the uniformity of pressure along the free surface, for which $\psi=-ch$, is

$$u^2+v^2=c^2-2g(y-h). \quad . \quad . \quad . \quad . \quad (2)$$

But now, instead of obtaining a differential equation approximately related to the free surface, proceed to eliminate y between (1) and (2), putting $\psi=-ch$.

The convergency of these expressions in the case of long waves is clear, and we easily obtain

$$\mathrm{F}'^2-(\mathrm{F}'\mathrm{F}'''-\mathrm{F}''^2)\frac{c^2h^2}{\mathrm{F}'^2}+\text{ etc.}$$

$$=c^2+2gh+2g\left[\frac{ch}{\mathrm{F}'}+\frac{c^3h^3\mathrm{F}'''}{3!\,\mathrm{F}'^4}+\text{ etc.}\right]. \quad . \quad . \quad (3)$$

This mode of treatment has the merit of simplicity and also of allowing the constants introduced in the integration by approximate methods to be treated with a feeling of security.

Write (3) to a first approximation

$$\left(c^2+\frac{c^3gh}{3\mathrm{F}'^3}\right)h^2\mathrm{F}'''=\mathrm{F}'^3-(c^2+2gh)\mathrm{F}'-2gch. \quad . \quad (4)$$

* Communicated by the Author.

† Phil. Mag., April 1876.