Clebsch.—4. Ueber die ebene Abbildung der geradlinigen Flächen vierter Ordnung, welche eine Doppel curve dritten Grades besitzen. *Ann. Clebsch & Neumann* (1870), pp. 445—466.

——— 5. Ueber die Abbildung einer Classe von Flächen fünfter Ordnung. *Gött. Abh.*, t. xv., 64 p.

Cremona.—Sulle superficie gobbe di quarto grado. *Mem. di Bologna*, t. viii. (30 April, 1868), 15 p.

De la Gournerie.—1. Memoire sur la surface engendrée par la révolution d'une conique autour d'une droite située d'une manière quelconque dans l'espace. . *Jour. de l'Ec. Polyt.*, t. xxiii. (1863), pp. 1—74.

——— 2. Memoire sur les lignes spiriquès. *Liouv.*, t. xiv. (1869), 92 p.

Geiser.—Ueber die Flächen vierten Grades, welche eine Doppelcurve zweiten Grades haben. *Crelle*, t. lxx. (1869), pp. 249—257.

Korndörfer.—Ueber die Ebene Abbildung, &c. *Clebsch & Neumann*, t. xi.

Kummer.—1. [Surfaces of the fourth order with sixteen conical points.] *Berl. Monatsb.* (1864), pp. 246—260, and 495—499.

——— 2. Ueber die algebraischen Strahlensysteme, insbesondere über die der ersten und zweiten Ordnung. *Berl. Abh.* (1866), pp. 1—120.

——— 3. Ueber die Flächen vierten Grades auf welchen Schaaren von Kegelschnitten liegen. *Berl. Monatsb.* (July, 1863). *Crelle*, t. lxiv. (1864), pp. 66—76.

Maxwell.—On the Cyclide. *Quart. Math. Jour.*, t. ix. (1867), pp. 111—126.

Schwarz.—Ueber die geradlinigen Flächen fünften Grades. *Crelle*, t. lxvii. (1867), pp. 23—57.

December 8th, 1870.

W. SPOTTISWOODE, Esq., F.R.S., President, in the Chair.

Prof. Kanikoff was present at the Meeting.

Mr. J. Hamblin Smith, M.A., of Caius College, Cambridge, was elected a Member of the Society.

The Auditor (Mr. S. Roberts, M.A.) certified that he had examined the Treasurer's accounts, and found them perfectly correct.

Prof. Smith, V.P., made a communication to the Society on the subject of Elliptic Integrals.

Prof. Cayley, V.P., then gave an account of the following

Note on the Theory of the Rational Transformation between Two Planes, and on Special Systems of Points.

In Prof. Cremona's theory of the transformation of plane curves, the fundamental equations are taken to be

$$\alpha_1+4\alpha_2+9\alpha_3+\ldots=n^2-1 \quad\ldots\ldots\ldots (1),$$

$$\alpha_1+3\alpha_2+6\alpha_3+\ldots=\tfrac{1}{2}(n^2+3n)-2 \quad\ldots\ldots\ldots (2);$$

and from these we have as a consequence

$$\alpha_2+3\alpha_3+\ldots=\tfrac{1}{2}(n-1)(n-1) \quad\ldots\ldots\ldots (3);$$

viz., the first equation expresses that any two curves of the system intersect in a single variable point; the second, that the curves form a *reseau*, or system containing two arbitrary parameters; and the third, that the curves are unicursal.

In the equivalent theory of the rational transformation between two planes, as given in my "Memoir on the Rational Transformation between Two Spaces," we have the equation (1); but instead of the equation (2), it would *primâ facie* appear to be sufficient if we had the inequation $\alpha_1+3\alpha_2+6\alpha_3+\ldots \overline{<} \ldots \tfrac{1}{2}(n^2+3n)-2$;

but on the ground there explained, the case

$$\alpha_1+3\alpha_2+6\alpha_3+\ldots<\tfrac{1}{2}(n^2+3n)-2$$

is excluded, and we thus have the equation (2), giving with (1) the equation (3).

I believe the better course is to assume (1) and (3) as the fundamental equations, from them deducing (2); and we thus also get over a difficulty presently referred to, but which did not occur to me when the memoir was written.

In fact, starting with the equations $x':y':z'=X:Y:Z$ (which are to give $x:y:z=X':Y':Z'$), we have in the first instance the equation (1). Moreover, establishing x', y', z' a linear equation $ax'+by'+cz'=0$, we have corresponding hereto a curve $aX+bY+cZ=0$, and the coordinates x, y, z of a point on this curve are proportional to $X':Y':Z'$; that is, substituting for z' the value $-\dfrac{1}{c}(ax'+by')$, they are proportional to rational and integral (homogeneous) functions of (x', y'), that is, to rational and integral functions of the single parameter $x':y'$; wherefore the curve $aX+bY+cZ=0$, is unicursal; whence the equation (3). The like change may be made in the theory of the rational transformation between two spaces; and it is in this case a more important one.

The difficulty is as follows: It is not self-evident that we are at liberty to assume $\alpha_1+3\alpha_2+6\alpha_3\ldots \overline{<} \tfrac{1}{2}(n^2+3n)-2$;

for imagine that we had a system of $(a_1, a_2, a_3, \ldots)$ points, such that $a_1+4a_2+\ldots$ being $=n^2-1$, and $a_1+3a_2+\ldots$ being $>\frac{1}{2}(n^2+3n)-2$, the points were such that the conditions in question (viz., the condition that the curve passes once through each of the points a_1, twice through each of the points $a_2 \ldots$) should be *less* than $a_1+3a_2+\ldots$, and in fact $=$ or $<\frac{1}{2}(n^2+3n)-2$; then the functions X, Y, Z would not of necessity be connected by a linear relation $\lambda X+\mu Y+\nu Z=0$, and the ground for the assumption in question, $a_1+3a_2+\ldots \overline{\gtrless} \frac{1}{2}(n^2+3n)-2$, would no longer exist. And except by the process now adopted of deriving the equation (2) from the equations (1) and (3), I do not know how the impossibility of such a system is to be established; viz., I do not know how we are to prove the following theorem:—There is not any system of $(a_1, a_2, a_3 \ldots)$ points, where

$$a_1+4a_2+9a_3 \ldots = n^2-1,$$
$$a_1+3a_2+6a_3 \ldots > \tfrac{1}{2}(n^2+3n)-2,$$

such that (a curve of the order n passing once through each point a_1, twice through each point $a_2, \ldots$) the number of conditions actually imposed on the curve is $=$ or $<\frac{1}{2}(n^2+3n)-2$.

A system of $(a_1, a_2 \ldots)$ points such that the number of actually imposed conditions is less than $a_1+3a_2+\ldots$, may be termed a special system; we have, of course, the well-known case $(a_1=n^2)$ of a system of n^2 points, such that any curve of the order n passing through $\frac{1}{2}(n^2+3n)-1$ of these passes through all the remaining points [or what is the same thing, where the number of conditions actually imposed is $=\frac{1}{2}(n^2+3n)-1$]; and we have the following special system, which presented itself to Dr. Clebsch, in his researches on the Abbildung of a quintic surface with two non-intersecting nodal lines; viz., "$a_1=12$, $a_2=2$. We may have 12 points and 2 points such that, for a quintic curve passing once through each of the 12 points and twice through each of the 2 points, the number of conditions actually imposed (instead of being $12+3.2, =18$) is $=17$." The construction is as follows: viz., Starting with the 2 points and any 10 points, we may draw a quartic passing twice through the first of the 2 points, once through the second of them, and through the 10 points; and another quartic passing twice through the second of the 2 points, once through the first of them, and through the 10 points: the two quartics intersect in the 2 points each twice, in the 10 points, and in 2 new points, forming, with the 10 points, a system of 12 points; and the first-mentioned 2 points and the 12 points form the system in question.

A more complicated case, $a_1=10$, $a_2=6$, $a_3=1$, occurs in Dr. Nöther's paper, "Ueber Flächen, welche Schaaren rationaler Curven besitzen," Leipzig, 1870. Except these two, I do not know any other case of a special system for which $a_2, a_3 \ldots$ are not all $=0$; the investigation of such systems would, I think, be very interesting.

[In my "Memoir on the Rational Transformation between two Spaces," p. 179, line 7 from the bottom, the factor $(3n-1)$ should be $(3n-2)$; and p. 180, eighth line from the bottom, the factor $3n-4r+6$ ought to have been $3n-4r+4$, viz., the whole term should be diminished by $r(r+1)$. But the "correction," sixth line from the bottom, should be increased by this same quantity, so that the value of the "postulation," as given fourth line from the bottom, is correct.]

A short discussion took place on the above paper; and then Professor Cayley gave an account of the following paper:—

A Second Memoir on Quartic Surfaces.

In my Memoir on Quartic Surfaces, *ante* pp. 19—69, although remarking (See No. 79) that the identification was not completely made out, I tacitly assumed that the symmetroid and the decadianome (each of them a quartic surface with 10 nodes) were in fact identical. There is yet a good deal which I cannot completely explain; but the truth appears to be, that the decadianome includes two cases of coordinate generality, say the sextic decadianome, and the bicubic decadianome = symmetroid: viz., in the first of these the circumscribed cone, having for vertex any one of the 10 nodes, is a proper sextic cone with 9 double lines; in the second it is a system of two cubic cones, intersecting, of course, in 9 lines, which are double lines of the aggregate sextic cone: or, in the notation of the Table No. 11, in the case of the sextic decadianome, the circumscribed cones are each of them 6_9; in that of the bicubic dianome = symmetroid, they are each of them (3, 3). We thus arrive at a very remarkable system of 10 points in space, viz., giving the name "ennead" to any 9 points *in plano*, which are the intersections of two cubic curves, or to any 9 lines through a point which are the intersections of two cubic cones; the 10 points in space are such that, taking any one whatever of them as vertex, and joining it with the remaining points, the 9 lines form an ennead. I purpose in the present short Memoir to consider the theories in question; the paragraphs are numbered consecutively with those of the Memoir on Quartic Surfaces.

Plane Sextic Curve with 9 Nodes.

110. A sextic curve contains 27 constants; and the number of conditions to be satisfied in order that a given point may be a node is = 3. Hence it would at first sight appear that the curve could be found so as to have 9 given nodes; this would be $9\times3, =27$ condi-