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An Experimental Test of the Normal Law of Error.

By J. W. Nixon, B.Sc.

There have been many attempts to show how results obtained at random conform to the normal law of error. There is Professor Pearson's analysis of the Monte Carlo Roulette figures (*Chances of Death*, vol. 1, p. 42), the late Professor Weldon's results of dice-throwing (cited by Professor Edgeworth in "Law of Error" (*Ency. Brit.*, 10th edition, p. 280), and various results in Westergaard's Grundzüge. The following figures are an attempt to show how aggregates of quantities which conform to scientific law may yet exhibit signs of fortuitousness as tested by the normal curve of error. Chambers's logarithm tables were selected for the purpose, and the digits used were those forming the seventh place of decimals of the 10,000 logarithms, from 801 to 10,800. These were summed successively in batches of 25; from each sum was subtracted 112·5 (25 × 4·5), and each difference so obtained was divided by 5, (the square root of 25). This experiment, it will be noticed, is identical with that given (for 1,200 logs.) by Professor Edgeworth in his Presidential Address (*Journal*, January, 1913, p. 189). The figures so obtained should conform approximately to a normal curve of centre zero and standard deviation $\sqrt{8{\cdot}25}$. The figures are grouped and compared with the normal curve in Table I.

Table I.

Deviations.		In groups of 25. Observed.	In groups of 25. Calculated.	In groups of 100. Observed.	In groups of 100. Calculated.	$\frac{(\text{Difference})^2}{\text{Calculated}}$
Negative	8—7 ...	1	1·32	0	} 1	} 0
	7—6 ...	2	2·76	1		
	6—5	10	6·56	1	2	·50
	5—4 ...	13	13·80	1	3	1·33
	4—3	17	24·76	9	6	1·50
	3—2 ...	39	38·70	9	10	·10
	2—1 ...	58	52·00	10	13	·69
	1—0 ...	62	60·10	13	15	·266
Positive	0—1	57	60·10	11	15	1·066
	1—2	48	52·00	16	13	·69
	2—3 ...	36	38·70	9	10	·10
	3—4	15	24·76	9	6	1·50
	4—5	10	13·80	5	3	1·33
	5—6	12	6·56	5	2	4·50
	6—7 ...	5	2·76	0 }	1	0
	7—8	9	} 1·32	0		
	8—9	5		0		$\chi^2 = 13{\cdot}572$
	9—	1	—	1		P = ·40
		400	400	100	100	

The same figures were then summed in batches of 100; the deviations from 450 were divided by $\sqrt{100}$ and compared with a normal curve of the same standard deviation as the former case. For these figures the criterion P (Pearson, *Phil. Mag.*, July, 1900) was evaluated. The results are shown in Table I.

Hence the chance is about 3 to 2 against a random sample leading to a system of deviations as great as that shown by the data.

It was noticed, however, while extracting the figures that they were not entirely independent. Many "runs" of figures were noticed, *e.g.*:

In the seventh decimal place of logs.		5494—5500	there is a run of	7 sevens.
"	"	7752—7759	"	8 eights.
"	"	5711—5718	"	8 twos.
"	"	8681—8690	"	10 eights.
"	"	8858—8869	"	12 sevens.

These sequences were due to the fact that the second differences of successive logarithms were so small as to render the first differences (up to the seventh decimal place) constant, and only after a succession of logarithms did these differences accumulate sufficiently to assert themselves in the seventh place. It is interesting at this point to note that in the *sixth* decimal place of logs., 8,669 to 8,703, there is a run of 35 nines! A similar reason accounts for runs of quantities in pairs, where 5 is the common difference, such as:—

7,2,7,2,7,2 . . .
8,3,8,3,8,3 . . .
0,5,0,5,0,5 . . .

To test the result when this cause was removed, a hundred totals (of 100 each) were obtained by summing the last digits of the logarithms of—

801, 901, 1001 . . . 10,701
802, 902, 1002 . . . 10,702

instead of 801, 802, 803 . . . 900

This was carried out by arranging the whole of the 10,000 figures in a square (100 × 100) of which the rows when summed would give the figures above quoted and the columns gave the totals now sought. A quarter of this square is printed as a specimen in Table II. The results were as follow:—

Table III.

	Negative deviations.						
	Over 6.	6—5.	5—4.	4—3.	3—2.	2—1.	1—0.
Observed	2	3	2	5	11	8	14
Theory	1	2	3	6	10	13	15
$\frac{\text{Difference}^2}{\text{Theory}}$	1	·5	·33	·16	·1	1·92	·066

TABLE III—*Contd.*

	Positive deviations.						
	0—1.	1—2.	2—3.	3—4.	4—5.	5—6.	Over 6.
Observed	11	10	10	10	7	3	4
Theory	15	13	10	6	3	2	1
$\frac{\text{Difference}^2}{\text{Theory}}$	1·06	·69	0	2·66	5·33	·5	·9

Whence $x^2 = 23{\cdot}32$, P = ·036 (about).

and the chance is thus about 26 to 1 against such a grouping being a random sample from a purely normal distribution. The unsatisfactory nature of this result is almost entirely due to the last batch of twenty-five deviations of which twenty-two were positive. The first fifty show a much closer correspondence with theory, as shown by the following tables (IV and V). As only fifty totals were used, the figures were not grouped in units of deviation, but according as they were between the upper and lower quartile, the quartile and the decile, &c. The quartile in the normal curve is at 1·9, the decile at 3·7.

TABLE IV.

	Between 0 and ±1·9.	Between ±1·9 and ±3·7.	Above ±3·7.
Observed	22	14	14
Calculated	25	15	10
$\frac{(\text{Difference})^2}{\text{Calculated}}$	·36	·06	1·6

Whence $\chi^2 = 2{\cdot}02$, P = ·36.

TABLE V.

	Below the lower quartile.	Between the median and the lower quartile.	Between the median and the upper quartile.	Above the upper quartile.
Observed	14	16	16	14
Calculated	12·5	12·5	12·5	12·5
$\frac{(\text{Difference})^2}{\text{Calculated}}$	·18	·98	3·38	·18

$\chi^2 = 4{\cdot}72$, P about ·2.

As a last test, the figures contained in the *first quarter* of the square above referred to were summed diagonally, *i.e.*, the central diagonal of 50 digits was summed, the adjoining diagonal of 49 digits was summed, together with 1 digit in the opposite corner, the next diagonal of 48, together with the 2 digits adjoining the corner digit, and so on. The 50 totals, each of a batch of 50, were grouped as in Tables IV and V. It will be seen that P is approximately of the same order.

	Between 0 and ± 1·9.	Between ± 1·9 and 3·7.	Above ± 3·7.
Observed	20	16	14
Calculated	25	15	10

$\chi^2 = 2{\cdot}22$, P = ·33.

	Below the lower quartile.	Between the median and the lower quartile.	Between the median and the upper quartile.	Above the upper quartile.
Observed	18	9	11	12
Calculated	12·5	12·5	12·5	12·5

$\chi^2 = 3{\cdot}58$, P = ·33.

The whole of the original tabulation of the figures which are summarised in this paper have been deposited with the Society. It only remains for me to thank Professor Edgeworth (at whose suggestion this experiment was undertaken) for the help and criticism he has afforded.

TABLE II.—*Extract from a larger table of 10,000 digits forming the seventh place of decimals of logs* 801–10,800.

54509054509054621399288290535103015350030164744979
85846238944882153758697077079796968686969708103260
17971055242540070822796094414277353884537735377353
36513168501822922810698354919464919353796120564908
56503393416873663787254105649117120576378614405664
43046526763787489848985895910612183541688503316998
10713317010724439243169095022060229489850332835640
79085033295899626886279085045428244293676493553047
36664059110848133294943315641602218379086268986279
35540590095045540580085048370119615788628245429480
78752724665283689974947898527135531725788639479097
15890974947909851602443173702331950468875283680086
11974947911085159234329504700097494802220749368009
17371456652950479009740593567642949257887528370345
03566531737146787530615802221962725801109639381355
63959368909864060470122107405935799875294936801109
20851615802333197394814678875306160357888752950481
73837135777653062725813443218517260256777541849482
43207406159246777653073837146789986418494825789009
63073837035788886529516159145677653184060581357777
11109741738371478011109752849492681234432196395049
74061604702455654319628493714680111198630738371580
07395048258012333219751849493692467888764206395048
88876530740616148146890000976318405159369245677776
73950504703579001009753074050504703578900098753074
51605825802345555431963062838260368023455443197418
63961616047035790111109864185173837259247901233321
11110986418518394837046913567777654318639627272604
17383715925701345666543208630628494937147924578899
14702467890099876429740739494826036913567888887643
91357899009987542974073940594825814680123444332197
03680246788888876431863073950605947147024689011111
61615937147924578901110098653085295273949371592580
87542974185173849382603702579123455555432197530741
70470357912245566665542197520740739405049370470357
44332097530851840627382715936924791345778888876543
40739516161593714703579134567788776543108631851840
05948269258136891234566665543219753085295173940594
26036925791356789900099876531975307417406172727150
36036925791346789900000998765319753074173051727272 6
72604815814691356890123333332109865319641851840627
73950516059482592581468024678901112211099765319752
97531863074173951727272615936036925702457801223344
35677899999887654310864196307406394061616150482592
93837260471470368135790234566777777665432097531863
87654219753186307306395062727271604826966924702468
26037047035803578023456778888888765432197531964185
66543219864208530741740739506172726150482592692570
61616160493704814703681357912456889001111110998764
02456890012222222221099765320864297418529528406283 8