



On some consequences of Gauss's principle in electrostatics

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I asked myself if the greater or less brightness of the sky has not an influence upon the distance sought. To ascertain this I requested my son-in-law to repeat the experiment at night, illuminating the white square by means of a lamp. He did so in the middle of September, at 9 in the evening, the sky being calm and without moon. To remove all lateral light, an assistant took away the lamp as soon as the observer, after contemplating the white square, directed his eyes to the sky. Now, to my great surprise, the result was sensibly the same as in the day experiment. Afterwards my second son operated, and found a value still of the same order. So the brightness appears to have no notable influence upon the appreciation of the distance at which the point observed in the celestial vault is instinctively placed.—*Bulletins de l'Académie Royale de Belgique*, sér. 3, tome ii. nos. 9 and 10, Sept. and Oct. 1881.

ON SOME CONSEQUENCES OF GAUSS'S PRINCIPLE IN ELECTRO-STATICS. BY M. CROULLEBOIS.

M. Bertrand* has deduced from Gauss's principle several important theorems relative to electrostatics; following the same path, I have obtained some interesting relations, and particularly the simplified demonstration of Clerk Maxwell's theorem.

I. Gauss's proposition $\Sigma MV' = \Sigma M'V$ is a pure identity if the potentials be replaced by the expressions which the definition furnishes. This relation can be arrived at independently of any analytical form attributed to the function V , by resting on the notion of electric energy. Let us consider a conductor of charge M , at the potential V , the potential energy of which is $\frac{MV}{2}$; let us vary the charge from M to M' , the potential will vary from V to V' , and the increase of energy, equal to the electrical work expended to bring the additional charge $M' - M$ from infinite distance to the conductor, will be

$$(M' - M) \frac{V + V'}{2}.$$

We shall therefore have for the final energy,

$$M'V' = MV + (M' - M)(V + V'),$$

from which, after simplification, and for a system of conductors,

$$\Sigma MV' = \Sigma M'V. \dots\dots\dots (1)$$

II. From the two sides of the equality (1) let us subtract MV and put $M' - M = \mu$ and $V' - V = u$, we shall have

$$\Sigma Mu = \Sigma V\mu, \dots\dots\dots (2)$$

* *Journal de Physique*, t. iii. p. 74.

or, if the simultaneous modification of the charges and potentials is infinitesimal,

$$\Sigma M \delta V = \Sigma V \delta M. \quad (2')$$

whence this theorem, a consequence of Gauss's principle explained according to § I. :—

In a system of fixed conductors, in which two distinct systems of equilibrium are considered, the sum of the products of the initial charge of each conductor and the variation of its potential from one state to the other is equal to the sum of the products of the initial potential and the variation of the charge.

III. *When conductors, maintained at constant potentials, are left to their mutual actions, the energy of the system tends towards a maximum.*

Clerk Maxwell has demonstrated this theorem* by means of the linear equations which exist between the potentials and the charges. The following process is more direct and speedy.

Let us suppose, at the beginning, each conductor A_1, A_2, \dots insulated, and impress on the system an *infinitely small deformation*: the charges M_1, M_2, \dots do not change; there are for the respective potentials the falls $\delta V_1, \delta V_2, \dots$; the loss of energy, equal to the external work accomplished, is

$$\delta W = -\frac{1}{2} \Sigma M \delta V.$$

Now, the conductors being fixed, let us connect them to constant batteries, in order to restore the potentials to their original values. This restoration of the potentials cannot be effected without additional charges $\delta M_1, \delta M_2, \dots$, regulated by the relation (2'). The positive variation of the initial energy will therefore be

$$\delta' W = +\frac{1}{2} \Sigma M \delta V.$$

We hence conclude that

$$\delta W + \delta' W = 0,$$

and

$$\delta' W - \delta W = \Sigma M \delta V.$$

Therefore (1) the work accomplished, during the displacement, by the electrical forces is equal to the augmentation of energy of the system; (2) the energy furnished by the sources is equal to twice one or the other of those quantities, and is expended *exactly*, half in mechanical work, half in electrical work or potential energy.

According to the equality (2), the preceding theorem applies to a *finite deformation*; but, for its application to the theory of electrometers, there is, as is known, only occasion to consider an elementary modification.—*Comptes Rendus de l'Académie des Sciences*, Jan. 9, 1882, t. xciv. pp. 74–76.

* 'Electricity and Magnetism,' vol. i. p. 96.