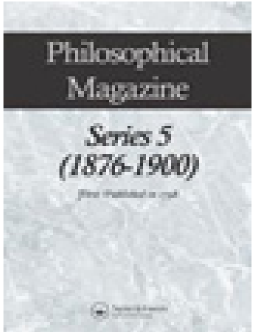

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II. A theory of the synchronous motor

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As such an hypothesis, if true, would be of fundamental importance to chemical and physical philosophy, it is to be desired that it should be thoroughly tested by experiment. Indeed it appears that the present need on the experimental side of thermochemistry is the determination of the heat of formation of all the typical organic compounds, and of the first few members of the homologous series derived from them, with some reliable estimate of the probable error of each value. The first grand general survey of the experimental region of thermochemistry has been carried out by such experimenters as Favre and Silbermann, Andrews, Berthelot, Thomsen, Stohmann, Louguinine, and others, to whom we owe the fine existing body of thermochemical data; but it is now time that those who wish to carry on their work should take up the details and establish for the thermochemical constants of bodies as reliable determinations as the skilled analyst can give of their percentage composition.

The existing data for the heat of combustion of a large number of organic compounds as liquids and solids, such as have been determined by Stohmann and his pupils, and Louguinine and others, could be made available for the theoretical study of their heats of formation as gases, by the calculation of their latent heats of vaporization, according to the equation given in the introduction to the present paper, and the use of approximate values of their specific heats as vapours, if it is desired to reduce all results to a temperature of 18° C. But the discussion of such results will probably be more profitable after the fundamentals of the subject have been more thoroughly investigated.

Melbourne, July 1894.

II. *A Theory of the Synchronous Motor.* By W. G. Rhodes, *M.Sc.**

1. SEVERAL foreign writers, notably Steinmetz †, have given theories of the synchronous motor, but most of them, by failing to see how the analysis could be simplified, add to the difficulties of the theory by mathematical intricacies which are apparently quite unnecessary. The author offers the following attempt to present a theory of the synchronous motor in as simple a way as possible, and as the mathematics for the most part consists of simple algebra, the difficulties are reduced to a physical conception of the subject. Many of the results have already been obtained, and the part for which the author

* Communicated by the Physical Society: read April 26, 1895.

† Trans. Am. Inst. Elec. Eng., December 1894.

chiefly claims originality is the method of attacking the problem.

2. We consider the case of an alternating-current machine whose field is excited by a direct current, while a simple alternating current passes round the armature.

Let p = output of motor ;
c = virtual value of armature current ;
R = resistance of armature ;
E = virtual value of impressed E.M.F. ;
e = " " counter E.M.F. ;
L = coefficient of self-induction of armature ;
n = frequency of armature current ;
I = impedance of armature = $\{R^2 + (2\pi n L)^2\}^{\frac{1}{2}}$;
S = reactance = $2\pi n L$;
ψ = phase-difference between c and E ;
ϕ = " " c and e ;
θ = " " c and Ic.

Then the input $= p + c^2 R$;

and also $= c\mathrm{E} \cos \psi$;

therefore $p + c^2 R = c\mathrm{E} \cos \psi$.

Solving for c we get

$$c = \frac{E}{2R} \cos \psi \pm \frac{1}{2R} \sqrt{\{E^2 \cos^2 \psi - 4pR\}}. \quad . \quad . \quad . \quad (1)$$

Since c is always real, we must have

$$E^2 \cos^2 \psi \geqq 4pR ;$$

therefore the maximum value of p is

$$p = \frac{E^2}{4R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

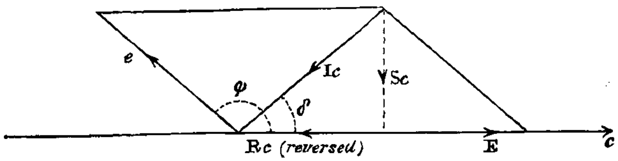
This occurs when $\psi = 0$; that is, when the current and the impressed E.M.F. are in phase with each other.

3. We notice that the maximum output is the same as the maximum energy which can be given to an external circuit by a generator of constant E.M.F., E. From (1) we get the corresponding value of the current

$$c = \frac{E}{2R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

To find the corresponding value of e we notice that E, e, and Ic (the resultant of Sc and Rc reversed) are in equilibrium

Fig. 1.



amongst themselves; so that taking components of these along and at right angles to the direction of E, we have

$$\left.\begin{aligned} -e\cos\phi &= \mathrm{E} - \mathrm{R}c \\ \text{and} \qquad e\sin\phi &= \mathrm{S}c \end{aligned}\right\};$$

$$\left.\begin{aligned} \text{therefore} \qquad e\cos\phi &= 2\mathrm{R}c - \mathrm{R}c = \mathrm{R}c \\ \text{and} \qquad e\sin\phi &= \mathrm{S}c \end{aligned}\right\}\text{from (3).}$$

Squaring and adding, we get

$$e^2 = (\mathrm{R}^2 + \mathrm{S}^2)c^2 = \mathrm{I}^2c^2,$$

therefore

$$e = \mathrm{I}c = \frac{\mathrm{IE}}{2\mathrm{R}}. \qquad (4)$$

Also, by division,

$$-\tan\phi = \frac{\mathrm{S}}{\mathrm{R}} = \tan\theta \text{ (see fig. 1).}$$

4. We thus find that when working at maximum output:—

(1) The impressed E.M.F. is in phase with the current in the armature.

(2) The maximum output is $p = \frac{\mathrm{E}^2}{4\mathrm{R}}$.

(3) The corresponding current in armature is $c = \frac{\mathrm{E}}{2\mathrm{R}}$.

(4) The corresponding counter E.M.F. is $e = \frac{\mathrm{IE}}{2\mathrm{R}}$.

(5) The angle of phase between the armature-current and the E.M.F. necessary to overcome the resistance and self-induction of the armature is equal and opposite to the angle between the current and the counter E.M.F.

(6) Also from (4) we see that

$$e \gtreqless \mathrm{E} \text{ according as } \mathrm{I} \gtreqless 2\mathrm{R},$$

$$\text{that is according as } (2\pi n\mathrm{L})^2 + \mathrm{R}^2 \gtreqless 4\mathrm{R}^2,$$

$$\text{,, \quad ,, } \quad \mathrm{L}^2 \gtreqless \frac{3\mathrm{R}^2}{4\pi^2 n^2},$$

$$\text{,, \quad ,, } \quad \mathrm{L} \gtreqless \frac{\mathrm{R}\sqrt{3}}{2\pi n}.$$

Running Light.

5. We have

$$\left.\begin{aligned} p + c^2\mathrm{R} &= c\mathrm{E}\cos\psi \\ \text{and} \qquad p &= ce\cos\phi \end{aligned}\right\}.$$

If we neglect the friction of the bearings &c., we may, in this case, put $p=0$; we then have

$$\phi = \pm\frac{\pi}{2}$$

and

$$c\mathrm{R} = \mathrm{E}\cos\psi.$$

Hence the maximum value of c is (putting $\psi=0$)

$$c = \frac{\mathrm{E}}{\mathrm{R}},$$

the same as would be produced by a constant E.M.F. E in a non-inductive circuit of resistance R.

Also putting $\psi = \pm\frac{\pi}{2}$, the minimum current is zero.

Now, from fig. 1, we get (of course E and c are not now in phase)

$$\mathrm{E}^2 = e^2 + \mathrm{I}^2c^2 + 2\mathrm{I}ce\cos(\theta - \phi);$$

when $\phi = \pm\frac{\pi}{2}$, this becomes

$$\begin{aligned} \mathrm{E}^2 &= e^2 + \mathrm{I}^2c^2 \pm 2\mathrm{I}ce\sin\theta \\ &= e^2 + \mathrm{I}^2c^2 \pm 2\mathrm{S}ce, \quad \ldots \ldots \quad (5) \end{aligned}$$

since $\sin\theta = \frac{\mathrm{S}}{\mathrm{I}}$.

The upper sign in (5) corresponds to the machine running as a generator, and the lower sign as a motor.

We also see from (5) that corresponding to $c = \frac{\mathrm{E}}{\mathrm{R}}$ we have

$$e = \mp\frac{\mathrm{ES}}{\mathrm{R}},$$

and, corresponding to $c=0$, we have

$$e=\pm E.$$

Now, solving equation (5) as a quadratic in c we get

$$c=\mp\frac{eS}{I^2}\pm\frac{1}{I^2}\sqrt{(I^2E^2-R^2e^2)}, \quad . \quad . \quad . \quad . \quad (6)$$

and, as c is real, we must have

$$I^2E^2 \geqq R^2e^2$$

or

$$IE \geqq Re;$$

therefore the maximum value of e is given by

$$e=\pm\frac{IE}{R}$$

and the corresponding current is

$$c=\mp\frac{eS}{I^2}$$

$$=\mp\frac{SE}{RI}.$$

The equation

$$e^2-2Sce+I^2c^2=E^2$$

is the characteristic curve of the motor running light. It may be written

$$e^2-2Sce+S^2c^2+R^2c^2=E^2$$

or

$$(e-Sc)^2+R^2c^2=E^2,$$

or

$$\frac{(e-Sc)^2}{E^2}+\frac{c^2}{\frac{E^2}{R^2}}=1,$$

which is the equation to an ellipse having as conjugate diameters the lines

$$e-Sc=0 \text{ and } c=0.$$

Similarly, the equation

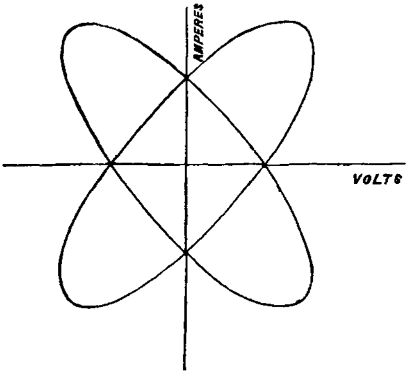
$$e^2+2Sce+I^2c^2=E^2,$$

which refers to the generator, may be written

$$\frac{(e+Sc)^2}{E^2}+\frac{c^2}{\frac{E^2}{R^2}}=1,$$

also the equation to an ellipse. These ellipses are represented in fig. 2.

Fig. 2.



Minimum Current at Given Power.

6. We have

$$p+c^2\mathrm{R}=\mathrm{E}c\cos\psi.$$

The current is a minimum when $\frac{dc}{d\psi}=0$. Now, differentiating with respect to ψ,

$$(2c\mathrm{R}-\mathrm{E}\cos\psi)\frac{dc}{d\psi}+\mathrm{E}\,c\sin\psi=0\,;$$

therefore, when $\frac{dc}{d\psi}=0$, we have

$$\sin\psi=0,$$

or

$$\psi=0\,;$$

that is, the current is a minimum when in phase with the impressed E.M.F., as is otherwise obvious. Putting, therefore, $\psi=0$, we have

$$p+c^2\mathrm{R}=\mathrm{E}c. \quad \ldots \quad (7)$$

This curve is of the second degree in c and p, and is satisfied by the following system of points:—

$$(a)\ \left.\begin{array}{l} c=0,\\ p=0,\end{array}\right\} \qquad (b)\ \left.\begin{array}{l} c=\frac{\mathrm{E}}{2\mathrm{R}},\\ p=\frac{\mathrm{E}^2}{4\mathrm{R}},\end{array}\right\} \qquad (c)\ \left.\begin{array}{l} c=\frac{\mathrm{E}}{\mathrm{R}},\\ p=0.\end{array}\right\}$$

That the equation is satisfied by (a) is obvious, and we see that it is satisfied by the points (b) and (c) by writing it in the form

$$\left(c-\frac{E}{2R}\right)^2+\frac{p}{R}=\frac{E^2}{4R^2}.$$

7. Thus we see that the curve of minimum current at given power passes through the points of

(a) zero current and zero power ;
(b) maximum power ;
(c) maximum current and zero power.

We notice that the maximum current at no load is $c=\frac{E}{R}$, whereas if the motor were at rest the current would be $c=\frac{E}{I}$; that is, the maximum current at no load is in all cases greater than the maximum current if the armature is at rest.

8. Again, from the equation

$$p+c^2r=cE\cos\psi$$

we have

$$\psi=\cos^{-1}\frac{p+c^2r}{cE},$$

therefore

$$\frac{d\psi}{dc}=\frac{c^2r-p}{c\sqrt{\{c^2E^2-(p+c^2r)^2\}}}$$
$$=0$$

when

$$p=c^2r.$$

ψ is then a maximum, and we see that the maximum difference of phase between the current and the impressed E.M.F. takes place when the electrical efficiency is $\frac{1}{2}$.

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9. *Example.*—Suppose we have a 50 kilowatt motor driven by a 1000 volt generator, and suppose that R=3 ohms and S=4 ohms, so that I=5 ohms.

Then maximum output . . $=\frac{10^6}{12}=$ 83·3 kilowatts.

Corresponding current . . $=\frac{1000}{6}=$ 166·7 amperes.

" counter E.M.F. $=\frac{5000}{6}=$ 833·3 volts.

Maximum current running light $=\frac{1000}{3}=$ 333·3 amperes.

Corresponding counter E.M.F. $=\frac{4000}{3}=$ 1333·3 volts.

&c.

10. To find the characteristic of the motor at any given load we must eliminate θ and ϕ between the equations

$$\left.\begin{aligned} p&=ce\cos\phi,\\ \sin\theta&=\frac{S}{I},\\ \text{and}\qquad E^2&=e^2+I^2c^2+2Ice\cos(\theta-\phi). \end{aligned}\right\}$$

The eliminant is

$$E^2-e^2-I^2c^2-2Rp=2S\sqrt{c^2e^2-p^2}, \quad . \quad . \quad . \quad (8)$$

which is the required general relation between e and c.

11. In the paper referred to above, Steinmetz calls equation (8) the Fundamental Equation of the Synchronous Motor: the equation is there developed and plotted; results are obtained directly from equation (8), but the development is so cumbrous that the writer thinks that his simple treatment may benefit those interested in the subject of Alternate Current Motors.

III. *Contribution to the Theory of the Robinson Cup-Anemometer.* *By* C. Chree, *M.A.**

§ 1. THE velocity of the wind during a hurricane is one of those items of information that, like the speed of express trains, is a source of general interest. The question of the accuracy to be expected in such definite statements as that the wind has blown at 60 miles an hour, though of much less general interest, is still of some practical importance. The usual instrument for the measurement of wind-velocities in this country is the Robinson† cup-anemometer. This consists fundamentally of four hemispherical cups attached to arms, inclined to each other at angles of 90° in a horizontal plane, the cups moving under the influence of the wind round a vertical axis. The number of revolutions of the cups in a given time is the information which the instrument, if properly constructed, could be relied on to give. The information desired, however, is what is the time-integral of the wind's velocity. To obtain this it is usually assumed that the wind's velocity is deducible by multiplying that of the centres of the cups by a constant "factor," and the recording mechanism is arranged so as to give the result of this multiplication. The value originally proposed for this factor by Dr. Robinson was 3, and this is the value generally employed in reducing results

* Communicated by the Author.

† So called after Dr. Robinson of Armagh.