



LIX. On the orientation of the slit in interference experiments

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Peters* involve an approximation equivalent to thus neglecting s in comparison with n , and the values he obtains for these nutations are therefore on the whole too small, that for the fortnightly, which is the most important, being about $\frac{1}{4}$ of what it should be.

Taking the more accurate expressions for $\dot{\theta}$, $\dot{\psi}$ in (20), (21), it appears that the semiaxes of the ellipse in the case of a rigid earth being calculated by Peters, with the above error as I think, to be $0''\cdot0885$ perpendicular to and $0''\cdot0812$ parallel to the plane of the ecliptic, those for the massless shell ($\epsilon = \frac{1}{300}$) would be, with the error corrected, $1''\cdot87$ perpendicular to and $2''\cdot03$ parallel to the plane of the ecliptic.

If the mass of the shell be taken into account, as its thickness gradually increases from zero the fortnightly nutation goes through a series of changes somewhat similar to that in the case of the half-yearly.

Dublin, Sept. 16, 1898.

LIX. *On the Orientation of the Slit in Interference Experiments.* By JAMES WALKER, M.A.†

IN a paper published in the Phil. Mag. for November 1898 I considered the question of the admissible width of the slit in interference experiments, as carried out with Fresnel's mirrors, the biprism, and the divided lens, on the assumption that the slit was in its most favourable position, that is, parallel to the intersection of the mirrors or to the edge of the biprism, or perpendicular to the plane through the principal axes of the two halves of the lens.

M. Fabry, in a general discussion on the visibility of Interference Fringes‡, has given an expression for the visibility in the case of a faulty orientation of the slit, and has shown that the effect is the same as that of a slit in its most favourable direction, having a width equal to the projection of the actual slit on a plane perpendicular to that direction§. This result assumes that the actual slit is so narrow that the visibility of the fringes, when it is properly adjusted, may be taken as unity, and also that it is not tilted either towards or away from the interference apparatus. The effect of such a tilt M. Fabry has indeed considered|| from a different point of

* "Numerus Constans Nutationis," *Mémoires de l'Académie Impériale des Sciences*, St. Petersburg (1843).

† Communicated by the Author.

‡ Thesis for the Degree of Doctor of Science, published at Marseilles, 1892.

§ *Loc. cit.* p. 87.

|| *Loc. cit.* p. 33.

view; but it may perhaps not be out of place to deduce the results from a general expression of the visibility of the fringes for the case of any orientation and any width of the slit.

In my former paper I showed that at a point x, y of the screen the relative retardation of the interfering streams from a point distant ξ from the central line of a properly orientated slit is (measured in length)

$$\alpha + \beta x + \gamma \xi,$$

the values of α, β, γ in the three cases of mirrors, biprism, and divided lens (neglecting the thickness of the two latter) being given by the following schedule:—

	Mirrors.	Biprism.	Divided Lens.
α .	0.	$\frac{1}{2} \frac{(\mu-1)^2 (\tan^2 \alpha_1 + \tan^2 \alpha_2) ab}{a+b}$.	0.
β .	$\frac{2a \sin 2\omega}{a \cos 2\omega + b}$.	$\frac{(\mu-1)(\tan \alpha_1 + \tan \alpha_2)a}{a+b}$.	$2\epsilon \frac{a}{ab - F(a+b)}$.
γ .	$\frac{2b \sin 2\omega}{a \cos 2\omega + b}$.	$\frac{(\mu-1)(\tan \alpha_1 + \tan \alpha_2)b}{a+b}$.	$2\epsilon \frac{b}{ab - F(a+b)}$.

Where a, b are the distances of the interference apparatus from the slit and the screen respectively;

2ω is the acute angle between the mirrors;

α_1, α_2 are the acute angles of the prism;

2ϵ is the separation of the halves of the lens;

F is the absolute value of the focal length of the lens.

Suppose now that the slit is turned first round the line bisecting its length through an angle ϕ , and then about its centre round a normal to its new plane through an angle θ ; then if u, v be the distances of a point of the slit from lines bisecting its breadth and its length, we have to write

$$\begin{aligned} a - \sin \phi (u \sin \theta + v \cos \theta) & \text{ for } \alpha, \\ u \cos \theta - v \sin \theta & \text{ for } \xi; \end{aligned}$$

and the intensity due to an element $du \cdot dv$ at this point is proportional to

$$\left[1 + \cos \frac{2\pi}{\lambda} \{ \alpha + \beta x + (\gamma \cos \theta - \beta' x \sin \theta \sin \phi) u - (\gamma \sin \theta + \beta' x \cos \theta \sin \phi) v \} \right] du dv^*,$$

* This neglects small terms arising from α in the case of the biprism, and $-\gamma' \sin \theta \sin \phi$ should be added to the coefficient of u and $\gamma' \cos \theta \sin \phi$ to that of v , where $\gamma' = 2 \frac{1}{(a+b)^2} \frac{(\mu-1)^2 (\tan^2 \alpha_1 - \tan^2 \alpha_2) b^2}{(a+b)^2}$.

where

$$\begin{aligned}\beta' &= \frac{2b \sin 2\omega}{(a \cos 2\omega + b)^2} \text{ for the mirrors,} \\ &= \frac{(\mu-1)(\tan \alpha_1 + \tan \alpha_2)b}{(a+b)^2} \text{ for the biprism,} \\ &= -\frac{2\epsilon F b}{\{ab - F(a+b)\}^2} \text{ for the divided lens.}\end{aligned}$$

Assuming, then, that each element of the slit acts as an independent source of light, the intensity due to the whole slit is proportional to

$$\begin{aligned}& \int_{-l/2}^{l/2} \int_{-k/2}^{k/2} \left[1 + \cos \frac{2\pi}{\lambda} \{ \alpha + \beta x + (\gamma \cos \theta - \beta' x \sin \theta \sin \phi) u \right. \\ & \quad \left. - (\gamma \sin \theta + \beta' x \cos \theta \sin \phi) v \} \right] du dv \\ &= kl \left\{ 1 + \frac{\sin \frac{\pi}{\lambda} (\gamma \cos \theta - \beta' x \sin \theta \sin \phi) k}{\frac{\pi}{\lambda} (\gamma \cos \theta - \beta' x \sin \theta \sin \phi) k} \cdot \frac{\sin \frac{\pi}{\lambda} (\gamma \sin \theta + \beta' x \cos \theta \sin \phi) l}{\frac{\pi}{\lambda} (\gamma \sin \theta + \beta' x \cos \theta \sin \phi) l} \right. \\ & \quad \left. \times \cos \frac{2\pi}{\lambda} (\alpha + \beta x) \right\},\end{aligned}$$

where k is the breadth and l is the length of the slit; so that the visibility of the interference-fringes is the absolute value of

$$\frac{\sin \frac{\pi}{\lambda} (\gamma \cos \theta - \beta' x \sin \theta \sin \phi) k}{\frac{\pi}{\lambda} (\gamma \cos \theta - \beta' x \sin \theta \sin \phi) k} \cdot \frac{\sin \frac{\pi}{\lambda} (\gamma \sin \theta + \beta' x \cos \theta \sin \phi) l}{\frac{\pi}{\lambda} (\gamma \sin \theta + \beta' x \cos \theta \sin \phi) l};$$

and thus, unless $\phi=0$, depends upon the order of the bands.

When $\theta=0$, the visibility is independent of the length of the slit at the point $x=0$, and is given by the absolute value of $\sin(\pi\gamma k/\lambda)/(\pi\gamma k/\lambda)$. On moving away from this point the bands become less and less distinct, disappear when $x=\lambda/(\beta' \sin \phi \cdot l)$, and then reappear as a set of bands complementary to the former, and so on.

At a given point x of the field the visibility is independent of the length of the slit only if

$$\tan \theta = -\frac{\beta'}{\gamma} x \sin \phi = -\frac{x}{d} \sin \phi,$$

where

$$\begin{aligned} d &= a \cos 2\omega + b \text{ for Fresnel's mirrors,} \\ &= a + b \text{ for the biprism,} \\ &= \frac{ab}{F} - (a + b) \text{ for the divided lens ;} \end{aligned}$$

and the visibility at the point is then the absolute value of $\sin(\pi\gamma k \sec \theta/\lambda)/(\pi\gamma k \sec \theta/\lambda)$.

It thus follows that if the slit be inclined with its upper part towards the interference apparatus, a rotation of the slit in its own plane in a direction from y towards x causes the point of maximum distinctness to move in the direction of positive x^* .

If $\phi=0$, the visibility is given by the absolute value of

$$\frac{\sin(\pi\gamma k \cos \theta/\lambda)}{\pi\gamma k \cos \theta/\lambda} \cdot \frac{\sin(\pi\gamma l \sin \theta/\lambda)}{\pi\gamma l \sin \theta/\lambda},$$

which is independent of the length of the slit if $\theta=0$, a result that is obvious from other considerations; and when k is a small fraction of λ/γ , the visibility is given by the absolute value of $\sin(\pi\gamma l/\sin \theta/\lambda)/(\pi\gamma l \sin \theta/\lambda)$, so that the slit acts as a slit properly orientated of width equal to the projection of the actual slit on a plane perpendicular to its most favourable direction. This is M. Fabry's result mentioned above.

If $\theta=0$, $\phi=0$, the bands disappear when $k=\lambda/\gamma$, and in this case a rotation of the slit increases the visibility, and would theoretically cause a reappearance of the bands. A consideration of the magnitude of the quantities involved shows, however, that the increase of the visibility is too slight to be noticed.

When the slit is properly adjusted ($\theta=0$, $\phi=0$), the bands attain their second maximum of distinctness when $k=1.4303\lambda/\gamma$, their visibility being about $\frac{1}{5}$; if then we take Lord Rayleigh's result † that the limit of visibility is reached when the ratio of the illuminations at the darkest and brightest parts of a system of bands is .975, which corresponds to a visibility of $\frac{1}{10}$, the bands will disappear when the slit is rotated

* It is obvious that these phenomena cannot be observed in all cases. Thus with a biprism and sodium light, if $l=1$ cm., $\phi=10^\circ$, the point at which the bands would first disappear falls outside the field common to the two streams, unless $b \tan \delta/(a+b)$ exceed .026, where δ is the deviation produced by the biprism. M. Fabry has, however, observed the phenomenon with Fresnel's mirrors (*loc. cit.* p. 33).

† Phil. Mag. (5) vol. xxvii. p. 484.

through an angle θ , given under ordinary experimental conditions by

$$\sin(\pi\gamma l \sin \theta/l)/(\pi\gamma l \sin \theta/\lambda) = \frac{1}{16}.$$

Such a rotation reduces the visibility of the prime maximum of the fringes to $\frac{1}{16}$, or to less than one third of the visibility of the second maximum with a properly adjusted slit. The actual reduction of distinctness does not, however, appear to be nearly so great; and the question arises, over what range of brightness of the field the limit of visibility may be regarded as constant. It might be of interest to test this point, employing Lord Rayleigh's method * of determining the limit of visibility, and controlling the brightness by a revolving disk with transparent and opaque sectors.

In the case of Lloyd's mirror the relative retardation of the interfering streams from a point distant ξ from the central line of a properly placed slit is at a point xy of the screen $2x(c+\xi)/d$, where c, d are the distances of the central line from the mirror and screen respectively.

If then the slit be turned as in the former cases, the visibility will be given by the absolute value of

$$\frac{\sin \frac{2\pi}{\lambda} \left\{ \frac{x}{d} \left(\cos \theta + \frac{c}{d} \sin \theta \sin \phi \right) k \right\}}{\frac{2\pi}{\lambda} \frac{x}{d} \left(\cos \theta + \frac{c}{d} \sin \theta \sin \phi \right) k} \cdot \frac{\sin \frac{2\pi}{\lambda} \left\{ \frac{x}{d} \left(\sin \theta - \frac{c}{d} \cos \theta \sin \phi \right) l \right\}}{\frac{2\pi}{\lambda} \frac{x}{d} \left(\sin \theta - \frac{c}{d} \cos \theta \sin \phi \right) l}$$

and this is independent of the length of the slit, if

$$\tan \theta = c \sin \phi / d,$$

a relation that holds for the whole field.

LX. *On the Construction of a Mechanical Model to Illustrate Helmholtz's Theory of Dispersion.* By J. H. VINCENT, D.Sc., A.R.C.Sc.†

Introduction.

IN a course of lectures recently delivered at the Cavendish Laboratory, Prof. J. J. Thomson described a mechanical model which obeyed the formula given by Helmholtz for the velocity of propagation of waves in a medium capable of absorption.

Prof. J. J. Thomson's Model.

The system contemplated consisted of a weighty cord stretched horizontally; from this cord depended a uniform

* *Loc. cit.*

† Communicated by Prof. J. J. Thomson.