

in the case of common minerals, where the number of published analyses is very great, a judicious selection of the best and most recent analyses has been made.

The statement of the optical constants and the physical characters of minerals has been treated in much the same fashion as the chemical data. The best and most trustworthy determinations have been selected, while measurements of doubtful value have been omitted.

It is on the crystallographic portion of the work, however, that Prof. E. S. Dana has expended the greatest amount of labour. We are informed in the preface that "an attempt has been made to trace back to the original observer the fundamental angles for each species, then the axes have been recalculated from them, and finally the important angles of all common forms have been calculated from these axes." The author is able to state that in every case this recalculation of the angles of all the forms of a mineral has been undertaken, and that no pains has been spared in the verification and correction of the results. The crystal forms are indicated by letters, and the symbols employed are in the first instance those of Miller, and in the second instance the modified form of Naumann's symbols familiar to all who have used the earlier editions of the work. The author gives it as his opinion that the former should eventually supplant the latter altogether. In the hexagonal and rhombohedral system, however, the Bravais-Miller system is adopted in preference to that of Miller.

With few exceptions, the figures of crystals (1400 in number) are new. Many have been drawn from original data, and those taken from other works have been redrawn so as to secure uniformity of projection; the habits of each species and the types of twinning in crystals have been very fully illustrated.

While the general account of the mode of occurrence and association of mineral species has been very carefully attended to, there has been no attempt to make this part of the work exhaustive, for to have done so would have greatly increased the bulk of the volume. The account of American localities—which has always been an important feature of Dana's work, and has made it for North America what the treatises of Kokscharov and Zepharovitch are for the Russian and Austrian Empires respectively—has been greatly added to. The works of Roth and Hintze, with the numerous books and memoirs devoted to the geology of particular regions, now supply all the information that is needed in respect to mineralogical distribution in other areas.

We have tested the volume in many ways as to the completeness and recent nature of the information given with respect to particular species, and always with satisfactory results. To pass such a voluminous mass of information through the press has required eighteen months of labour, and notices of important contributions to our knowledge that have appeared since the earlier pages of the book were printed off have been relegated to a supplement. This supplement, which extends to 28 pages, also contains brief accounts of minerals of unknown composition, and of doubtful species having little or no claim to recognition.

In conclusion, we must congratulate both the original author of the "System," and the writer of the volume

in its present form, on the completion of their useful labours. It is not too much to say that the publication of each successive edition of this work has constituted an epoch in the history of mineralogical science; and the present edition, coming from the hands of a new author, completely maintains the prestige of former ones.

J. W. J.

MODERN INFINITESIMAL CALCULUS.

An Introduction to the Study of the Elements of the Differential and Integral Calculus. From the German of the late Axel Harnack, Professor of Mathematics at the Polytechnicum, Dresden. (London and Edinburgh: Williams and Norgate, 1891.)

MR. G. L. CATHCART'S translation forms a handsome volume, and will prove acceptable to those engaged in mathematical teaching, as a storehouse of suggestive methods and ideas for analytical exegesis.

But let us examine the work from the standpoint of the student approaching the subject of the Calculus for the first time, supposing this book to be put into his hands to acquire his first acquaintance with the method and reasoning.

Until very recently the Classics, Greek and Latin, as taught at school, were looked upon chiefly as collections of grammatical examples, and the subject-matter was lost sight of in the careful parsing and analysis of the sentences. Boys were taught on a system which implied that they were all, in their turn, to become schoolmasters and instructors; and the interests of the majority, who would profit intellectually from the literary study of the ancient masterpieces, were completely neglected.

So, too, in Mathematics: the ordinary text-books give an excellent schoolmaster's training in the subject; but the large and increasing class of students, brought into existence recently by the commercial developments of scientific application, who are required to put into immediate practice the theory which they find indispensable, cannot afford the time to be dragged the whole length of the quagmire of the Convergency of Series, of Inequalities, of Discontinuity, and of the so-called Failure of Taylor's Theorem. These are the quagmires in which the mere mathematician delights to lose himself, and also to lure in others after him.

To one who is already very familiar with the notation and operations of the Calculus the present treatise will prove, not repellent, but even fascinating to minds who pursue the subject for its purely analytical interest. Having been over the road before, they will be prepared to appreciate the strictly logical order in which the theorems are developed, starting in Chapter I. with the fundamental conceptions of Rational Numbers, of their Addition, Subtraction, Multiplication, and Division—the subject of Arithmetic in short; and passing on in Chapter II. to Radicals and Irrational Numbers in general. The next three chapters treat of the Conceptions of Variable Quantities, of Functions of a Variable, their Geometric Representation and Continuity; and it is not till the sixth chapter that the Differential Coefficient is introduced and determined for the simplest functions.

But the beginner, who has had the courage to read thus far, will wonder what on earth the subject is all

about, even when he has reached the end of Book I., which covers the ground of the subject usually called the Differential Calculus: there are no illustrations, except for one or two meagre geometrical applications, for the mind to hold on by; no diagrams, and no examples to test the soundness of the student's knowledge.

It is true that these collections of examples are derided in certain lofty quarters of the mathematical hierarchy; but the humbler priests of the science, who are in touch with the novice mind of human nature, know their practical value; and these collections of problems, formerly a feature of our text-books unknown abroad, are now being extensively copied and adopted in other countries. "In scientiis ediscendis prosunt exempla magis quam præcepta" (Newton).

The Second Book considers Functions of Complex Numbers: we make another fresh start with the operations of Arithmetic, as it is called here; not that any resemblance can be traced to what generally goes by that name. In this book the questions of Convergence, of Single- and Multiple-valued Functions, as illustrated by a Riemann surface, and of their Zeros and Infinities, are gone into at great length; but at the same time the reader will have an impression that the information is given in a very condensed form, and that an attempt has been made to give a brief *résumé* of a subject which requires a large volume to itself.

This Morbid Pathology of the Mathematical Function, as we may call it, requires a very clear, concise, and cosmopolitan terminology, which, as Mr. Cathcart points out on p. 148, it does not yet possess; it is unfortunate that the nomenclature has mostly been formed originally in the agglutinate German language, and in many cases is only very imperfectly translatable.

This part of the subject, although principally known to us from the researches of later writers, such as Cauchy, Riemann, Dirichlet, and Weierstrass, owes very much to Gauss; but Gauss deserves to lose the credit of priority, from his baneful habit of bottling up his discoveries, after announcing that he had obtained the solution, so as to warn off all other investigators from his preserves of research.

The Integral Calculus is developed in Book III.; here also the treatment, though complete, is very condensed; and but few simple problems and applications are provided to show the use of the subject when the analysis is established.

The author never employs the hyperbolic functions, although their use can be traced back to Newton ("Principia," Lib. II., Prop. ix.); but in the reductions of the integral of $F(x, \sqrt{R})$ where R is the quadratic $a + 2bx + cx^2$, the use of \sqrt{R} as the argument in conjunction with the circular and hyperbolic functions enables us to present the different results which arise in a more systematic manner than that employed in the present work. A very short sketch is also given of the method of reduction of the integrals when R is of the third or fourth degree; the elliptic integrals are now introduced, but no mention is made of the elliptic functions, introduced by Abel by the inversion of the elliptic integrals.

The Fourth Book, which treats of the integrals of complex functions and of the general properties of analytic functions, is probably the sole presentation of this modern and difficult subject in our language. To a mathematician

of Mr. Cathcart's development the treatment will appear very concise and elegant, but for our part we miss the footholds afforded by the physical applications of the general theorems of functions; say to Hydrodynamics, such as those recently published by Prof. W. Burnside in the Proceedings of the London Mathematical Society, on Riemann's Theory and on Automorphic Functions, determined from their discontinuities.

The book will recommend itself, as we said at the outset, to the advanced student, who pursues mathematical study as an end to itself, by reason of the strict logical order in which the subjects are presented; but is this strict logical order the most suitable arrangement for a beginner?

Herbert Spencer says that "in each branch of instruction we should proceed from the empirical to the rational." In the operatic version of "Manon" the events are presented in chronological order; but in the original "Histoire de Manon Lescaut" the story begins in the middle, so as to excite the reader's curiosity as to the preceding events which led up to the point at which the characters appear on the scene.

According to Prof. Harnack's preface, the present work may be considered the operatic version of his lectures, while the simple story would appear in the lectures delivered in the Dresden Polytechnicum to his technical students, who required a knowledge of Analysis chiefly as an instrument for the solution of mechanical problems.

Mr. Cathcart explains in his Translator's Note the desire he had to make these lectures accessible to the English reader, and records the regret he felt at the news of the death of Prof. Harnack, while engaged on a revision of his notes for a new edition. The thanks of the mathematical world are due to Mr. Cathcart for the care and trouble he has taken in this valuable piece of work.

A. G. GREENHILL.

ALTERATIONS OF PERSONALITY.

Les Altérations de la Personnalité. Par Alfred Binet. Bibliothèque Scientifique Internationale. (Paris: Ancienne Librairie Germer Baillière et Cie., 1892.)

IN what is in ordinary parlance called somnambulism, or sleep-walking, the patient rises in the night, performs a number of seemingly intelligent actions directed to some special end, answers questions with regard to such actions with a variable amount of coherence, returns to bed, and generally, but not in all cases, wakes in the morning with no remembrance of that which he has done during the night. Such is somnambulism in its narrower sense. It exhibits the individual in an abnormal psychological condition, the actions performed in this abnormal condition being generally unconnected in memory with the normal sequence of events in waking life. The word somnambulism is, however, now used in a wider and at the same time more technical sense, being applied to all cases where the individual, either spontaneously or through hypnotic suggestion, falls into an abnormal condition distinguishable from the normal condition of his or her waking life. It is with the alterations of personality exhibited during the state of somnambulism in this wider sense that M. Binet's volume chiefly deals.