

THE COMPARATIVE BEHAVIOR OF FLOATING AND BOOSTER-CONTROLLED BATTERIES ON FLUCTUATING LOADS

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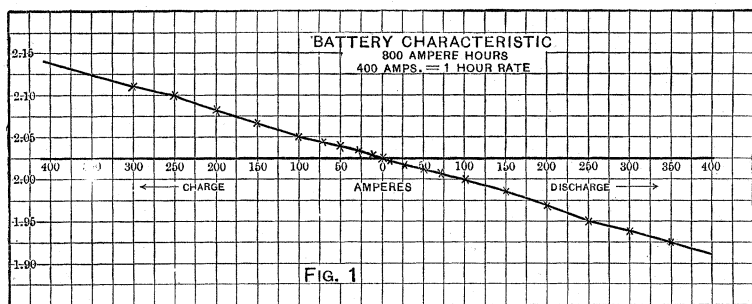
Methods of Application.—The object of using the storage-battery as a regulator on variable loads is to absorb, as far as possible, all fluctuations above or below the average current. When the external load is light, the generator sends current into the battery as well as to the external circuit. When the demand is heavy, the battery discharges in parallel with the generator, relieving the latter of the excess load. In some instances it is desired to maintain the generator current constant, eliminating the shock and strain of excessive and suddenly-imposed loads, and to secure the fuel-economy incident to steady loads as compared with fluctuating. In other cases, the object is to maintain a constant voltage on a circuit or rather to prevent it from varying more than a predetermined amount. Obviously, the maintenance of absolutely constant load, or voltage, is the ideal condition, and the more nearly this is approached the more nearly is the object attained.

Where the load is applied some distance from the power-station, there are five possible methods of installing a storage-battery to maintain the load on the generator reasonably constant, which are:

- (1) Floating battery in power-station.
- (2) Floating battery, out on line, at point of application of load.
- (3) Battery with booster, in power-station.
- (4) Battery and booster, out on line, at point of application of the load.

(5) Battery cut on line, at point of application of the load, and booster in the power-station.

Cell Characteristic.—Fig. (1) shows the characteristic of an 800-ampere-hour cell one-quarter discharged, at 70° F., in good condition. This will change with temperature, the characteristic more nearly approaching the horizontal with higher, and the perpendicular with lower, temperature. It will also change with state of charge, density of electrolyte, and condition of plates. The ordinates above the horizontal line which intersects the characteristic curve at zero represent the voltage of a cell on charge, and those below this line the volts on discharge, at various rates of current flow. The abscissas are amperes, the curve being extended on either side of the zero to that point where the current flow is equal to the one-hour rate of charge or discharge. Each reading was taken 20 seconds after beginning of the corresponding current



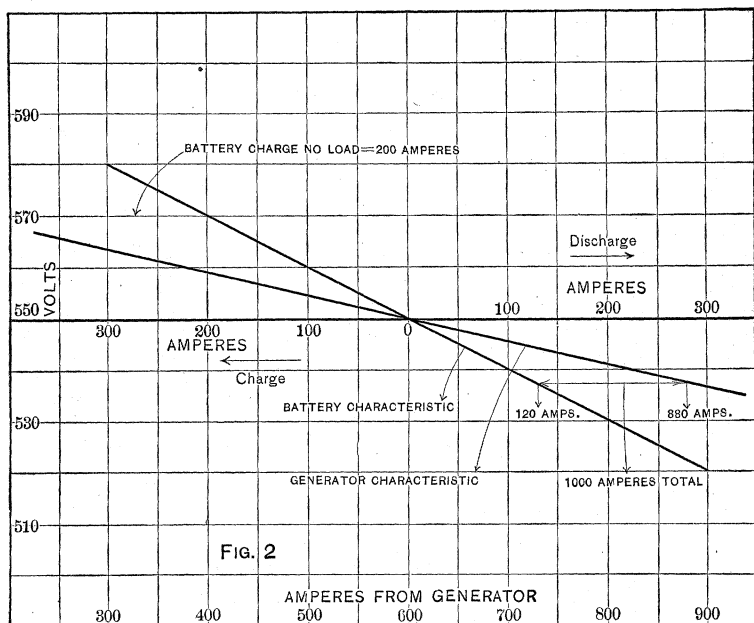
flow. The zero point shows the voltage of a cell when current flows neither into nor out of it.

This is the general form of characteristic for nearly any type or size of cell, the conditions of charge, temperature, density of electrolyte, etc., being the same. The data then to construct the characteristic for any size of battery of same state of charge, temperature, and density of electrolyte as given are:

Volts per cell at zero, 2.025; at the one-hour discharge rate 1.915, and at the one-hour charge rate 2.14. Connect these points by a straight line. The one shown is practically a straight line, and repeated tests on many sizes and types of cells indicate that if the conditions be maintained constant throughout a test, and due allowance be made for the error in reading the small changes in the voltmeter deflections, the characteristic is a straight line between the points of one-hour charge and one-hour discharge rates.

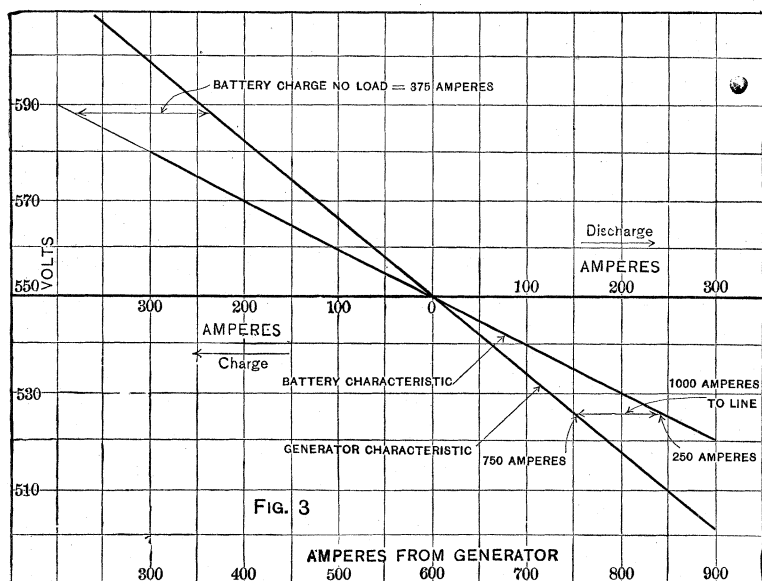
Conditions Necessary for Floating.—If a storage-battery be “floated” on a system on which the load is fluctuating, the voltage of the circuit across which it is connected must obviously vary with the load; rising above that of the battery when the demand is light and sending in a charging current; falling when the demand is heavy and allowing the battery to discharge and assist the generator.

In order that the system may work properly there should be a certain relation between the battery characteristic and that of the generator. The greater the inclination of the generator characteristic to the horizontal; that is, the greater the change in



voltage with change in load, the greater will be the battery discharge on loads in excess of the normal or average load. Referring to Figs. 2 and 3. In Fig. 2 the generator characteristic has a certain inclination to the horizontal. The battery characteristic is superimposed on it, the point of zero current flow intersecting the generator curve at the point of normal load—600 amperes. For simplicity both are here assumed to be straight lines. When 1,000 amperes are sent over the line, the generator furnishes 880 and the battery 120 amperes, the increase in load on the generator being 280 amperes or 47 percent.

In Fig. 3 the inclination of the generator characteristic is greater than in Fig. 2, and for 1,000 amperes station output, the generator furnishes 750 amperes, the battery 250, the increase of generator load above normal being 150 amperes or 25 percent. In the latter case, however, on light load, the voltage rises so greatly that an excessive current will pass into the battery, probably injuring it. As the one-hour rate of charge should never be exceeded—and even this should last but a very short time—the relative inclination of the two characteristics must come within certain limits. Furthermore, large shunt generators of high efficiency cannot be made—or at least certainly are not made—so that the full-load voltage decreases more than 15

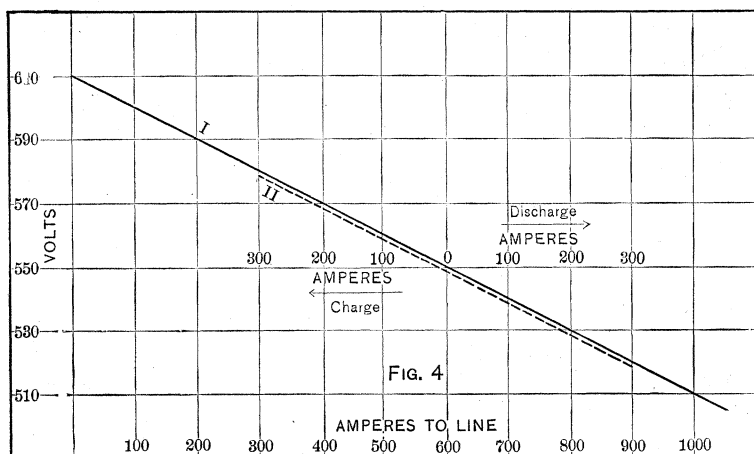


percent below the no-load voltage. This limits the inclination of the generator characteristic.

CASE I.—FLOATING BATTERY IN STATION.

Data of System.—Consider the case of a fluctuating load such as is shown in Curve A, Fig. 7. The minimum current is 260, maximum 1,000, and average 600 amperes. Such a load coming directly on the generating equipment must result in low efficiency and high fuel-consumption per h.p. hour; and will require generators with large overload capacity and heavy engines to withstand the strains imposed. It may here be mentioned that the load-curve shown is not an assumed one, but represents the actual readings made on an operating electric railway.

Assume that this load is carried over one feeder to a point 2 miles from the power-station; that the minimum voltage at any time is to be 350 volts; that the track is of 70 lb. rail, well bonded. Still assuming both characteristics as straight lines, Fig. 4 is drawn. Line "I" is the generator characteristic. The dotted line running just underneath "I" is the characteristic of a 600-ampere-hour battery, the one-hour charge or discharge rate of which is 300 amperes. This line is in reality coincident with the generator curve. The voltage of the generator at normal load, 600 amperes, is 550 volts, which is also the open-circuit voltage of the battery. The ordinates are volts as indicated. The lower set of abscissas are amperes of generator output; the set on the hori-



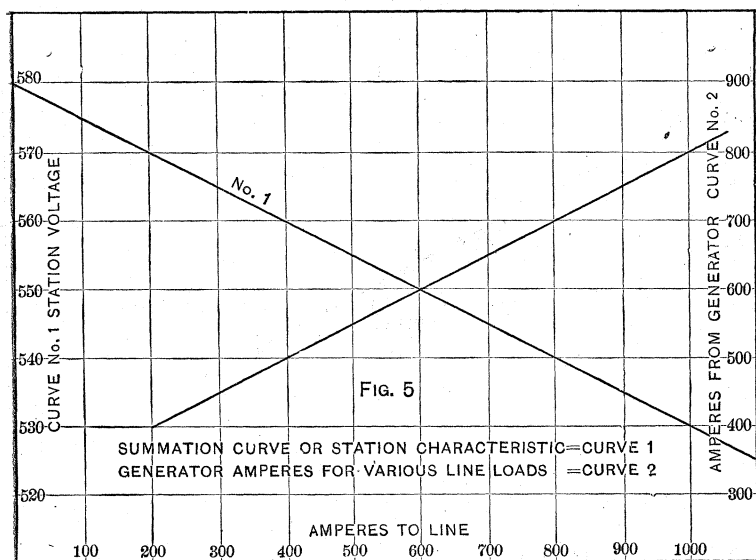
zontal line at 550 volts are amperes of charge or discharge of the battery.

Summation Curve.—Fig. 5 shows a summation curve which gives the station voltage at any output to the external circuit and is constructed as follows:

At voltage of 530 the generator output is 800 amperes, the battery discharge 200; total, 1,000 amperes to the line at this voltage. Similarly, at 540 volts the sum of the two outputs = $700 + 100 = 800$; at 560 volts the generator output = 500 amperes and the current going into the battery = 100 amperes hence the line current = $500 - 100 = 400$. Likewise, at 570 volts, line current = $400 - 200 = 200$ amperes. In this way the points on the summation curve are located which give line

current as the abscissas and station voltage as the ordinates. Since the characteristics are both straight lines, the summation curve is also a straight line. In this case only two points have to be located and joined together, but in many cases where the generator characteristic is a curve, the summation line becomes a curve and several points must be located to trace it. For this reason the method of tracing is here set forth.

Line Constants.—From Fig. 5 it may be seen that the station voltage at 1,000 amperes is 530 volts. Since the minimum voltage at the delivery point 2 miles distant must not fall below 350 volts, the total line-drop must not exceed 180 volts, which fixes the resistance at .18 ohm. The resistance of two miles



of well-bonded 70 lb. rails is .0582 ohm. This leaves .1218 ohm as the copper resistance or .0609 ohm per mile, corresponding to 940,000 cir. mils. Taking this resistance, the curve of line-drop at various loads is plotted, which is shown in Fig. 6. This is, of course, a straight line.

Derived Diagrams.—From the foregoing data and figures the curves in Fig. 7 are derived. Curve A is the load-diagram before mentioned; Curve B, the generator load; Curve C, the current to or from the battery; Curve D, the variation in station voltage with load, and curve E, the change of voltage at the feeding point, plotted by subtracting from the ordinates

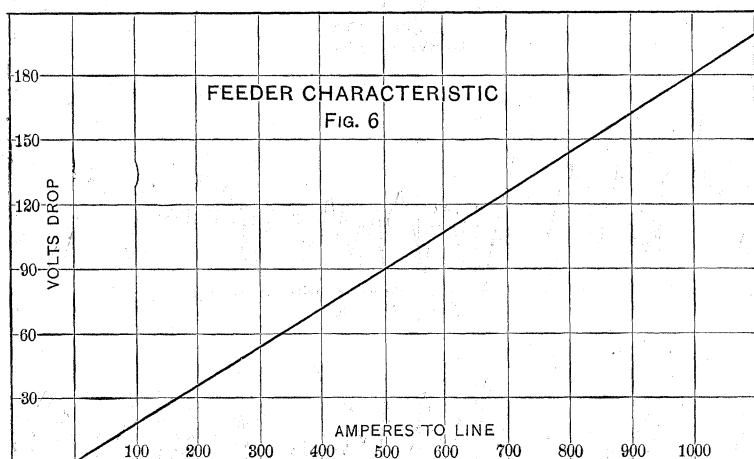
of D the feeder-drop for the various currents. Since all the relations are of the first order, the various curves are of the same form as the load-curve. Curves D and E are also of the same form, but reversed, being inverse functions of the load-curve.

A study of these curves reveals the following:

(1) The voltage at the station falls off rapidly with increase of load (Curve D, Fig. 7).

(2) Only two thirds of the capacity of the battery to discharge are utilized. It can discharge at the rate of 300 amperes, and only 200 amperes are drawn from it at the instant of the maximum peak (Fig. 4).

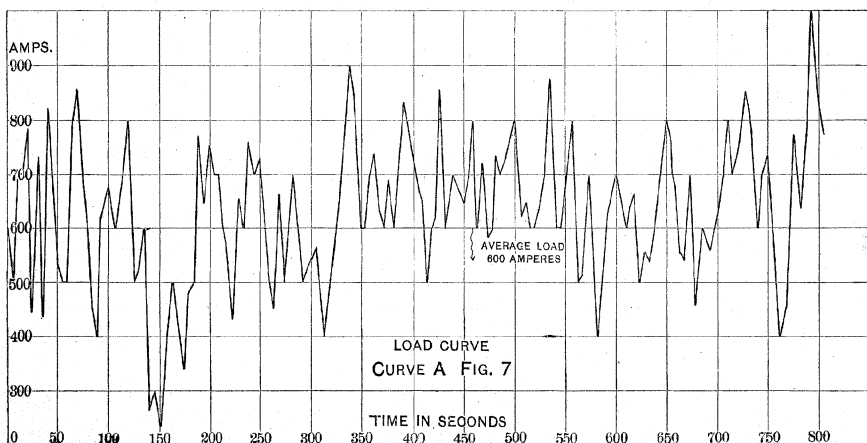
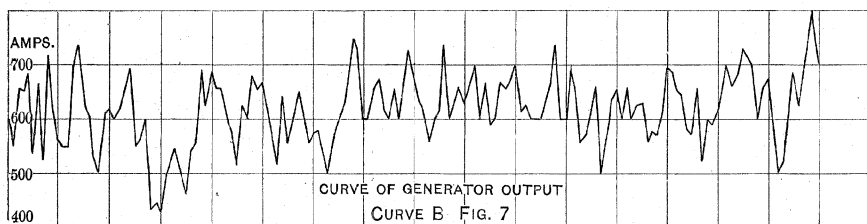
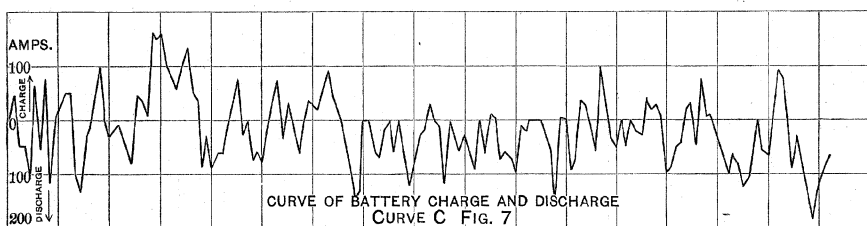
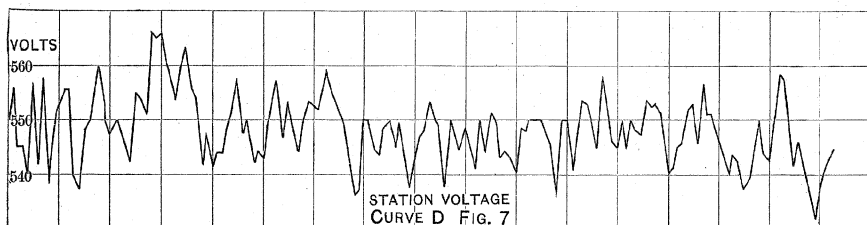
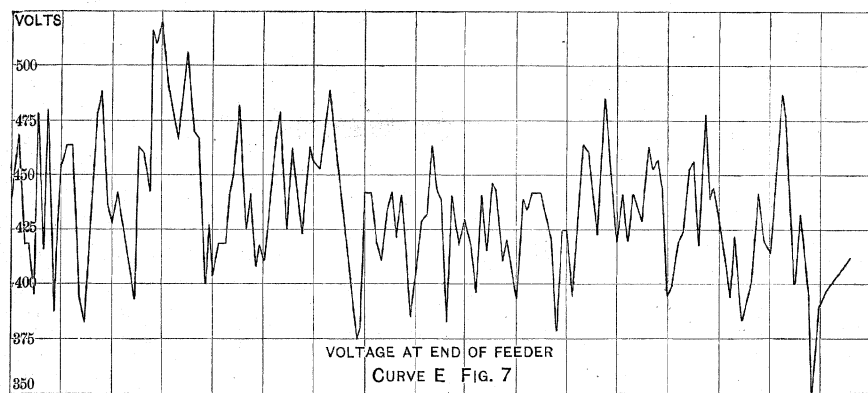
(3) Because of the fall in station voltage with load, the copper



required on the line is excessive, and the voltage at the feeding-point unsatisfactory, as shown by Curve E.

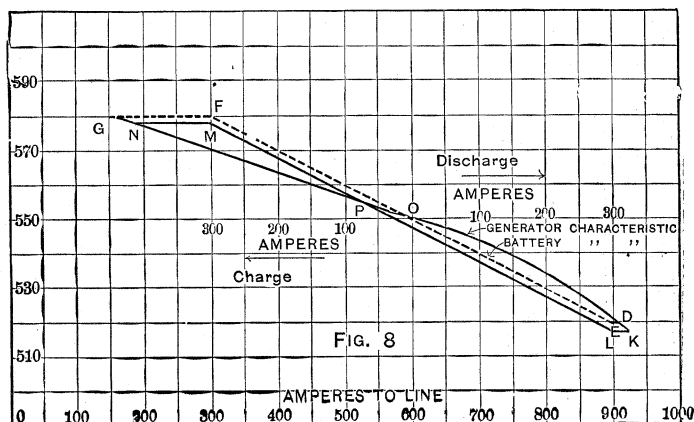
(4) The fluctuations on the generator are greatly reduced, and the capacity of the generating equipment may be decreased in the ratio of 8 to 10, or 20 percent.

Drooping Generator Characteristic.—All the foregoing is based on straight generator characteristic. As a matter of fact, however, it would be of the form shown in Fig. 8. This change, from a theoretical straight characteristic to the actual drooping one, introduces complications. If the battery characteristic intersect that of the generator at the point of normal load—600 amperes—as indicated by the dotted line, the discharge on heavy loads will be greater than in the previous case, since the inclination of any element of the curved portion to the horizontal is



greater than that of the straight-line characteristic, and a larger portion of the load will be taken by the battery. But it may also be seen that with the same light loads as before, the battery-charging current is less than in the previous case.

Relation between Input and Output.—In any battery used for regulation, it is necessary that the input approximately equal the output, otherwise the state of battery charge is changed and the efficiency of the regulation decreased. If the input exceed the output, the excess energy will be lost, going off in the form of free gases from the cells, and the high voltage due to overcharging decreases the current which may be sent into the battery or light load, thus permitting more fluctuation of the generator load. Furthermore, continuous overcharge will ruin a battery, if persisted in.

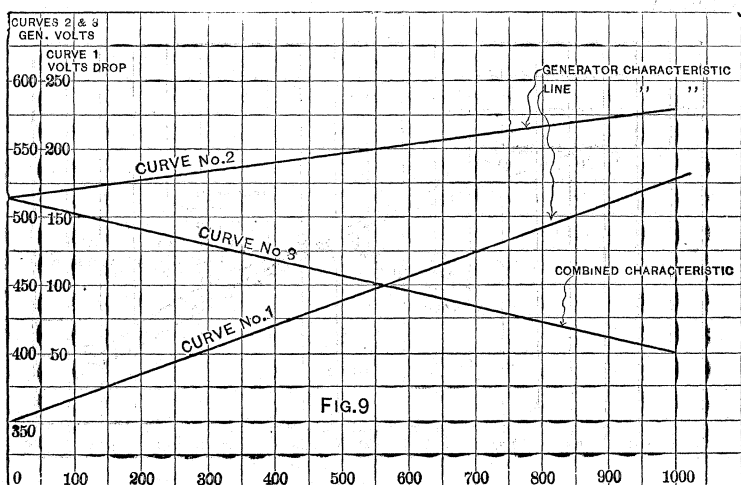


If the output exceed the input, electrical bankruptcy follows and the battery is in no condition to help the generator when most needed.

From Fig. 8 it may be seen that the input and output are to each other inversely as the two areas on either side of the intersection of the generator and battery characteristics, which are formed by the spaces included between the two characteristics and closed by horizontal lines which are drawn through the points of voltage at maximum charge and discharge. That is, the areas ODE and OFG are to each other inversely as the battery input is to the output. The point O is not necessarily the intersection with the line of average external load, but is the point of intersection of the two characteristics. Obviously, therefore, the battery characteristic must be moved downwards,

increasing $o f c$ and diminishing $o d e$ until these two areas are equal, the resulting areas being $p m n$ and $p l k$. This means that the normal battery voltage is lowered, its zero point being shifted downwards. With the normal load of 600 amperes on the line, there will now be a charging current into the battery of 20 amperes, making the generator current 620 amperes and lowering the station voltage to 548 volts.

Conditions with Drooping Characteristic.—The conditions are then manifestly less advantageous in the actual case than in the theoretical case first discussed and in either case are worse than they would be with an over-compounded generator in the power-station working without a battery. The mere reduction of mechanical strains and fuel-consumption are of little avail if the



cars on the system cannot be accelerated, carried over grades and schedules maintained.

Compound Generator.—For comparison with the previous cases, Figs. 9 and 10 are shown. In Fig. 9 are plotted the characteristic of an over-compounded generator, the line characteristic with resistance .18 ohms as before and a combination of these showing the total characteristic for the feeding-point. From this last, the curve in Fig. 10, which is the voltage curve at the feeding-point, is plotted. This is drawn to the same scale as Curve "E," Fig. 7, and a comparison of these two curves shows the superiority of the over-compounded generator working alone. The minimum voltage in the former case is 350, in the latter 401; and the maxima are 522 and 482 volts respectively. If 350 volts

are sufficient for satisfactory operation, the resistance of the circuit may be increased from .18 to .23 ohm. Deducting the track resistance, .0582 ohm, the copper resistance becomes .1718 ohm which corresponds to 665,000 cir. mils, as against 946,000 cir. mils. in the first case—a reduction of 29 percent.

CASE II.—FLOATING BATTERY OUT ON THE LINE.

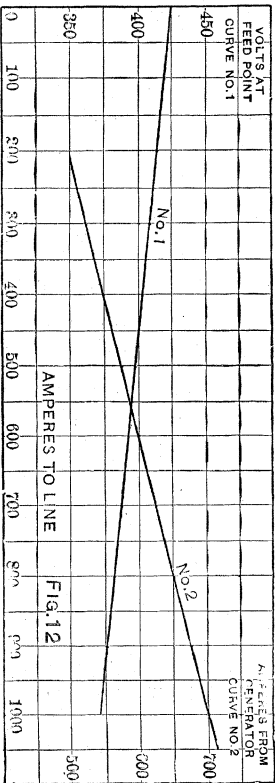
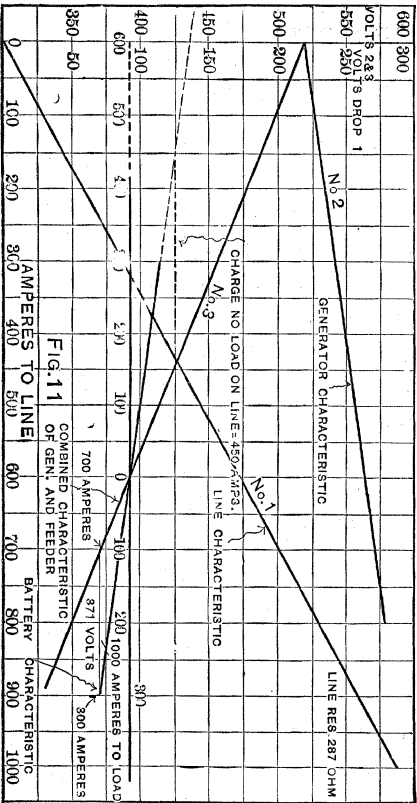
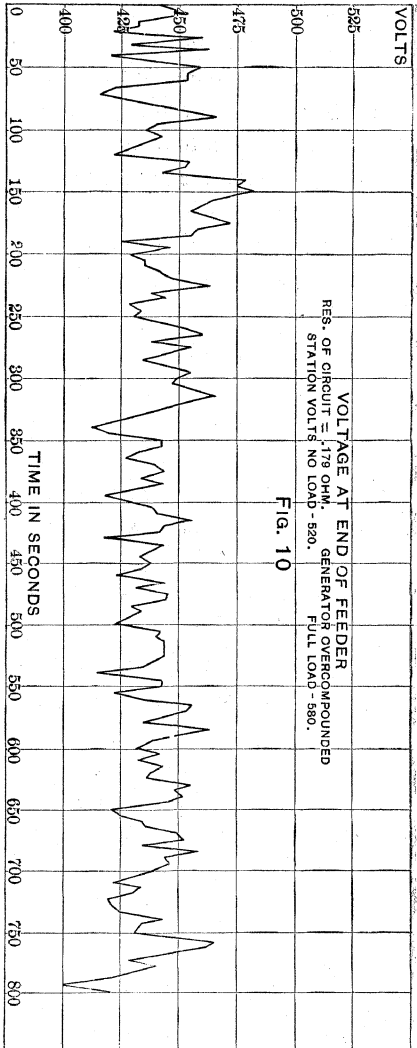
Data of System.—Consider now the case of a floating battery located at the feed-point—2 miles away from the station: Assume an over-compounded generator giving 520 volts at no load, and 530 volts at full load of 800 amperes, and assume further that the battery will discharge 200 amperes at maximum load of 1,000 amperes, giving 800 amperes as the maximum transmitted over the line. If the minimum voltage allowable be 350 volts, the resistance = .287 ohm or .2288 ohm for the copper resistance, corresponding to 495,000 cir. mils. or 25 percent less copper than that required in the case of the compound-wound generator working alone.

Fig. 11 shows the generator characteristic, the line characteristic, the combined characteristic, and the characteristic of a 600-ampere-hour, 392-volt battery. This shows that when the maximum load of 1,000 amperes comes on the system the generator supplies 700 amperes, the battery 300, and the voltage is 371.

Derived Diagrams.—In Fig. 12 are shown the summation curve of Fig. 11, giving the voltage at the feeder-point for any current to the line, and the line No. 2 which shows the relation between the amperes flowing over the line and the current from the generator. Fig. 13 shows the three curves: B the load on the generator; C the current to or from the battery, and D the voltage at the feed-point all for the same load curve shown in A, Fig. 7. All these are to the same scale as the corresponding curves in Fig. 7 and may be readily compared therewith.

Excessive Rate of Charge.—The only objectionable feature in this system is, that the charging current, if the load on the line should reduce to zero, would be far in excess of the permissible rate. This is shown in both Figs. 11 and 12. In Fig. 12 the voltage at zero line-load is seen to be 425 volts, which on the combined characteristic in Fig. 11 corresponds to a current-flow over the feeder of 450 amperes. If there is any possibility of the line ever being relieved of load, some provision must be made for avoiding this excessive charge. In the system under discussion, however, this never occurs.

Results.—The location of a floating battery at the feeding-point shows manifest and undoubted improvement in that



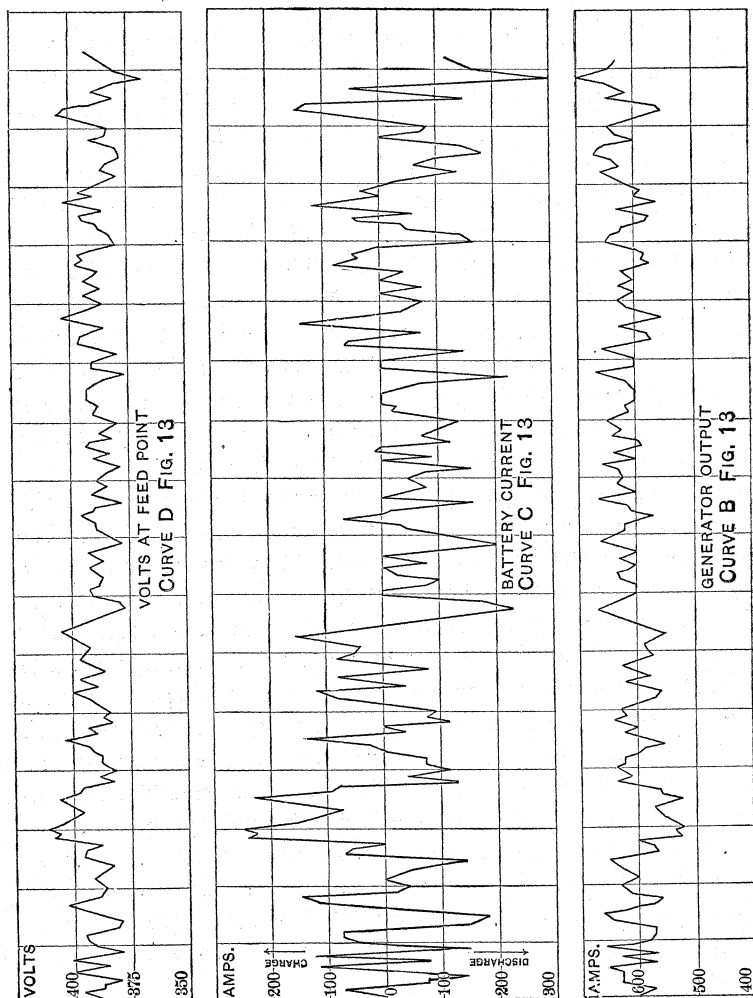
it not only performs all the functions of equalization with the production of the accompanying benefits, but the copper and size of generator are so much decreased that the saving is greater than the cost of the battery, so that the first cost of the system is less than in the case of a generator without the battery auxiliary.

It is to be noted that the voltage at the feeding-point, with average line-load, is 392 volts, which is also the open-circuit voltage of the battery. In the case of the battery located in the power-station, the floating voltage required is 550. The number of cells in series in the former instance is 194, and in the latter 271; so that the cost of the battery is diminished in this ratio, or 28 percent. Also this changes the slope of the battery characteristic. The 550-volt battery shows 30 volts increase above normal for the one-hour charge rate, and the same amount below normal for the one-hour discharge rate. In Fig. 11 it may be seen that the variation above or below normal for the one-hour rate of charge or discharge, is, 21.5 volts due to the smaller number of cells in series. The change in the voltage of the individual cells is, of course, the same in either case.

Overcharge.—In practice it is necessary to overcharge or “boil” the cells occasionally to maintain them in good condition. The voltage on overcharge rises to about 2.6 volts per cell or 30 percent more than the normal voltage. Thus a battery which has a floating voltage of 550 volts would require a potential of 715 volts at the terminals to give a proper overcharge. Since it is seldom possible to increase the station voltage to such a figure, it becomes necessary when “boiling” to connect the battery so that it is divided into two portions, each having an equal number of cells in series. These two halves are then connected in parallel across the line. As the voltage required to overcharge is now halved, proper resistances must be inserted between each set of cells and the line. Generally, the ordinary water-barrel rheostat is used. Since overcharge need only be given ten or twelve times a year, this is not a continuous condition of operation.

Where the battery floats at the end of a long feeder of fairly high resistance, which is heavily loaded, the normal battery voltage is lower than that of the station, and during the early hours of the morning, when for half or three quarters of an hour the load may be thrown off the feeder, the decrease in feeder-drop, together with full generator excitation, may bring the

voltage at the battery terminals up sufficiently high to give it the required overcharge. In the case just cited, the voltage at the feed-point with 75 amperes passing over the line (which current is ample for boiling a 600-ampere-hour battery) is 502



volts. The potential required at the battery is $194 \times 2.6 = 503$ volts. A very slight increase in the generator voltage will, therefore, meet the requirement, if the feeder be entirely relieved of other load.

COMPARISON CASES I. AND II.

The results, then, of locating a battery out on the line as compared with the use of the generator working alone are:

(1) Load on generator is maintained much more constant as indicated in Curve B, Fig. 13.

(2) Copper cost is greatly reduced.

(3) Voltage varies much less, and with less copper the minimum voltage is increased.

Also the full capacity of the battery is utilized, which was not the case when floated in the station.

BOOSTER SYSTEMS.

In discussing the operation of the two batteries previously considered with booster auxiliaries, it is proper here to indicate how, and under what conditions, automatic boosters operate. There are two generic types of regulating boosters: One in which slight changes of line voltage cause changes in field excitation to produce charge or discharge; in the other the field excitation is varied by the changes in external load.

Fig. 14 shows an example of the first-mentioned type: *G* is a shunt-wound generator, *B* the booster armature, *E* the battery, and *M M* the motors or variable load on the system. The driving-motor is not shown. The operation of this booster is as follows: At normal load, the voltage of the battery and line are equal, and as no current flows either into or out of the former, the series field, *s*, is not excited and the booster voltage is zero. On increase of external load, the generator tends to send out additional current, causing a slight decrease in line voltage. The battery then begins to discharge and the field, *s*, being energized by the outgoing current, induces an e.m.f. to assist the discharge, or in other words, adds an e.m.f. to the battery e.m.f., equal to the drop due to discharge. This added e.m.f. compensates for the battery-drop whatever the rate of discharge, since both vary directly as the outflowing current. Conversely, if the load decreases, the line voltage will rise slightly, causing the beginning of a charge. The direction of the current now being reversed in the coils, the e.m.f. produced in the booster armature is also in the opposite direction and assists in forcing the charge into the battery. Since the rise in battery voltage on charge and the booster e.m.f., both increase proportionally to the current, the rise in battery e.m.f. is practically compensated for. Therefore, within small percentages the voltage is maintained constant, regardless of the rate of battery charge or discharge,

and from this it follows that the load on the generator is also kept constant. Obviously, this booster is operative only on systems having a falling characteristic with load increase; that is, with a shunt generator, if the battery is located in the power-house or on a feeder, the drop through which exceeds the increase in voltage at the generator terminals, if the system is fed by a compound-wound dynamo.

The second type of booster is shown in Fig. 15. In this the battery voltage is equal to that of the line, and at normal load the excitation of the series field s , which is in the external circuit, is exactly equal to that of the shunt field f . These being oppositely wound, neutralize, and the booster voltage is zero. If the external load be increased, the excitation of s overpowers that of f and there results a booster e.m.f. proportional to the increase in external load and in a direction to cause the battery to discharge.

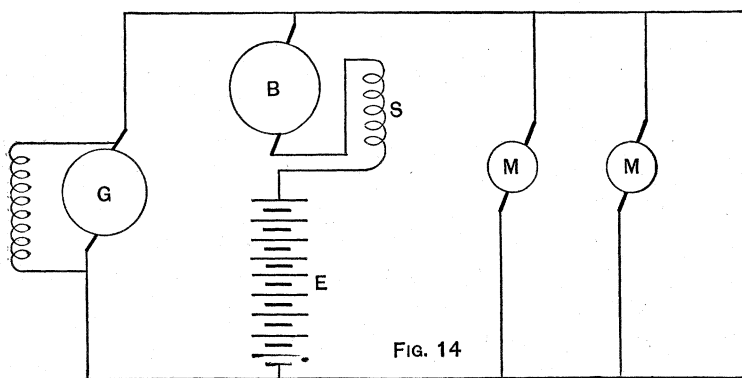


FIG. 14

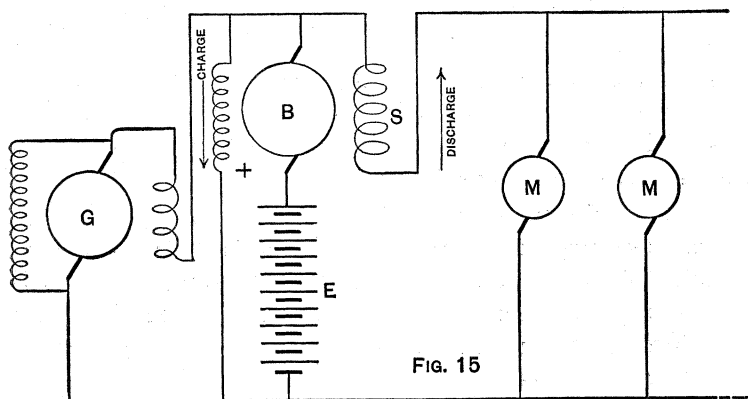
Should the load decrease, the shunt field is the predominant one and produces an e.m.f. in the booster armature to cause a charging current to flow into the battery. This booster can be used either with shunt or compound-wound machines.

These diagrams merely illustrate the salient principles of booster operation, and are not complete practical devices, as additional windings are required to produce points of stable equilibrium and prevent excessive charge or discharge. There is a large number of different types having various windings and connected in ways peculiar to the ideas of the inventors, or the limiting factors of existing patents. These two, however, show the principles on which they all work.

CASE III.—BATTERY WITH BOOSTER IN POWER-STATION.

Since with a booster-controlled battery, the generator load is kept

practically constant, it matters little what kind of generator is used in the power-station, it being only necessary to settle on the best average station voltage. Assuming 550 volts as the desired e.m.f. at the station, a 550-volt battery, the same load and general conditions as in Case I, the size of battery is to be determined. Since the average load is 600 amperes, and the maximum 1,000, a battery to discharge 400 amperes is required. If the one-hour rate is not to be exceeded, an 800-ampere-hour battery is necessary. The maximum load, however, comes on the system but infrequently, as shown by the load-diagram, and as it lasts only 5 seconds, a 700-ampere-hour battery, having a one-hour rate of 350 amperes, will be ample as it will only have to give out 17 percent in excess of its rating, and then for a negligible time. Even a 600-ampere-hour battery would work satisfactorily on this load.



Assuming a 700-ampere-hour battery in the station, practically all the fluctuations in load will be absorbed and the generator load-curve becomes a straight, or rather a wavy line. The station voltage being constant, the feed-point e.m.f.'s at various line-loads are not quite so satisfactory as are furnished by a compound-wound generator. If the minimum voltage at the feed-point is to be 350, the resistance must not exceed .2 ohm or .1418 ohm for the copper resistance, requiring 805,000 cir. mils. as against 665,000. The generating equipment, however, is reduced in size from 550 kw to 350 kw and the fuel economy greatly increased.

COMPARISON OF CASES I. AND III.

As compared with the shunt-wound generator working with a floating battery in the power-station, the generating equipment

is 330 kw as against 440; the copper, 805,000 circular mils as against 945,000; the load variation 6 percent as against 47 percent; the voltage variation at the feed-point 19 percent as against 20 percent—these two factors being practically equal, because of the greater amount of copper in the line fed from the floating battery.

CASE IV.—BATTERY AND BOOSTER OUT ON LINE.

If the floating battery out on the line has a booster auxiliary working with it, the current passing over the feeder will be maintained constant, and the average load only will be transmitted from the power-station. If, as in the previous cases, the voltage is not to fall below 350 volts, the resistance (assuming 550 as the generator voltage) will be .333 ohm total or .2751 ohm for the copper; equivalent to 415,000 cir. mils. This, however, is not so advantageous as it seems, for the voltage at the point where the booster is located is constant and the e.m.f. will neither fall below 350, nor will it rise above it. Taking 420 volts as a satisfactory pressure, the resistance of the line becomes .217 ohm = .1588 ohm for the copper = 720,000 cir. mils. This allows the maintenance of a satisfactory potential which varies so slightly as to be practically constant.

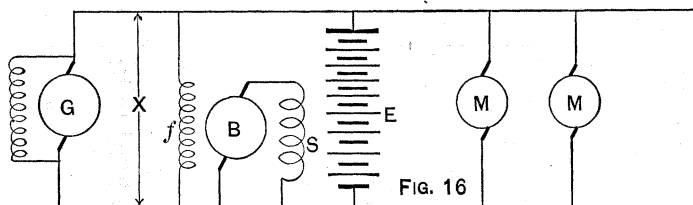
COMPARISON OF CASES II. AND IV.

Compared with the floating battery on the line, the booster-controlled system gives a higher and more satisfactory voltage; the copper is 720,000 cir. mils. as against 495,000 cir. mils. This is not a real comparison, however, in view of the higher voltage maintained. The load on the generating equipment is kept constant within 6 percent as against a variation of 17 percent; the power-house capacity required is 330 kw against 385 kw.

While the booster-controlled battery out on the line is theoretically superior to a similar equipment in the power-station it requires extra attendance, which in many cases offsets its advantages and makes it advisable to put the booster and battery in the station and add a little more copper to the line, or install a floating battery at the feed-point.

Furthermore, the usual condition in practice is that of a power-plant supplying current to numerous feeders, no single one of which carries sufficient current, or is enough longer than the others to warrant a battery installation only to take care of its load and voltage. Generally, therefore, in order to regulate the station load, the battery and booster must be in the power-station.

The advantages shown by the booster-controlled batteries over those floating across the circuit, do not include a most important one; namely, the ability to obtain full battery discharge even when the stored energy in the cells is nearly exhausted and the voltage decreased below normal. The battery characteristic (Fig. 1) shows that the minimum voltage, when discharging at the hour rate, is 1.915 volts per cell. This is the case only when the battery is worked nearly fully charged, and the output and input are so nearly equal, that the state of battery charge is practically unchanged. If, however, any unusual condition or event should raise the average load on the system, the output might exceed the input for several hours, and the battery becomes so far exhausted that the voltage would fall to 1.7 per cell when discharging at the one-hour rate. This is a decrease of .215 volts per cell—or 58 volts in a 550-volt battery—below the minimum voltage of discharge when the battery is worked near the point of full charge. Obviously, without a booster the battery in this con-



dition would not be of any assistance at all in the power-station, and its regulating ability out on the line greatly impaired.

Another advantage offered by the booster is, that the necessary overcharge is not attended by a troublesome manipulation of water-barrels, with the usual accompaniment of heavy grate-bars and a jug of sulphuric acid.

CASE V.—BOOSTER IN STATION; BATTERY ON LINE.

The final method of installing the battery, that is, with the booster in the power-house and the battery out on the line, is peculiar, in that only one type of booster may be used, and there must be a separate feeder from the booster to the point where the battery is located. In the foregoing discussions, a single feeder from the power-station to the feed-point has been assumed, to simplify the comparison, which is just as valid in its results as if the load had been distributed over the length of various feeders. If the circuits in the cases discussed were merely a simple trolley and track with distributed loads, any of the four

described applications could be used, except that the booster system shown in Fig. 15 would not be suitable as the auxiliary for the battery floating on the line.

Character of Booster.—A particular arrangement of the type of booster shown in Fig. 15, causes it to allow only a practically unvarying current to pass through its armature. Referring to Fig. 16, *G* is the generator; *B* the booster armature; *f*, a shunt field; *s* a series field; *E* the battery, and *M M* the motors or variable load. The voltage across the mains at *x* is obviously the generator voltage, while the potential at the terminals of *M M* is equal to the voltage at *x* plus that of the booster. As in the case of the booster shown in Fig. 15, the shunt and series fields are in opposition; but at normal load, the magnetization of the shunt field exceeds that of the series field, and an e.m.f. is generated in the booster armature, which is added to that of the generator to

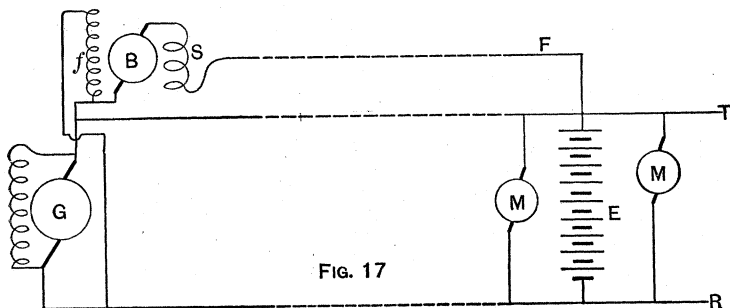


FIG. 17

produce normal voltage across the mains. The battery being connected directly across the mains, its normal voltage is equal to that of the generator, plus booster.

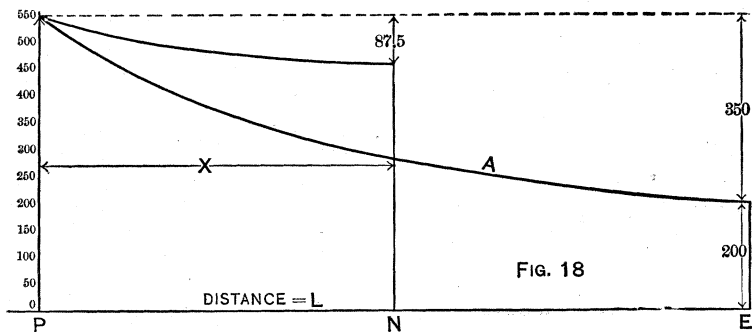
If an excess load should come on the system, the generator tends to send additional current to the line, strengthening *s* and reducing the booster voltage and, consequently, the potential across the mains. The battery then discharges. The converse occurs on increase of load; the booster voltage rises, and current flows into the battery. This system allows only a constant current to pass through the booster armature and is therefore known as the constant-current booster.

In Fig. 17 is shown the method of applying this machine to the case in hand. The symbols are the same as in Fig. 16; *t* is the trolley, *R* the track and *F* the feeder from the booster. The shunt coil *f* is connected across the line and coils in series with the feeder *F*.

Line-Drop.

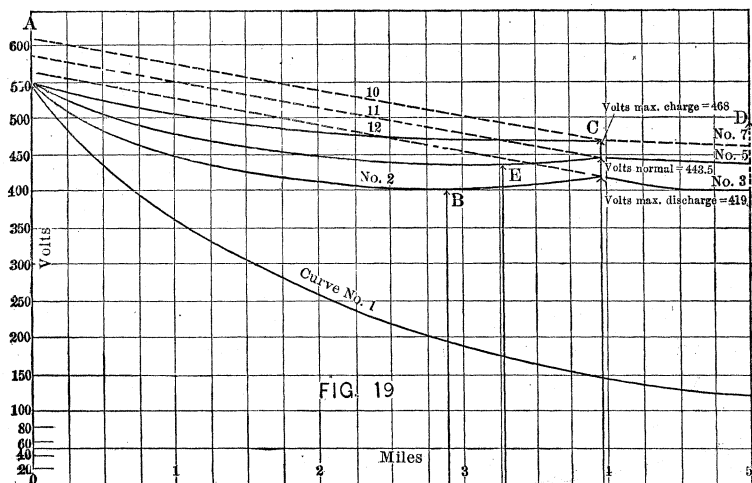
In Fig 18. the curve A represents the voltage which obtains along a line fed from one end, and with the load uniformly distributed along its length.

In the figure, the drop at the distance L, from the source of is seen to be 350 volts. If from P only one half the circuit is supplied with current, there will be a decrease in drop due to lowered resistance, and a further decrease due to the fact that only one half the current flows over the feeder. If the distributed load from P to E is 1,000 amperes and the drop 350 volts, the load over one-half the distance is 500 amperes and the resistance of the circuit one half that from P to E, so that the drop is one fourth of 350 or 87.5 volts, if the circuit extend only to N. This shows that the drop at the end of a main supplying a distributed load varies directly as the square of the distance over which it feeds.

*Data of System.*

Assume the same conditions as in previous cases and shown in Fig. 19, taking the load as being equally distributed along the line, which in this case, however, is 5 miles in length instead of 2 miles, as before. Computations for drop on a 2-mile line with this system show the cross-section of the copper to be so small that it has not the requisite current-carrying capacity. The copper which extends along the entire length of the circuit will be taken as 400,000 cir. mils. inclusive of the trolley wire. The resistance per mile of circuit is .1424 ohm for the copper and .0291 for the track, making .1715 ohm per mile, or .8575 ohm total. The drop on distributed load is 429 volts at the far end of the line when 1,000 amperes pass over it. This is shown by Curve No. 1 in Fig. 19. It is desired so to place the battery on

the line as to reduce the drop to 150 volts, putting the booster in the power-station, and adding such an amount of copper as may be necessary.



Location and Voltage of the Battery.

Since the drop from the power-station varies as the square of the distance over which a distributed load is supplied, the distance, X , to which the original copper can feed at a drop of 150 volts, is computed by the proportion:

$$X^2 : (5)^2 :: 150 : 429, \text{ from which}$$

$$X = 2.89 \text{ miles, which is shown at point B.}$$

The remaining distance, or 2.11 miles, is to be divided into two equal parts and the battery so located that it will feed in each direction 1,055 miles, which is its distance from the end of the line. Point c. shows its position. Its voltage is fixed by the condition that the line voltage must not fall below 400 volts. Applying the above proportion:

$$(5)^2 : 429 :: (1.055)^2 : Y$$

$$Y = \frac{429 \times (1.055)^2}{(5)^2} = 19.1 \text{ volts}$$

= drop for the distributed load on either side the battery, which is located at c. $19.1 + 400 = 419.1 =$ battery voltage when discharging at maximum rate or 1,915 volts per cell.

$$\frac{419.1}{1.915} = 219 = \text{No. of cells in series.}$$

$219 \times 2.025 = 443.47 =$ voltage at c, with normal load on system, battery neither charging nor discharging, 2.025 being the open-circuit voltage per cell as shown in the characteristic (Fig. 1). Voltage per cell is 2.14 when charging at maximum rate.

$2.14 \times 219 = 468 =$ volts at battery at maximum charge.

Changes with Load Variations.

As the load decreases and the voltage rises, the distance that the line will feed toward the battery is increased as shown by curves 4 and 6, while curves 5 and 7 show the corresponding rise of potential at the end of the line. At normal load of 600 amperes total, the minimum voltage is 440 and the point of lowest potential has moved outward to E, the station now feeding from A to E. When the minimum load of 260 amperes is supplied the current from A to c is $\frac{260 \times 3.96}{5} = 207$ amperes

The drop at this load is only 71 volts, making the line voltage at c, 479, which is 11 volts higher than that of the battery on charge. Therefore, the line will supply all current from A to c, and also send enough into the battery to bring the voltage of the line at c down to 468; which extra current to the battery is

$\frac{11}{0.6774} = 16.3$ amperes, 0.6774 being the resistance of the circuit from A to c.

The point E and the voltage at that point are computed as follows: The general formula for distance to which each of two sources of current will feed where both supply the same line at a fixed distance from each other and where the line carries a distributed load is

$$X = \frac{L}{2} + \frac{V_1 - V_2}{arL} \text{ in which}$$

V_1 is the voltage of the source having the higher potential.

V_2 the voltage of the source having the lower potential.

L = distance between the two sources in feet.

a = amperes of distributed load per foot of line.

r = resistance per foot of circuit taken over single distance of transmission.

X = distance in feet to which the higher voltage source will feed.

$L - X$ = distance to which the lower voltage source will feed.

In the case under consideration the quantities are as follows:

$L = 3.96$ miles = 20,880 feet.

$V_1 = 550$ volts.

$V_2 = 443.47$ volts.

$$a = \frac{600}{5 \times 5280} = 0.02278 \text{ amperes per foot}$$

$$r = \frac{0.1715}{5280} = 0.0000325 \text{ ohm per foot.}$$

$$X = \frac{20880}{2} + \frac{550 - 443.47}{0.02278 \times 0.0000325 \times 20,880}$$

$$= 17270 \text{ ft.} = 3.27 \text{ miles.}$$

which is the distance of *E* from *A*. The current distributed

over the line to *E* is $\frac{600 \times 3.27}{5} = 392.4$ amperes. The re-

sistance of this length of circuit is $0.1715 \times 3.27 = 0.56$ ohm.

The drop then to *E* = $\frac{392.4 \times 0.56}{2} = 110$ volts, making the voltage at *E* 440 volts, when the normal current flows over the line.

Feeder to Battery.

The total current to that portion of the system past *E* away from *A*, must be supplied by the feeder *F* (Fig. 17), and this current *Z* is determined by the proportion.

$$(5 - 3.27) : Z :: 5 : 600, \text{ whence}$$

$$Z = 207.6 \text{ amperes.}$$

The size of the feeder *F* is now to be decided on. If its current-carrying capacity be sufficient to allow 207.6 amperes to flow, its resistance is of little moment so far as the operation of the system is concerned, since the voltage of the booster can be made sufficiently high to compensate for any feeder-drop. This resistance, however, is commercially a serious factor, since the booster energy input is lost. The question then is to determine the most economical cross-section of copper, which is beyond the scope of this paper. For purposes of the discussion assume it to be 400,000 cir. mils.; resistance .1424 ohm per mile. Resistance of circuit = $.1424 + .0291 = .1715$ ohm per mile; or $.1715 \times 3.95 = .6774$ ohm, total. The drop then to *c* = $.6774 \times 207.6 =$

140 volts. $443 + 140 = 583$ = voltage of generator plus booster at the station. $583 - 550 = 33$ = booster voltage when average load is on the line. Since the current is practically constant, the drop to c may be considered constant, hence for variation of battery voltage to cause charge or discharge, the booster voltage must vary as indicated at A. The straight lines 10, 11, 12 show the drop along the feeder.

Battery Capacity.

Since the battery regulates 2.11 miles of the system, the average current as shown is 207.6 amperes, while the maximum

$$\frac{1000 \times 2.11}{5} = 422 \text{ amperes.} \quad \text{Therefore, the one-hour dis-}$$

charge rate should be $422 - 207.6 = 214.4$ amperes, making the battery capacity 430 ampere-hours. The constant current from the booster, however, must be taken care of. On light load the generator is furnishing all current up to c, and also 16.3 amperes to the battery, making 223.3 amperes flowing to this point. The

$$\text{current consumption from c to d is only } \frac{260 \times 1.055}{5} = 54.8$$

amperes; $223.3 - 54.8 = 168.5$ amperes, going to the battery which should not be in excess of the one-hour rate of charge. This necessitates the use of a 340-ampere-hour battery, which is less than the capacity required for the maximum discharge.

Results.

This system then distributes current over five miles with a maximum current of 1,000 amperes: the minimum voltage is 400, the average 440, the fluctuations on the system are from 260 to 1,000 amperes, on the generator from 431 to 785.6. This is accomplished by the use of 9 miles of 400,000 cir. mil. feeder, a $12\frac{1}{2}$ kw. booster, a 450 kw. generator, and 261 cells of 430-ampere-hour capacity.

This paper has covered, it is true, only one set of conditions, but from it can be learned the method of investigation applying to any particular set of conditions; and it also shows that the setting forth of any general formulas or deductions to be applied indiscriminately is practically impossible.

APPENDIX.

Deduction of formula

$$X = \frac{L}{2} + \frac{V_1 - V_2}{arL}$$

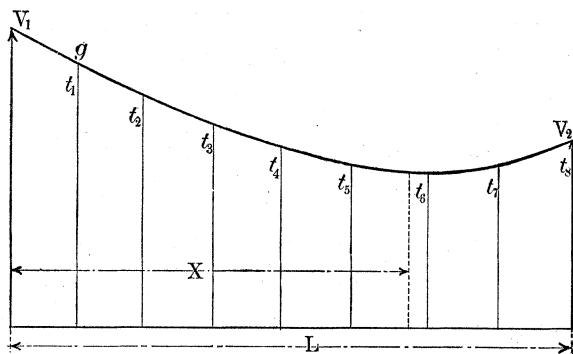


FIG. 20.

In Fig. 20 the heights of V_1 and V_2 represent the respective voltages of two sources of e.m.f.; t_1, t_2, t_3, t_4 , etc., taps or points at which current is taken from the line. L is the distance apart of the two sources. X is the distance over which the source V_1 will feed, and $L - X$ the distance over which V_2 will feed. Let t_x represent the point which is X distance from V_1 and $L - X$ distance from V_2 . Obviously at the point t_x the voltage of V_1 minus the drop from it to t_x = voltage of V_2 minus drop from it to t_x . Assume d as the distance apart of the taps; r as the resistance of the circuit from one tap to the next succeeding one, and a as the number of amperes taken from the line per tap.

The drop then from V_1 to t_1 represented by the curve $V_1 g$ is equal to the total amperes supplied by V_1 , multiplied by the resistance from V_1 to t_1 or drop to $t_1 = n ar$, n being the number of taps from V_1 to t_x and na the total current flow from V_1 to t_1 .

The drop from t_1 to t_2 is equal to $(n - 1) ar$ and from t_2 to $t_3 = (n - 2) ar$, and so on, the last drop being $= ar$. The total drop from V_1 to t_x is the sum of all these. Since d is the distance apart of the taps $\frac{X}{d}$ = number of taps from V_1 to t_x

and the drop from V_1 to $t_x = ar + 2ar + 3ar + \frac{X}{d} ar$. This being an arithmetical progression the sum is equal to one-half the sum of the first and last terms multiplied by the number of

terms; that is, $D_1 = \frac{\left(ar + \frac{X}{d} ar\right) \frac{X}{d}}{2} = \frac{ar}{2} \left(\frac{X}{d} + \frac{X^2}{d^2}\right)$

D_1 being the drop from V_1 to X

Calling D_2 the drop from V_2 to X

$V_1 - D_1 = V_2 - D_2$, from which

$$D_1 = V_1 - V_2 + D_2$$

$$D_2 = \frac{ar}{2} \left\{ \left(\frac{L-X}{d} \right) + \left(\frac{L-X}{d} \right)^2 \right\} \text{ this being derived in the same}$$

way as was the value of D_1 .

Substituting;

$$\frac{ar}{2} \left(\frac{X}{d} + \frac{X^2}{d^2} \right) = \frac{ar}{2} \left\{ \left(\frac{L-X}{d} \right) + \left(\frac{L-X}{d} \right)^2 \right\} + (V_1 - V_2)$$

whence

$$X = \frac{L}{2} + \frac{(V_1 - V_2) d^2}{ar(1+L)}$$

If the distance apart of the taps be taken as unity, say one foot; d^2 becomes equal to one and disappears from the numerator, and since d is negligible compared to L , it may, in practice, be omitted and the formula becomes

$$X = \frac{L}{2} + \frac{V_1 - V_2}{arL}$$

If it be assumed that t_x coincides with some tap point, which receives one-half its current from V_1 and the other half from V_2 , and a = amperes per tap, the drop from V_1 is

$$\frac{1}{2} ar + \frac{3}{2} ar + \frac{5}{2} ar \cdots \left(\frac{X}{d} - \frac{1}{2} \right) ar,$$

$$\text{and the sum} = D_1 = \frac{\left\{ \frac{1}{2} + \left(\frac{X}{d} - \frac{1}{2} \right) \right\} \frac{X}{d} ar}{2} = \left(\frac{X^2}{d^2} \right) \frac{ar}{2}$$

Similarly:

$$D_2 = \left(\frac{L-X}{d} \right) \frac{ar}{2}, \text{ and}$$

$$\frac{ar(X^2)}{2(d^2)} = \frac{ar(L-X)^2}{2} + V_1 - V_2$$

whence

$$X = \frac{L}{2} + \frac{(V_1 - V_2) d^2}{arL} \text{ and if } d \text{ be taken as unity—say one foot}$$

$$X = \frac{L}{2} + \frac{V_1 - V_2}{arL}$$

a and r being respectively the amperes supplied and the resistance of the circuit per foot of distance between the two sources of e.m.f.