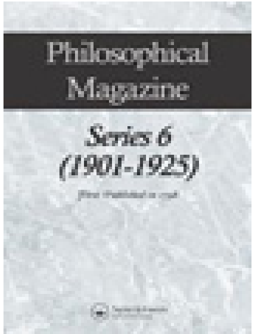

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XX. On a determination of the ratio of the specific heats at constant pressure and at constant volume for air and steam

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the *chemical induction* and *chemical deduction* periods (see p. 391):—

Since velocity of reaction follows the law of action of mass, when the molecules taking part in the reaction have attained, under the influence of light, a constant value of their chemical potentials, the same law of mass-action must also be the governing principle for the velocity of reaction at any given moment of the chemical induction and deduction periods, only the velocity constant in the equation for velocity of reaction will vary as the chemical potentials of the reacting substances change.

Davy Faraday Laboratory of the
Royal Institution, November 1902.

XX. *On a Determination of the Ratio of the Specific Heats at Constant Pressure and at Constant Volume for Air and Steam.* *By* WALTER MAKOWER, *B.Sc., University College, London*[*].

[Plate I.]

1. *Introduction and General Method.*

THE method employed was similar to that used by Lummer and Pringsheim (Smithsonian Contributions to Knowledge, 1898), which consists in allowing the gas under investigation to expand adiabatically and measuring the lowering of temperature caused by such expansion.

In these experiments the initial and final pressures of the gas were measured on a sulphuric acid gauge, and the change of temperature deduced from the variation of the electrical resistance of a fine platinum-bolometer strip immersed in the gas under investigation. The gases experimented upon were air, oxygen, carbon dioxide, and hydrogen, for which the values of the ratio of the two specific heats were found to be 1·4025, 1·3977, 1·2995, 1·4084 respectively.

The chief modifications introduced in the present investigation consist in the substitution of a platinum-thermometer with compensating leads, for the bolometer-strip of Lummer and Pringsheim, who employed a somewhat different device for eliminating errors due to conduction of heat along the leads. Also, at the suggestion of Prof. Callendar, the electrical contacts were made by means of a specially constructed automatic mercury switch, instead of by hand. It was also hoped that it might be possible to use smaller quantities of gas than Lummer and Pringsheim had used, and it was

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[*] Communicated by the Physical Society: read November 14, 1902.

partly with the object of testing this point that the present investigation was undertaken.

If θ_1 and θ_2 be the initial and final temperatures of the gas, and p_1 and p_2 the initial and final pressures respectively, then according to the well-known relation

$$\gamma=\frac{\log(p_1/p_2)}{\log(p_1/p_2)-\log(\theta_1/\theta_2)} \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

If then the gas be allowed to expand in such a manner that p_1, p_2, θ_1 and θ_2 can be measured, the ratio (γ) of the specific heat at constant pressure to the specific heat at constant volume can be calculated.

In the case of steam, which could not be considered as a perfect gas at the temperatures at which the present experiments were made, the characteristic equation proposed by Callendar (Proc. R. S. lxvii. 1900) was employed. On this assumption the adiabatic relation is still given by equation (1).

PART I.

2. *Experiments with Air.*

The apparatus employed is shown on Plate I. It consisted of a large spherical copper vessel (not shown in the figure) which we will call the "air-vessel," of about 50 litres capacity, connected to a tube C for admitting the air to be experimented with; into the "air-vessel" passed a platinum thermometer by means of which the fall of temperature on expansion of the air at a point near the centre of the vessel was measured. Into the neck of the vessel was soldered a side tube of 1·8 cm. diameter. By withdrawing a rubber stopper fitting tightly into this tube the pressure in the vessel was allowed to fall from a value (p_1) previously adjusted to the atmospheric pressure (p_2). By means of the tube D the "air-vessel" was connected to an oil manometer M which could be placed in communication with the experimental vessel or cut off from it at will by means of the glass tap E. The usual arrangement for measuring the resistance of the platinum thermometer is also shown in the figure. In connexion with the "air-vessel" was a mercury-gauge N which served as an automatic key for closing the battery-circuit at a definite instant after releasing the pressure of the air. The gauge N was connected by rubber *r* to a T-piece in the tube D, through which passed a platinum wire *w*, just dipping into the mercury when the pressure inside the apparatus was equal to the pressure of the atmosphere. Dipping into the other arm of the gauge was a wire *x* passing out through a loosely fitting cork *c*, through which also passed a glass tube with a platinum wire *p* sealed through it, electrically connected to *w*. (This was employed in the chronograph measurements to be described below.) When the pressure in the "air-vessel" was equal to the atmospheric pressure, the wire *w* was in electrical connexion with the wire *x*; on raising the pressure the contact between the wire *w* and the mercury was broken, thus breaking the electric circuit from the wire *w* through the mercury to the wire *x*. If the pressure in the "air-vessel" was now suddenly released, contact was made between the mercury and the wire *w* after a definite time had elapsed. This time (which we will denote by τ) could be varied at will by raising or lowering the limb containing the wire *x*, and also by means of a screw pinch-cock (not shown in the figure) which served to constrict, to a greater or less extent, the rubber tubing joining the two limbs of the gauge. The two wires *x* and *w* were connected respectively to the two terminals of the key K, thus putting the gauge in parallel with this key.

3. *Measurement of Temperature.*

From formula (1) it appears that it is necessary to measure both the temperature (θ_1) before opening the vessel and the temperature (θ_2) to which the gas has fallen, measured at an instant as soon as possible after opening the vessel, as the gas begins to heat up, by conduction from the walls of the vessel, almost at once after releasing the pressure. In order, therefore, to obtain reliable results it is necessary that the thermometer which is used should be able to follow as nearly as possible the variations of temperature of the gas. On this account a platinum thermometer of a pattern similar to that employed by Callendar in his steam-engine experiments of 1895 was constructed.

A piece of pure platinum wire (*p*) (Pl. I. fig. 2) of diameter ·001 inch was soldered* on to the platinum leads *l* sealed through one end of the glass tubes *g*; these in turn were soldered on to the copper leads L, passing out of the glass tubes through the other ends which were left open. Close to the thermometer leads were placed compensating leads to which were soldered a piece of fine platinum wire *p′* of the

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* In the air-experiments ordinary soft solder was used. In the steam experiments to be described below the fine platinum wires were attached with silver solder.

same diameter as the thermometer wire, but of shorter length, sufficiently long, however, to eliminate any end-effect error due to conduction of heat from the stout platinum leads to the fine platinum-thermometer wire. The four glass tubes were placed closely side by side, and introduced into the "air-vessel" through the stopper B. To measure the resistance of the thermometer, the thermometer and compensating leads were connected to the two arms of the Wheatstone-bridge, as shown in figure 1. In order to keep the heating effect in the thermometer, due to the passage of the electric current, below ·01° C. the current used in the resistance measurements was made sufficiently small, being supplied by one Leclanché-cell through 240 ohms in the battery-arm. The resistances of the two ratio arms were 3 ohms each. To obtain the balance position, a Thomson galvanometer was used, which, however, was rendered astatic to avoid unsteadiness caused by magnetic disturbances.

In all resistance measurements the galvanometer circuit was kept permanently closed, the battery circuit being broken or made by means of the keys. In this way trouble due to thermoelectric E.M.F.'s was avoided.

Measurement of θ_1.—Air was pumped into the "air-vessel" until the pressure inside exceeded that of the atmosphere by a definite amount (about 67 cms. of oil), time being allowed for the air to assume a constant temperature. The resistance of the thermometer was read off by adjusting the resistance R and the sliding contact *s* with sufficient accuracy to give the temperature of the thermometer to ·01° C. The battery circuit was closed by hand by means of the key K.

Measurement of θ_2.—The resistance R was then diminished and the sliding-contact adjusted by judgment nearly to the position where there would be no current through the galvanometer at the instant when the battery circuit was made by means of the automatic gauge-key N. If the sliding-contact was adjusted exactly to the right position, the galvanometer-needle remained at rest for an instant and then gradually moved off as the thermometer heated up again. If, however, the shift was too small the needle gave a kick in the opposite direction to that corresponding to the heating up of the thermometer, came to rest, and then changed the direction of its motion, and gradually moved off as before as the gas heated up. In making the observations that position of the slider was sought for which the kick of the galvanometer just vanished.

To determine the time which elapsed between the instant of removing the stopper *b* and that at which the mercury in the gauge made contact with the wire w, the contact e was disconnected from f and connected to g, thus cutting out the Wheatstone-bridge and placing the gauge-key N in series with a storage-cell S and a chronograph. The platinum point p, which was electrically connected to w, was brought just into contact with the top of the mercury-column, when the air in the "air-vessel" was adjusted to the initial pressure (p_1). On releasing the pressure the chronograph circuit was broken at p and made again through the wire w, after the expiration of a certain time (depending on the rate at which the mercury fell) which was measured on the chronograph to about ·01 second.

This time was varied from ·5 second up to about 5 seconds.

4. *Pressure Measurements.*

The excess pressure (p_1-p_2) in the "air-vessel" before opening to the atmosphere was measured on a manometer M filled with Fleuss pump-oil. The density and coefficient of expansion having been carefully determined the excess pressure could be obtained in centimetres of water by means of the formula

$$\text{density} = {\cdot}8826 - {\cdot}000644\,t$$

(where t = temperature centigrade)[*].

As the oil used was exceedingly viscous some trouble was experienced at first, owing to the long time taken by the oil in running down the sides of the tube when the pressure was altered. For this reason the position of the oil in the manometer was prevented from shifting more than two or three centimetres by closing the tap E immediately before opening the stopper b.

The pressure (p_2) was obtained by reading the barometer.

5. *Observations.*

The following is a series of observations.

The resistances are given in arbitrary units, of which 100 = 1·31 ohms approximately.

The kicks of the galvanometer-needle are given in terms of the micrometer-divisions in the eyepiece of the reading microscope.

By plotting K against r the change of resistance corresponding to no kick of the galvanometer is found to be 10·47

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[*] The density and coefficient of expansion of the oil were determined by Mr. N. Eumorfopoulos, of University College, who very kindly supplied me with the oil used in these experiments.

TABLE I.

Resistance (R) before opening vessel.	Resistance (R) after opening vessel.	Kick of galvr. in micrometer divisions. K.	Change of Resistance r.
651·00	640·54	no kick	10·46
651·19	640·84	3	10·35
651·27	641·04	5	10·23
651·38	641·24	10	10·14
651·61	641·44	10	10·17
651·62	651·34	5	10·28
651·82	641·49	4	10·33
651·97	641·54	2	10·43

Barometric pressure = 767·6 mm. mercury at 0° C.
= 1044 cms. of water at 0° C.

Excess pressure = 67·1 cms. oil at 18° C. = 58·3 cms. water. The coefficient of the platinum wire used in the thermometer was ·003835 and its resistance at 0° C. was 610·58. Hence 2·34 units of resistance correspond to 1° pt.; therefore a shift of 10·47 units corresponds to a change of temperature of $4^{\circ}{\cdot}48$ pt. = $4^{\circ}{\cdot}43$ C.

$$\therefore \quad \gamma = \frac{\log \dfrac{1102{\cdot}3}{1044}}{\log \dfrac{1102{\cdot}3}{1044} - \log \dfrac{289{\cdot}83}{285{\cdot}4}} = 1{\cdot}396.$$

For this experiment $\tau = {\cdot}76$ second.

6. *Corrections and Results.*

To the value of γ found above there are two corrections to be applied:—

(1) The air in the immediate neighbourhood of the thermometer has risen in temperature by conduction and convection during the time (τ) which elapses between opening the stopper b and closing the battery circuit.

(2) The final temperature as indicated by the thermometer will be higher than the temperature of the air surrounding it on account of direct radiation from the walls of the vessel.

(1) In order to find how much the air had heated up before the battery-circuit was closed by the automatic key, a number of observations were taken similar to those given above, but with different values of τ. A series of values of γ was thus obtained for different values of τ, from which it was possible to deduce the value γ_0 which would have been obtained had

no time been allowed for the air round the thermometer to heat up. For since

$$\log\left(\frac{\theta_1}{\theta_2}\right)=\frac{\gamma-1}{\gamma}\log\left(\frac{p_1}{p_2}\right), \quad . \quad . \quad . \quad . \quad (2)$$

we see, by expanding by the logarithmic series and neglecting all terms except the first, that

$$(\theta_1-\theta_2)=\theta_2\frac{p_1-p_2}{p_2}\frac{\gamma-1}{\gamma}. \quad . \quad \text{(approximately)} \quad (3)$$

Hence $\frac{\gamma-1}{\gamma}$ is proportional to the fall of temperature. $\frac{\gamma-1}{\gamma}$ as calculated from (2) was plotted against τ, and by extrapolating back to $\tau=0$ the value of $\frac{\gamma-1}{\gamma}$ corresponding to no heating of the thermometer, due to conduction and convection, was obtained.

In Table II. are given the values obtained:—

Table II.

Time of closing circuit in seconds (τ).	γ.	$\frac{\gamma-1}{\gamma}$.
0·76	1·396	·2837
1·12	1·396	·2837
1·90	1·386	·2785
3·33	1·381	·2759
1·83	1·386	·2785
1·45	1·392	·2816
3·00	1·380	·2754
2·30	1·386	·2785
5·00	1·380	·2754
4·01	1·380	·2754
0·95	1·396	·2837
5·15	1·380	·2754
1·90	1·389	·2801
1·65	1·389	·2801
2·13	1·391	·2811
2·15	1·391	·2811

Assuming the variation of $\frac{\gamma-1}{\gamma}$ with τ to be linear over the small range considered, the value of $\frac{\gamma-1}{\gamma}$ corresponding to $\tau=0$ is ·285. Hence $\gamma=1·399$.

(2) The error due to radiation was allowed for by coating the thermometer with platinum black. Assuming that the absorption by a platinum-blacked surface is 15 times as powerful as that of a bright surface*, the error due to radiation could be estimated.

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* Lummer and Pringsheim, *loc. cit.*

The value of γ obtained with a platinum-blacked thermometer was 1·360 for $\tau = 0·86$ second.

Since the value of γ corresponding to $\tau = 0·86$ sec. is 1·394, the correction to be applied for radiation is

$$\frac{1·394 - 1·360}{14} = ·0024.$$

Hence the corrected value of the ratio of the two specific heats of air is $\gamma = 1·401$.

Part II.

7. *Experiments with Steam.*

It will be readily understood that in order to determine the ratio of the two specific heats for steam, the use of vessels of the size employed in the experiments with air just described would be exceedingly inconvenient; and indeed the large size of the vessel does not seem to present the same advantage as in experiments made by the method of Clement Desormes. In the latter method the whole of the gas contained in the vessel is being experimented with, and consequently any error due to the heating of the gas close to the walls produces serious errors in the value of γ obtained; it is therefore desirable to reduce the surface of the vessel compared with its volume. In the present method, however, it is merely with the variation of temperature at the point where the thermometer is situated with which we are concerned, and any heating of gas near the walls of the vessel is unimportant. It therefore seemed likely that results of equal accuracy to those obtained with a large vessel might be obtained with a far smaller one.

To test this point experiments were made with air contained in smaller vessels, and the following apparatus was finally constructed for the steam experiments.

A cylindrical copper vessel with coned ends of about 9·3 litres capacity (Pl. I. fig. 3) was constructed having a wide tap A, by opening which the steam could be allowed to expand adiabatically.

A tube D, provided with a tap through which the vessel could be filled with steam, passed through the lower extremity. On either side of the vessel tubes (B and C) were attached; through B a platinum thermometer was inserted; C communicated through a tap and a fine tube E with a glass tube which was connected to the oil manometer and automatic key. The whole was inclosed in a copper jacket filled with steam maintained at an excess pressure of about half an atmosphere, so that the steam in the inner vessel was superheated about 10° C.

The pressure in the jacket was kept constant by means of an automatic gas-regulator devised by Callendar, which controlled the supply of coal-gas reaching the burner employed for heating the boiler which generated the steam. It was found that by this device the pressure could be kept constant to 1 mm. of mercury. To keep the temperature as constant as possible the whole vessel was packed in cotton-wool.

To prevent the condensation of steam in the tube E a small metal tap T was attached close to the vessel: this tap was not opened until the pressure in the vessel had become constant. By pumping in air the pressure in the tube E was raised slightly above that of the steam in the experimental vessel, so that on opening the tap T a small quantity of air passed into the vessel preventing steam from passing into the tube E and condensing there. In order to roughly determine the pressure of the steam before opening the tap T a small auxiliary mercury gauge was attached to C close to the vessel; when the pressure as registered by this gauge was constant and had been adjusted to about the value required for taking an observation, the tap T was opened, thus putting the oil manometer in connexion with the vessel. As the tube E was fine very little steam diffused into it, and no trouble was experienced from this cause when the experiment was carried out as described. To get rid of any small quantity of moisture which might collect after the apparatus had been working for several hours a T-piece F provided with a drain-tap was attached through which such moisture could be expelled. To carry out an experiment the jacket was filled with steam under pressure, the tap A being open, and the tap D was then opened and steam allowed to enter the vessel, until all the air had been expelled; the taps D and A were then closed. It was found that the pressure in the vessel rose for a short time on account of a small quantity of water carried over by the steam entering through D. The pressure of the steam was then adjusted to a suitable value (about 56 cms. of water above the atmospheric pressure) and allowed to become constant. The tap T was then opened, after which the tap A was quickly opened and the pressures and temperatures registered, as in the experiments with air. The initial temperature (θ_1) was always observed just before opening the tap A, only a few seconds being allowed to elapse between taking this observation and opening the tap. Temperatures were read to ·02° C. and pressures to the nearest millimetre of oil.

Before proceeding to a discussion of the results obtained for steam, the values of γ obtained for air using the same apparatus are given as an indication of its sensitiveness.

TABLE III.

Time of closing the circuit in seconds (τ).	γ.
0·58	1·397
1·21	1·392
2·00	1·392

The value obtained with a platinum-blacked thermometer was 1·374; the correction to be applied for radiation is therefore ·017,

$$\therefore\ \gamma=1{\cdot}399.$$

The striking agreement of this value with that obtained with the large vessel demonstrates conclusively that it is possible by the method here employed to work with quantities of gas far smaller than has hitherto been supposed.

8. *Observations and Results.*

The observations were taken in a manner similar to that adopted in the case of air; it was, however, found inconvenient to take all observations between exactly the same pressure limits. The excess pressure was therefore adjusted approximately to the same value in each experiment, and, as in the air-experiments, the sliding-contact was adjusted by judgment *nearly* to the position where there would be no current through the galvanometer at the instant when the battery-circuit was closed; from the initial and final temperatures and pressures a value of $\frac{\gamma-1}{\gamma}$ was calculated, which, as has been shown above, is proportional to the fall of temperature for a constant excess pressure. The kicks of the galvanometer-needle were recorded and a correction applied to the value of $\frac{\gamma-1}{\gamma}$ obtained, in order to allow for the fact that the sliding-contact had not been *exactly* adjusted to the correct position.

In the first series of observations a kick of 1 scale-division corresponded to ·0012 on $\frac{\gamma-1}{\gamma}$; in the second series of observations a kick of 1 scale-division corresponded to ·0016 on $\frac{\gamma-1}{\gamma}$. The observations are given in full below.

TABLE IV.—Series I.

θ_1.	θ_2.	p_1 in cm. water at 0° C.	p_2 in cm. water at 0° C.	Kick of galvanometer in scale-divisions.	$\frac{\gamma-1}{\gamma}$ uncorrected for kick of galv.	$\frac{\gamma-1}{\gamma}$ corrected.
383·30	379·02	1081·2	1028·0	9	·2250	·2358
383·30	378·50	1084·6	1028·0	3	·2349	·2385
383·20	378·37	1084·0	1028·0	no kick	·2395	—
383·50	378·86	1085·4	1030·9	2	·2325	·2349
383·40	378·79	1094·5	1036·0	10	·2200	·2320
383·40	378·69	1091·6	1036·0	no kick	·2367	—
383·40	378·63	1093·5	1036·0	3 or 4	·2314	·2356
383·30	378·53	1092·7	1036·0	no kick	·2349	—
383·30	378·61	1091·6	1036·0	1 or 2	·2355	·2373
383·30	378·79	1090·9	1036·0	3	·2284	·2320
383·30	378·76	1088·4	1035·0	1	·2372	·2384
383·30	378·87	1087·9	1035·0	3	·2331	·2367
383·30	378·83	1090·7	1035·0	17	·2243	·2447
383·30	378·79	1090·4	1035·0	4	·2272	·2320
383·20	378·99	1086·0	1035·0	3	·2296	·2332
383·20	379·02	1086·9	1035·0	2	·2248	·2372
383·20	378·93	1091·0	1035·0	17	·2157	·2361
383·20	378·93	1087·2	1035·0	just a kick	·2308	·2308
383·40	378·82	1090·7	1035·0	7	·2296	·2380
383·40	378·72	1090·3	1035·0	2	·2355	·2379
383·30	378·67	1090·5	1035·0	2 or 3	·2326	·2356
383·10	378·83	1075·8	1021·0	10	·2145	·2265
383·00	378·64	1075·2	1021·0	2	·2212	·2236
383·00	378·46	1077·1	1021·0	5	·2230	·2290
383·00	378·29	1077·7	1021·0	1	·2290	·2302
383·00	378·25	1073·0	1017·0	2	·2331	·2355
383·00	378·21	1073·4	1017·0	1	·2331	·2343
382·90	378·21	1069·4	1015·0	3	·2367	·2403
382·70	377·89	1066·0	1009·0	3 or 4	·2302	·2344
382·70	377·89	1063·8	1009·0	2	·2390	·2414
382·90	378·03	1073·8	1018·0	no kick	·2401	—
383·10	378·23	1076·9	1021·0	no kick	·2395	—
383·40	378·42	1091·2	1035·0	no kick	·2401	—
383·40	378·51	1090·9	1035·0	no kick	·2441	—

For these observations $\tau = 0{\cdot}67$ second.

Mean value of $\frac{\gamma-1}{\gamma} = {\cdot}2349$,

whence $\gamma = 1{\cdot}307$.

TABLE V.—Series II.

θ_1.	θ_2.	p_1 in cm. water at 0° C.	p_2 in cm. water at 0° C.	Kick of galvanometer.	$\frac{\gamma-1}{\gamma}$ uncorrected for kick of galv.	$\frac{\gamma-1}{\gamma}$ corrected.
383·10	378·49	1084·9	1029·0	6	·2290	·2386
383·20	378·49	1084·2	1029·0	no kick	·2372	—
383·20	378·72	1082·5	1029·0	no kick	·2320	—
383·00	378·48	1091·0	1035·0	4	·2255	·2319
383·00	378·34	1091·6	1035·0	4	·2302	·2366
383·10	378·51	1089·5	1035·0	1	·2349	·2365
383·90	379·47	1098·2	1043·0	1	·2248	·2364
383·90	379·08	1100·4	1043·0	no kick	·2355	—
383·90	379·35	1098·7	1043·0	no kick	·2308	—
383·90	379·50	1099·1	1043·0	3	·2200	·2248
383·90	379·44	1098·9	1043·0	4	·2243	·2307
383·90	379·32	1098·8	1043·0	no kick	·2302	—
383·90	379·22	1099·7	1043·0	1	·2320	·2336
383·90	379·10	1100·6	1043·0	no kick	·2343	—
383·90	379·42	1100·6	1045·0	1	·2266	·2282
383·90	379·38	1100·5	1045·0	1	·2290	·2306
383·90	379·28	1101·5	1045·0	no kick	·2302	—

For these experiments $\tau=0·50$ second.

Mean value of $\frac{\gamma-1}{\gamma}=$ ·2328,

whence $\gamma=1·303$.

The values of γ obtained in these two series of experiments are given for clearness

	τ in seconds.	γ.
Series I.	0·67	1·307
Series II.......	0·50	1·303

The agreement of these observations was not sufficiently close to necessitate the application of the small correction for radiation applied in the air-experiments (correction (2) above). An attempt was made to apply a correction for the heating up of the steam round the thermometer in the time τ (correction (1) above). The discrepancies were, however, found to be too great to render it possible to plot a curve and extrapolate to the value $\tau=0$.

It is worthy of mention that the movement of the galvanometer needle was more rapid in these experiments than in those with air, indicating a quicker rate of heating up of the thermometer. This indication was further confirmed by a third series of experiments which was taken, for which τ was

1·14 seconds; the value of γ obtained in this series was 1·291, showing that the thermometer had heated up considerably after 1·14 seconds.

I desire to express my best thanks to Prof. Callendar for his advice and encouragement throughout the course of the work; also to Prof. Porter I am indebted for many valuable suggestions.

XXI. *On the Spectrum of an Irregular Disturbance.*

By Lord Rayleigh, *O. M., F.R.S.*[*]

In my paper " On the Character of the Complete Radiation at a given Temperature "[†], I have traced the consequences of supposing white light to consist of a random aggregation of impulses of certain specified types, and have shown how to calculate the distribution of energy in the resulting spectrum. The argument applies, of course, to all vibrations capable of propagation along a line, and it is convenient to fix the ideas upon the transverse vibrations of a stretched string. Suppose that this is initially at rest in its equilibrium position and that velocities represented by $\phi(x)$ are communicated to the various parts. The whole energy is proportional to $\int_{-\infty}^{+\infty}\{\phi(x)\}^2\,dx$; and it is desired to know how this energy is distributed among the various components into which the disturbance may be analysed. By Fourier's theorem,

$$\pi\,\phi(x)=\int_0^\infty f_1(k)\cos kx\,dk+\int_0^\infty f_2(k)\sin kx\,dk, \quad . \quad (1)$$

where

$$f_1(k)=\int_{-\infty}^{+\infty}\cos kv\,\phi(v)\,dv, \qquad f_2(k)=\int_{-\infty}^{+\infty}\sin kv\,\phi(v)\,dv. \quad (2)$$

It was shown that the desired information is contained in the formula

$$\int_{-\infty}^{+\infty}\{\phi(x)\}^2\,dx=\frac{1}{\pi}\int_0^\infty[\{f_1(k)\}^2+\{f_2(k)\}^2]dk. \quad . \quad (3)$$

As an example, we may take an impulse localized in the neighbourhood of a point, and represented by

$$\phi(x)=e^{-c^2x^2}. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

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[*] Communicated by the Author.

[†] Phil. Mag. xxvii. p. 460 (1889); Scientific Papers, iii. p. 268.

Phil. Mag. Ser. 6, Vol. 5, Pl. I.

Fig. 2.

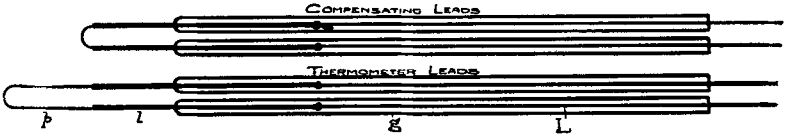

Fig. 3.