73. On the Circles Touching Three Given Tangential Circles

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Source: *The Mathematical Gazette*, Vol. 1, No. 18 (Oct., 1899), pp. 278-279
Published by: Mathematical Association
Stable URL: http://www.jstor.org/stable/3604525
Accessed: 10-01-2016 14:23 UTC

By the aid of this principle, choosing as base points MN (*Gazette*, p. 257), the extension of Feuerbach's Theorem given there can be established.

(f) Let S denote the circumcircle of a triangle.

If P coincide in turn with the circumcentre, incentre, and excentres, the corresponding values of (PS) are

$$-\tfrac{1}{2}R,\ -r,\ r_1,\ r_2,\ r_3.$$

To prove the second: $R^2-d^2=2Rr$ by Euler's Theorem; where d is the distance of the circumcentre from incentre. Hence

$$(PS)\equiv(d^2-R^2)/2R=-r.$$

It is obvious that we may substitute "sphere" for circle, and "plane" for straight line in (a), (b), (c), (d), (e). C. E. M'VICKER.

OBITUARY.

The Mathematical Association has lost an efficient officer in the sudden death of Samuel Oliver Roberts, who had been one of our honorary secretaries since 1897. His mathematical tastes were undoubtedly inherited from his father, Mr. Samuel Roberts, F.R.S., whom he has predeceased. Mr. S. O. Roberts went to Cambridge as a scholar of St. John's College in 1879, and was seventh wrangler in 1882. At Cambridge he made many fast friends, some of whom he constantly revisited there, thus ever retaining a close touch with his Alma-Mater. After leaving Cambridge he was for three or four years at the Royal Grammar School, Newcastle-on-Tyne. Thence he was appointed in 1888 to the second mathematical mastership at the Merchant Taylors' School, and retained the post till the time of his death. He was an active member of the Physical Society and the London Mathematical Society, and an ardent student of modern languages and modern history. But it was to his school work that he devoted his enthusiasm and abundant energies. He was pre-eminent as a teacher, and often took delight in contrasting the mathematical teaching which he himself experienced at school with that of the present day, to the glorification of the latter. He also took a great interest in school sports, and won the admiration of his pupils by his mastery of chess. He was the admiration of all his friends for his strenuous devotion to work. He laboured hard for his school, possibly so hard as to have an adverse influence on his health, though we would fain believe not. He had been suffering from an apparently slight indisposition for some little time; this suddenly took a serious form just before Whitsuntide, and ended fatally on the evening of May 31st. A more detailed account is given in the school magazine, *The Taylorian*, Vol. XXI., No. 6, July 1899.

MATHEMATICAL NOTES.

73. *On the circles touching three given tangential circles.*

If O, O_1, O_2 be the centres of the three given circles; r, r_1, r_2 their radii; I the centre of one of the circles touching each of the three; θ, θ_1, θ_2 the angles $O'IO''$, etc., and R the radius of the circle, centre I, then the sides of the triangle $O'IO''$ are r_1+R, r_2+R, r_1+r_2.

Then $\sin^2\dfrac{\theta}{2}=r_1r_2/(r_1+R)(r_2+R)$, etc.

But $$\Sigma \sin^4\frac{\theta}{2} - 2\Sigma \sin^2\frac{\theta}{2}\sin^2\frac{\theta_1}{2} + 4\Pi \sin^2\frac{\theta}{2} = 0 ; \quad \because \quad \Sigma\theta = 2\pi.$$

Substituting the values of $\sin^2\frac{\theta}{2}$, etc., we have a quadratic for R.

If O_1, O_2, for instance, touch each other externally and O internally, the value of R is obtained from the quadratic by writing $-r_1$, $-r_2$ for r_1, r_2.

E. N. BARISIEN.

74. *Asymptotes in polar coordinates.*

If $u \equiv 1/r = f(\theta)$ is the equation of a plane curve, it is possible to approximate as follows to its form in the neighbourhood of $\theta = \alpha$, where $f(\alpha) = 0$.

For, if $\theta = \alpha + \phi$ and ϕ is a small quantity of the first order of infinitesimals, then by Taylor's Theorem,

$$u = f(\alpha + \phi) = \phi f'(\alpha) + \frac{\phi^2}{2\,!} f''(\alpha) + \ldots .$$

Thus, if $f'(\alpha)$ is not zero or infinite, $u = \phi f'(\alpha)$ to the first order. This is equivalent to $u = \sin\phi f'(\alpha)$ to the same order, and leads to the equation of the linear asymptote in the form $r\sin(\theta - \alpha) = 1/f'(\alpha)$.

If $f'(\alpha)$ vanishes, and $f''(\alpha)$ is neither zero nor infinite, then $u = \frac{1}{2}\phi^2 f''(\alpha)$ to the same order $= 2\sin^2\frac{\phi}{2} . f''(\alpha)$ to the same order. We thus have the equation of the parabolic asymptote, viz., $r\{1 - \cos(\theta - \alpha)\} = 1/f''(\alpha)$. H. T. GERRANS.

EXAMINATION QUESTIONS AND PROBLEMS.

Our readers are earnestly asked to help in making this section of the GAZETTE *attractive by sending either original or selected problems.*

Solutions should be sent within three months of the date of publication. They should be written clearly on one side of the paper. Contractions not intended for printing should be avoided. Figures should be drawn with the greatest care on as small a scale as possible, and on a separate sheet.

The question need not be re-written, but the number should precede every solution.

The source of problems when not otherwise indicated is shown by—C. (Cambridge), O. (Oxford), D. (Dublin), W. (Woolwich), Sc. (Science and Art Department), etc.

320. [L¹. 16. b.] (*a*) A fixed point P is joined to a point M on a conic, and a circle is described on PM as diameter. Find the envelope of the circles as M moves on the conic, and discuss the cases in which P is (1°) at the centre; (2°) a focus; (3°) a vertex.

[L¹. 4. c.] (*b*) Given two conics C, C', a tangent to C cuts C' in A and B. Find the locus of the intersections of the tangents to C from A and B. E. N. BARISIEN.

321. [J. 1. c.] If H_r denote the sum of the homogeneous products of r dimensions of n quantities α, β, γ, ... then any function of their differences satisfies

$$n\frac{du}{dH_1} + (n+1)H_1\frac{du}{dH_2} + \ldots = 0.$$

E. P. BARRETT.