

MINOR CONTRIBUTIONS.

AN EXAMPLE IN THERMOMETRY.¹

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SINCE the publication of the methods of procedure employed at the International Bureau of Weights and Measures, at Breteuil, the ordinary method of obtaining the corrections of a thermometer by comparison with another thermometer whose errors are known, is neither defensible on the score of precision nor on the score of economy in time. The determination of the absolute corrections of a thermometer at a large number of points requires but little more time than the determination of the relative corrections at a few points.

The details of the absolute method are scattered through three volumes of the *Memoirs* of the Bureau, and, being written in French, they are not readily available to the English student. The study of thermometer Gerhardt No. 2403 is given in the following paper as an illustration of the new method of procedure.

The observed reading of a thermometer is subject to the following corrections:—

- (1) A correction for calibration.
- (2) Corrections for external and internal pressure.
- (3) A correction for the fundamental interval between the 0° point and the 100° point. These corrections will be considered in the order in which they are named.

The Calibration of a Thermometer.

We take, as an example, the calibration of the 10° points of thermometer Gerhardt No. 2403. If the graduations upon the stem are exactly equal in value, the corrections to these values will be indicated by the lengths of a column of mercury placed successively end to end for each of the 10° points. Since it is not easy to break off a column having a definite length, we must measure the excess of the length of the column over the length of the successive 10° graduations. If we designate the excess of the length of the column over the length of the 10° spaces by λ , we have only to apply to

¹ Contribution from the Shannon Physical Laboratory of Colby University.

each of the 10° graduations successively the values $\frac{\lambda}{10}$, $\frac{2\lambda}{10}$, ..., $\frac{10\lambda}{10}$, in order to obtain the corrected values of the 10° points of the thermometer.

But we cannot assume that all the subdivisions of the scale are equal. (1) Let $x_1, x_2, x_3, x_4, \dots, x_{11}$ represent the *corrections* to the 10° points of the scale. Represent the observed excess of the length of the column by $a_1, a_2, a_3, a_4, \dots, a_{10}$; then, for any space, *e.g.* between the line 0° and line 10° , the true relation will be expressed by the equation:—

$$x_1 - x_2 + \lambda = a_1.$$

Following the notation given on p. C 37, Vol. II., we shall have a series of equations of the form:—

$$\begin{array}{ll} (2) & x_1 - x_2 + \lambda_{10} = a_1, \\ & x_2 - x_3 + \lambda_{10} = a_2, \\ & \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ & x_{10} - x_{11} + \lambda_{10} = a_{10}. \end{array}$$

Breaking off successively columns having the lengths $10^\circ, 20^\circ, 30^\circ, 40^\circ, 50^\circ, 60^\circ, 70^\circ, 80^\circ$, and 90° , and measuring the excess of these columns over the corresponding spaces, we have a series of fifty-four equations to be solved by the process of least squares.

It is to be noted that the signification of λ , which was assigned to it under the supposition of equal subdivisions of the entire length of the scale, does not hold when we assume that the divisions have the corrections x_1, x_2, x_3, x_4 , etc. In this case, this constant will be affected by the assumption that the scale graduations themselves require corrections.

We take, as an example of calibration, thermometer Gerhardt No. 2403. The readings were all made with a telescope of short focus and a filar micrometer, in which one division equals 0.00066 . The method of making the observations will appear from the following example:—

Let R_i = the micrometer reading of the graduated scale, *e.g.* for 0° .

R_u = the micrometer reading of the upper end of the division,
e.g. for 10° .

R_m = the micrometer reading of the miniscus of the mercury column.

n = the number of tenths of the graduations between which the miniscus falls.

| By estimation. | Measurement with the micrometer. | |
|---|--|--|
| | For 0°. | For 10°. |
| $l = 0.025$ $n = 10.110$ $n - l = 10.085$ | $R_l = 82$ div. $R_m = 131$ div. $R_u = 261$ div. $n = 0.000$ $\frac{49}{179} \times 0.1 = 0.027$ Sum = 0.027 | $R_l = 44$ div. $R_m = 62$ div. $R_u = 224$ div. $n = 0.10$ 1 div. = 0.00066 $\frac{18}{180} \times 0.1 = 0.010$ Sum = 0.110 $0.110 - 0.027 = +0.083.$ |

All the measurements for this series of observations were made in the manner indicated above. We take as an example of the method of reducing the observations given in the first column of Table I.

| Interval. | Observed reading. | Relative errors. | Relative corrections. |
|--------------|-------------------|------------------|-----------------------|
| 0- 10 | 10.019 | -0.001 | +0.001 |
| 10- 20 | 10.034 | +0.014 | -0.014 |
| 20- 30 | 10.006 | -0.014 | +0.014 |
| 30- 40 | 10.030 | +0.010 | -0.010 |
| 40- 50 | 10.034 | +0.014 | -0.014 |
| 50- 60 | 10.008 | -0.012 | +0.012 |
| 60- 70 | 10.012 | -0.008 | +0.008 |
| 70- 80 | 10.020 | +0.000 | +0.000 |
| 80- 90 | 10.075 | +0.055 | -0.055 |
| 90-100 | 10.012 | -0.008 | +0.008 |
| Mean | 10.020 | | |

At this point two courses are open to us. Either we may take the results obtained by subtracting 10 from each of the values, or we may obtain a set of residuals by subtracting the mean value of column (1) from each separate value giving column (2). In the former case we must follow formulæ 1 and 8 given on pp. 8 and 12, Vol. V. In the latter case, the values of s and Σ become zero, and the reduction becomes more simple. We shall in this example follow the latter method. [See a paper on the Cumulative Errors of a Graduated Scale, by W. A. Rogers, *Proceedings Mechanical Engineers*, Vol. XV. The correction is here noted that the denominations S given on p. 134 should be written in small type.]

The numerical data given in Table I. has been obtained from the mean of independent observations taken by each of us.

(3)

TABLE I.

| λ_{10} | | | λ_9 | | | λ_8 | | | λ_7 | | | λ_6 | | |
|----------------|--------|---|-------------|--------|---|-------------|--------|---|-------------|--------|---|-------------|--------|---|
| o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |
| 0-10 | +0.001 | | 0-20 | -0.027 | | 0-30 | +0.014 | | 0-40 | -0.001 | | 0-50 | -0.022 | |
| 10-20 | -0.014 | | 10-30 | -0.005 | | 10-40 | -0.016 | | 10-50 | -0.055 | | 10-60 | -0.036 | |
| 20-30 | +0.014 | | 20-40 | +0.003 | | 20-50 | -0.038 | | 20-60 | -0.035 | | 20-70 | +0.004 | |
| 30-40 | -0.010 | | 30-50 | -0.039 | | 30-60 | -0.040 | | 30-70 | -0.004 | | 30-80 | -0.001 | |
| 40-50 | -0.014 | | 40-60 | -0.012 | | 40-70 | +0.016 | | 40-80 | +0.025 | | 40-90 | +0.014 | |
| 50-60 | +0.012 | | 50-70 | +0.048 | | 50-80 | +0.035 | | 50-90 | +0.037 | | 50-100 | +0.041 | |
| 60-70 | +0.008 | | 60-80 | +0.036 | | 60-90 | +0.034 | | 60-100 | +0.043 | | | | |
| 70-80 | +0.000 | | 70-90 | -0.009 | | 70-100 | -0.005 | | | | | | | |
| 80-90 | -0.005 | | 80-100 | +0.005 | | | | | | | | | | |
| 90-100 | +0.008 | | | | | | | | | | | | | |

| λ_5 | | | λ_4 | | | λ_3 | | | λ_2 | | | | | |
|-------------|--------|---|-------------|--------|---|-------------|--------|---|-------------|--------|---|---|---|---|
| o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |
| 0-60 | -0.038 | | 0-70 | +0.017 | | 0-80 | +0.014 | | 0-90 | +0.002 | | | | |
| 10-70 | -0.002 | | 10-80 | -0.012 | | 10-90 | -0.018 | | 10-100 | -0.002 | | | | |
| 20-80 | +0.002 | | 20-90 | +0.001 | | 20-100 | +0.004 | | | | | | | |
| 30-90 | +0.013 | | 30-100 | -0.006 | | | | | | | | | | |
| 40-100 | +0.025 | | | | | | | | | | | | | |

Since the coefficients of the unknown quantities are all unity, we have only to take the sums of the coefficients, in order to obtain the solution by least squares. Table II. is obtained by writing the figures in vertical columns, commencing at the left in the horizontal columns. The same figures are then written below the line of division with their signs changed. (See Table II. on following page.)

FORMULÆ FOR COMPUTING THE QUANTITIES D AND R .

$$\begin{aligned}
 D_{11} &= t_{11} - t_1 & R_{10} &= \frac{1}{11} (D_{10} - D_{11}) \\
 D_{10} &= t_{10} - t_2 & R_9 &= \frac{1}{11} (D_9 - D_{10}) \\
 D_9 &= t_9 - t_3 & R_8 &= \frac{1}{11} (D_8 - D_9) \\
 D_8 &= t_8 - t_4 & R_7 &= \frac{1}{11} (D_7 - D_8) \\
 D_7 &= t_7 - t_5
 \end{aligned}$$

CALCULATION OF D AND R .

| | | | | |
|-------------------|----------------------|--------------------|-------------------|-------------------|
| $t_{11} = -0.113$ | $t_{10} = -0.061$ | $t_9 = -0.027$ | $t_8 = -0.101$ | $t_7 = +0.198$ |
| $-t_1 = +0.040$ | $-t_2 = +0.151$ | $-t_3 = +0.004$ | $-t_4 = +0.120$ | $-t_5 = -0.078$ |
| $D_{11} = -0.073$ | $D_{10} = +0.090$ | $D_9 = -0.023$ | $D_8 = +0.019$ | $D_7 = +0.120$ |
| | $-D_{11} = +0.073$ | $-D_{10} = -0.090$ | $-D_9 = +0.023$ | $-D_8 = -0.019$ |
| | 11 $R_{10} = +0.163$ | 11 $R_9 = -0.113$ | 11 $R_8 = +0.042$ | 11 $R_7 = +0.001$ |
| | $R_{10} = +0.014$ | $R_9 = -0.010$ | $R_8 = +0.004$ | $R_7 = +0.000$ |

FORMULÆ FOR THE COMPUTATION OF THE QUANTITY Q .

$$Q_{10} = R_{10} + Q_{11} - \frac{2}{10} Q_{11} \qquad Q_8 = R_8 + Q_9 - \frac{2}{3 \times 8} (Q_{11} + Q_{10} + Q_9)$$

$$Q_9 = R_9 + Q_{10} - \frac{2}{2 \times 9} (Q_{11} + Q_{10}) \qquad Q_7 = R_7 + Q_8 - \frac{2}{4 \times 7} (Q_{11} + Q_{10} + Q_9 + Q_8) \qquad Q_{11} = 0$$

TABLE II.

COMPUTATION OF $t_1, t_2, t_3, t_4, \dots, t_{11}$.

| x_1 | x_2 | x_3 | x_4 | x_5 | x_6 | x_7 | x_8 | x_9 | x_{10} | x_{11} |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|----------|----------|
| +0.001 | -0.014 | +0.014 | -0.010 | -0.014 | +0.012 | +0.008 | +0.000 | -0.005 | +0.008 | +0.002 |
| -0.027 | +0.005 | +0.003 | -0.039 | -0.012 | +0.048 | -0.036 | -0.009 | +0.005 | -0.002 | -0.004 |
| +0.014 | -0.016 | -0.038 | -0.040 | +0.016 | +0.035 | +0.034 | -0.005 | -0.014 | +0.018 | +0.006 |
| -0.001 | -0.055 | -0.035 | -0.004 | +0.025 | +0.037 | +0.034 | -0.017 | +0.012 | -0.001 | +0.006 |
| -0.022 | -0.036 | +0.004 | -0.001 | +0.014 | +0.014 | +0.038 | +0.002 | -0.002 | -0.013 | -0.025 |
| -0.038 | -0.002 | +0.002 | +0.013 | +0.025 | +0.022 | +0.036 | -0.004 | +0.001 | -0.014 | -0.041 |
| +0.017 | -0.012 | +0.001 | -0.006 | +0.001 | +0.055 | +0.035 | +0.004 | -0.025 | -0.037 | -0.043 |
| +0.014 | -0.018 | +0.004 | -0.014 | +0.016 | +0.038 | +0.040 | -0.016 | -0.035 | -0.043 | +0.005 |
| +0.002 | -0.002 | +0.027 | -0.005 | -0.003 | +0.039 | +0.012 | -0.048 | +0.036 | +0.009 | -0.005 |
| -0.010 | -0.001 | +0.014 | -0.014 | +0.010 | +0.014 | -0.012 | -0.008 | -0.000 | +0.005 | -0.008 |
| | -0.151 | -0.004 | -0.120 | +0.078 | +0.341 | +0.198 | -0.101 | -0.027 | -0.061 | -0.113 |
| t_1 | t_2 | t_3 | t_4 | t_5 | t_6 | t_7 | t_8 | t_9 | t_{10} | t_{11} |

CHECK.

$$-0.040 - 0.151 - 0.004 - 0.122 + 0.073 + 0.311 + 0.198 - 0.101 - 0.027 - 0.061 - 0.113 = 0.$$

FORMULÆ FOR THE COMPUTATION OF THE QUANTITY P .

$$\begin{array}{lll}
 S_{11} = t_{11} + t_1 & 11 P_{11} = S_{11} + 2X & 11 P_{11} = 0 \\
 S_{10} = t_{10} + t_2 & 11 P_{10} = S_{10} + 2X & \text{Since } P_{11} = X_{11} + X_1 \\
 S_9 = t_9 + t_3 & 11 P_9 = S_9 + 2X & \therefore 2X = -S_{11} \\
 S_8 = t_8 + t_4 & 11 P_8 = S_8 + 2X & \\
 S_7 = t_7 + t_5 & 11 P_7 = S_7 + 2X & \\
 S_6 = t_6 + t_6 & 11 P_6 = S_6 + 2X &
 \end{array}$$

CALCULATION OF S AND P .

| | | | | | |
|-------------------|----------------------|-------------------|-------------------|-------------------|-------------------|
| $t_{11} = -0.113$ | $t_{10} = -0.060$ | $t_9 = -0.027$ | $t_8 = -0.101$ | $t_7 = +0.198$ | $t_6 = +0.341$ |
| $t_1 = -0.040$ | $t_2 = -0.151$ | $t_2 = -0.004$ | $t_4 = -0.120$ | $t_5 = +0.078$ | $t_6 = +0.341$ |
| $S_{11} = -0.153$ | $S_{10} = -0.212$ | $S_9 = -0.031$ | $S_8 = -0.221$ | $S_7 = +0.276$ | $S_8 = +0.682$ |
| $2X = +0.153$ | $2X = +0.153$ | $2X = +0.153$ | $2X = +0.153$ | $2X = +0.153$ | $2X = +0.153$ |
| $11 P_{11} = 0$ | $11 P_{10} = -0.059$ | $11 P_9 = +0.122$ | $11 P_8 = -0.068$ | $11 P_7 = +0.429$ | $11 P_6 = +0.835$ |
| $P_{11} = 0$ | $P_{10} = -0.005$ | $P_9 = +0.011$ | $P_8 = -0.006$ | $P_7 = +0.039$ | $P_6 = +0.076$ |

CHECK.

$$\begin{array}{l}
 P_{11} + P_{10} + P_9 + P_8 + P_7 + \frac{1}{2} P_6 = x. \\
 0 - 0.005 + 0.011 - 0.006 + 0.039 + 0.038 = + 0.077.
 \end{array}$$

CALCULATION OF Q .

| $Q + R - \frac{1}{p} \Sigma Q$ | ΣQ | p | $\frac{1}{p} \Sigma Q$ |
|---|------------|-----|------------------------|
| $Q_{11} = +0.000$ | | 5 | |
| $Q_{10} = +0.014$ | +0.014 | 9 | -0.002 |
| $Q_9 = -0.010 + 0.014 + 0.002 = +0.006$ | +0.020 | 12 | +0.002 |
| $Q_8 = +0.004 + 0.003 - 0.002 = +0.007$ | +0.027 | 14 | +0.002 |
| $Q_7 = +0.000 + 0.006 - 0.002 = +0.004$ | +0.031 | | |

FORMULÆ FOR COMPUTING $x_1, x_2, x_3, \dots, x_{11}$.

$$\begin{aligned}
 x_{11} &= \frac{1}{2}(P_{11} + Q_{11}) & x_1 &= \frac{1}{2}(P_{11} - Q_{11}) \\
 x_{10} &= \frac{1}{2}(P_{10} + Q_{10}) & x_2 &= \frac{1}{2}(P_{10} - Q_{10}) \\
 x_9 &= \frac{1}{2}(P_9 + Q_9) & x_3 &= \frac{1}{2}(P_9 - Q_9) \\
 x_8 &= \frac{1}{2}(P_8 + Q_8) & x_4 &= \frac{1}{2}(P_8 - Q_8) \\
 x_7 &= \frac{1}{2}(P_7 + Q_7) & x_5 &= \frac{1}{2}(P_7 - Q_7) \\
 x_6 &= \frac{1}{2}P_6
 \end{aligned}$$

CALCULATION OF $x_1, x_2, x_3, \dots, x_{11}$.

| Indices. | P | Q | $P+Q$ | $P-Q$ | | |
|----------|--------|--------|--------|--------|-------------------|----------------|
| 11 | +0.000 | +0.000 | +0.000 | +0.000 | $x_{11} = 0.0$ | $x_1 = 0.0$ |
| 10 | -0.005 | +0.014 | +0.009 | -0.019 | $x_{10} = +0.005$ | $x_2 = -0.010$ |
| 9 | +0.011 | +0.006 | +0.017 | +0.005 | $x_9 = +0.009$ | $x_3 = +0.002$ |
| 8 | -0.006 | +0.007 | +0.001 | -0.013 | $x_8 = +0.001$ | $x_4 = -0.007$ |
| 7 | +0.039 | +0.004 | +0.043 | +0.035 | $x_7 = +0.021$ | $x_5 = +0.018$ |
| 6 | +0.076 | | +0.076 | | $x_6 = +0.038$ | |

CHECK.

Sum of $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} = -\frac{1}{2}S_{11}$.
 $0.000 - 0.010 + 0.002 - 0.007 + 0.018 + 0.038 + 0.021 + 0.001 + 0.009 + 0.005 + 0.000 = +0.077$.

FORMULÆ FOR THE COMPUTATION OF $\lambda_2 \dots \lambda_{10}$.

$$\begin{aligned}
 2 \lambda_2 &= S_1 + Q_{11} + Q_{10} & \lambda_2 &= +0.007 \\
 3 \lambda_3 &= S_3 + Q_{11} + Q_{10} + Q_9 & \lambda_3 &= +0.006 \\
 4 \lambda_4 &= S_4 + Q_{11} + Q_{10} + Q_9 + Q_8 & \lambda_4 &= +0.007 \\
 5 \lambda_5 &= S_5 + Q_{11} + Q_{10} + Q_9 + Q_8 + Q_7 & \lambda_5 &= +0.006 \\
 6 \lambda_6 &= S_6 + Q_{11} + Q_{10} + Q_9 + Q_8 + Q_7 & \lambda_6 &= +0.005 \\
 7 \lambda_7 &= S_7 + Q_{11} + Q_{10} + Q_9 + Q_8 & \lambda_7 &= +0.004 \\
 8 \lambda_8 &= S_8 + Q_{11} + Q_{10} + Q_9 & \lambda_8 &= +0.003 \\
 9 \lambda_9 &= S_9 + Q_{11} + Q_{10} & \lambda_9 &= +0.002 \\
 10 \lambda_{10} &= S_{10} + Q_{11} & \lambda_{10} &= +0.000
 \end{aligned}$$

Substituting in the original equations,

$$x_1 - x_2 + \lambda_{10} = a_1, \quad x_1 - x_3 + \lambda_9 = b_1, \quad x_1 - x_4 + \lambda_8 = c_1, \text{ etc.},$$

we have

CALCULATION OF RESIDUALS.

| | | | | | | | | |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| +0.009 | +0.027 | -0.004 | -0.013 | -0.011 | -0.015 | -0.011 | -0.017 | +0.000 |
| +0.002 | +0.004 | -0.009 | +0.011 | +0.010 | -0.003 | +0.000 | +0.009 | -0.001 |
| -0.005 | -0.017 | +0.004 | +0.020 | +0.002 | -0.003 | +0.003 | +0.004 | |
| +0.015 | -0.004 | +0.015 | +0.000 | -0.010 | -0.019 | +0.006 | | |
| -0.006 | +0.011 | +0.003 | -0.012 | +0.004 | -0.001 | | | |
| +0.005 | -0.009 | -0.003 | +0.000 | +0.002 | | | | |
| +0.012 | -0.002 | -0.015 | -0.018 | | | | | |
| -0.008 | +0.007 | +0.009 | | | | | | |
| +0.009 | +0.006 | | | | | | | |
| -0.003 | | | | | | | | |

As a check upon the values of the corrections thus obtained, it may be found profitable to compare these results with those found by the method in use at the Shannon Physical Laboratory of Colby University.

This method consists in the *independent* measurement of the corrections for the scale divisions and of the equal-volume points of the mercurial column for any selected length.

It is obvious that the final corrections will be found by subtracting the equal-scale corrections from the equal-volume corrections for the same points, since the graduations upon the scale must correspond in position with the position of equal-volume points.

This method seems to possess three distinct advantages.

(1) In the ordinary method only a low magnifying power can be used in the telescope or the microscope employed, on account of the necessity of observing the scale and the column at the same time. In this method each focus is obtained independently.

(2) It can only be assumed that the equal-volume points hold true for the middle point of the column. This method allows the zero point of the scale corrections to be coincident with the zero of the equal-volume corrections.

(3) Since the value of the equal volume corrections will, in a good thermometer, have a considerable degree of symmetry, it will be possible to obtain a smooth curve for single degrees from observation at a few points. Since the operation of obtaining the corrections of the single degree points of the scale involves but slight labor, it will be found feasible to obtain the calibration corrections for a large number of points by taking the corrections for the equal-volume points from the constructed curve.

These advantages will be more clearly seen in the example given below.

The data are taken by the stop method. The stops of the comparator are set a distance apart equal approximately to the length of the spaces to be compared. The microscope carriage having been brought into contact with the stop on the left, the micrometer of the microscope is brought into coincidence with, *e.g.*, the line for 0° . The carriage is then brought into contact with the stop on the right and the reading for the coincidence with line 5° is taken. The difference in these readings will give the deviation from the distance between the stops expressed in divisions of the micrometer. In the same way the deviation of the remaining spaces from the constant distance between the stops will be obtained. Column 1 of Table III. contains the values derived from the observation in the manner described. Column 2 of this table contains the results obtained after subtracting the mean of the values in the first column from each value of the screw, since in this method, an increasing reading of the micrometer screw indicates that the space measured is too short. We thus obtain the relative corrections with respect to the mean value of the corrections. The values in column 3 are obtained by the summation of the values in column 2. These values represent the derived corrections which are to be applied to the actual divisions of the scale in order to obtain the subdivision of the total length of the scale into equal parts.

In the same way the corrections for the equal volume points of the corresponding points of the tube are obtained.

It is to be noted that when this method of discussion is applied to the measured excess of the mercury column over the distance selected between the scale divisions, we shall obtain *exactly* the same results as are derived from the same data by the more complex formulæ in the method of Kohlrausch.

The next column of this table contains the determined corrections to the scale readings for the 5° points. It will be seen that these values show a close agreement with the values derived by the method of Marek which are given in the column next following.

Thus far we have assumed that the equal volume corrections hold true for every part of the space occupied by the mercurial column in each of the 10° subdivisions. It is doubtful whether any reliable evidence upon this point can be derived from an examination of these observations, but the following method of investigation may throw a little light upon the subject. In Fig. 1 the two sets of corrections are laid off as ordinates, with the value of one small square for two units in the third decimal place. The equal-space corrections are represented by the full line curve, and the equal-volume corrections by the dotted curve. The values from which the curves are derived, are taken from Table I.

TABLE III.

| | Scale divisions. | Relative corrections | Summed series = Σ . | Column calibrations | Relative corrections. | Summed series = Σ . | Final corrections. | By method of Marek. | Zero of Scale at 5°. | | |
|--------|------------------|----------------------|----------------------------|---------------------|-----------------------|----------------------------|--------------------|---------------------|----------------------|-------------------|--------|
| | | | | | | | | | From computation. | From observation. | Mean. |
| | | | | | | | | | | | |
| 0-5 | 5.058 | +0.028 | +0.028 | 5.076 | +0.036 | +0.036 | +0.008 | 0 | -0.029 | -0.019 | -0.024 |
| 5-10 | 5.057 | +0.027 | +0.055 | 5.057 | +0.017 | +0.053 | -0.002 | -0.010 | +0.000 | +0.000 | +0.000 |
| 10-15 | 5.054 | +0.024 | +0.079 | 5.065 | +0.025 | +0.078 | -0.001 | 0 | +0.028 | +0.019 | +0.024 |
| 15-20 | 5.061 | +0.031 | +0.110 | 5.080 | +0.040 | +0.118 | +0.008 | +0.002 | +0.054 | +0.042 | +0.048 |
| 20-25 | 5.044 | +0.014 | +0.124 | 5.061 | +0.021 | +0.139 | +0.015 | 0 | +0.086 | +0.073 | +0.079 |
| 25-30 | 5.041 | +0.011 | +0.135 | 5.036 | -0.004 | +0.135 | +0.000 | -0.007 | +0.102 | +0.092 | +0.097 |
| 30-35 | 5.014 | -0.016 | +0.119 | 5.017 | -0.023 | +0.112 | -0.007 | 0 | +0.114 | +0.108 | +0.111 |
| 35-40 | 5.024 | -0.006 | +0.113 | 5.063 | +0.023 | +0.135 | +0.022 | +0.018 | +0.100 | +0.103 | +0.102 |
| 40-45 | 5.007 | -0.023 | +0.090 | 5.017 | -0.023 | +0.112 | +0.022 | 0 | +0.096 | +0.098 | +0.097 |
| 45-50 | 5.016 | -0.014 | +0.076 | 5.054 | +0.014 | +0.126 | +0.050 | +0.038 | +0.074 | +0.089 | +0.081 |
| 50-55 | 5.014 | -0.016 | +0.060 | 5.038 | -0.002 | +0.124 | +0.064 | 0 | +0.061 | +0.072 | +0.066 |
| 55-60 | 5.023 | -0.007 | +0.053 | 5.013 | -0.027 | +0.097 | +0.044 | +0.021 | +0.047 | +0.059 | +0.053 |
| 60-65 | 5.017 | -0.013 | +0.040 | 5.019 | -0.021 | +0.076 | +0.036 | 0 | +0.041 | +0.042 | +0.041 |
| 65-70 | 5.015 | -0.015 | +0.025 | 5.093 | -0.047 | +0.029 | +0.004 | +0.001 | +0.030 | +0.033 | +0.032 |
| 70-75 | 5.012 | -0.018 | +0.007 | 5.027 | -0.013 | +0.016 | +0.009 | 0 | +0.016 | +0.023 | +0.019 |
| 75-80 | 5.024 | -0.006 | +0.001 | 5.033 | -0.007 | +0.009 | +0.008 | +0.009 | +0.000 | +0.012 | +0.006 |
| 80-85 | 5.044 | +0.014 | +0.015 | 5.049 | +0.009 | +0.018 | +0.003 | 0 | +0.006 | +0.008 | +0.001 |
| 85-90 | 5.040 | +0.010 | +0.025 | 5.061 | +0.021 | +0.039 | +0.014 | +0.005 | +0.011 | +0.013 | +0.012 |
| 90-95 | 5.019 | -0.011 | +0.014 | 5.015 | -0.025 | +0.014 | +0.000 | 0 | +0.022 | +0.015 | +0.018 |
| 95-100 | 5.016 | -0.014 | +0.000 | 5.026 | -0.014 | +0.000 | +0.000 | +0.000 | +0.013 | +0.011 | +0.012 |
| Mean. | 5.030 | | Mean. | 5.040 | | | | | +0.000 | +0.000 | +0.000 |

In Fig. 2 the equal-volume curve is laid off for the midway points of the 5° subdivisions.

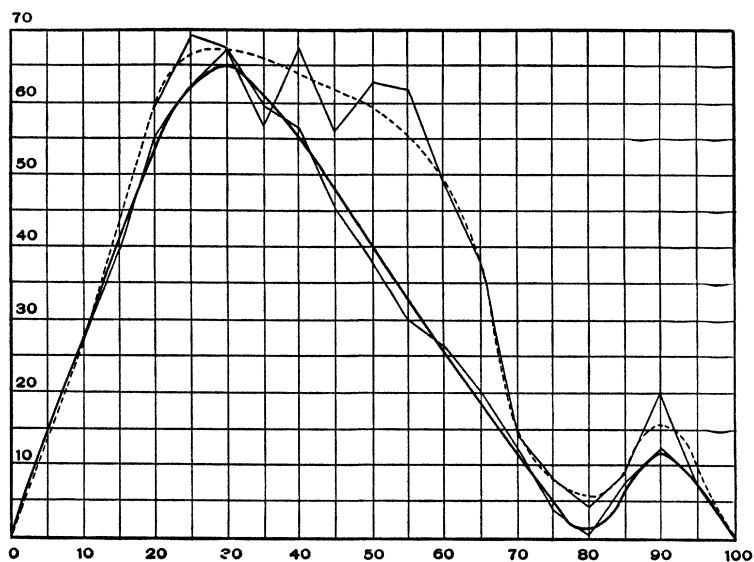


Fig. 1.

In Fig. 3 the zero of the equal space curve is taken at 5° . There are two methods of obtaining the data for this comparison. First, we can

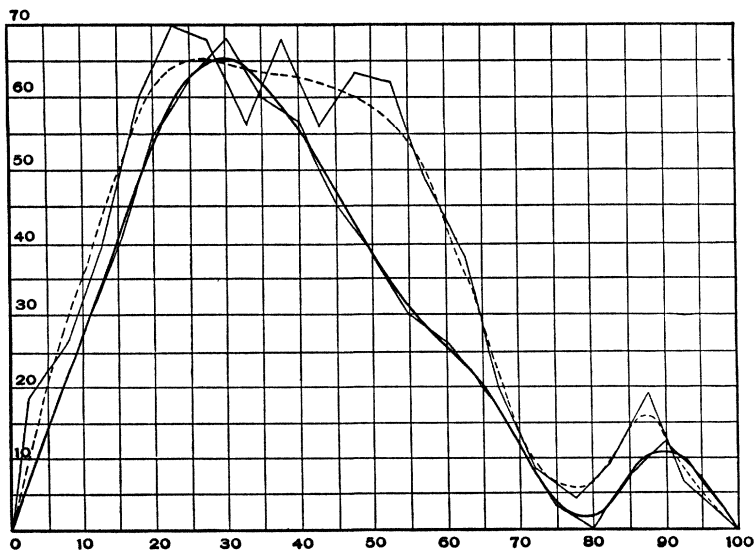


Fig. 2.

start the observed measures from this point, obtaining the results given in the last column but one of Table III. Secondly, we may employ the method given on page C 68, Vol. I., of the *Memoirs* for the conversion from one system to another.

Let x = the correction for any point whatever in any given system. Two points a and b in this system have the corrections x_a and x_b . These

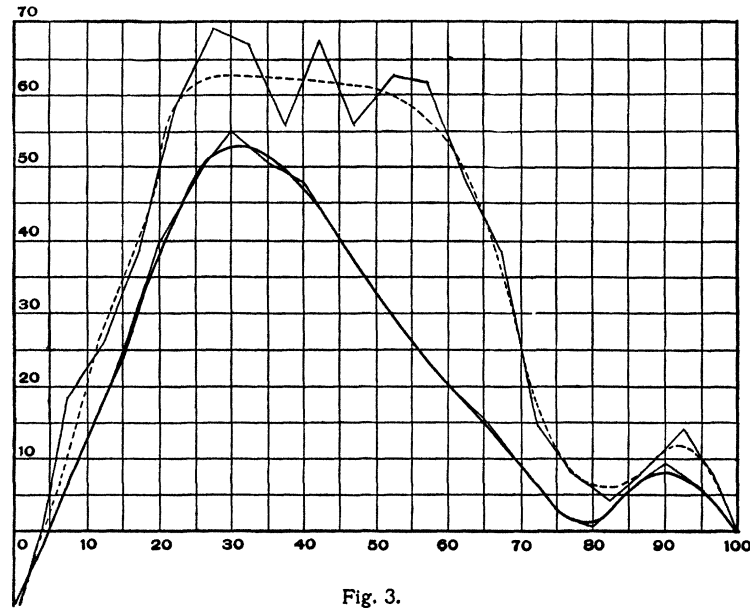


Fig. 3.

corrections are transformed into another system y_m , giving the corrections y_a and y_b by the following formula: —

$$(4) \ y_m = \frac{x_n - x_a - A(m - a)}{B} + y_a = x_n - x_a - A(m - a) + y_n \text{ nearly,}$$

$$\text{in which } A = \frac{(x_b - x_a) - (y_b - y_a)}{(b + y_b) - (a + y_a)}, \quad B = \frac{(b + x_b) - (a + x_a)}{(b + y_b) - (a + y_b)} = 1 \text{ nearly.}$$

In order to transform the zero point of the scale division to the 5° point we have the following: —

$$\begin{array}{llllll} a = 5 & x_a = +28 & y_a = 0 & a + x_a = 5.028 & a + y_a = 5.00 \\ b = 100 & x_b = 0 & y_b = 0 & b + x_b = 100.000 & b + y_b = 100.00 \end{array}$$

$$A = \frac{-0.028}{100 - 5} = -0.030 \quad y_n = x_n - x_m + 0.030 x_m.$$

$$\text{For } n = 70^\circ \quad y_n = +110 - 0.028 + 15 \times 0.030 = +0.086.$$

$$\text{For } m = 0^\circ \quad y_n = 0 - 0.028 - 0.001 = 0.029.$$

$$\text{For } x_n = 60^\circ \quad y_n = +55 - 0.028 + 55 \times 0.030 = +0.041.$$

In this manner the values in the second column from the last of Table III. were computed, and the means of the computed and the observed values are given in the last column. The mean values thus obtained were used in laying off the equal space-curve in Fig. 3. The measured differences between the two curves in this figure will give the corrections given in Table I.

As far as these observations indicate, the values in column I. are a little nearer the correct values than those given in columns II. and III., since the sum of the squares of the vertical column for I. is less than from either II. or III.

Internal and External Pressure.

We now take up the corrections due to external and internal pressure upon the mercury in the bulb of a thermometer.

- (5) Let d = the density of the water in which a thermometer is supposed to be immersed to a depth, below the surface, a distance h .

u = the distance of the middle point of the bulb to the zero point of the scale.

u_1 = the length of one degree expressed in millimeters.

δ_0 = the density of mercury at 0°C .

B = the observed reading of the barometer.

B' = the value of B reduced to 760 mm. at the level of the sea and at the latitude of 45° , by the formula

$$B' = B \frac{1 + 0.0000081 \frac{T}{T'}}{1 + 0.0001815 \frac{T}{T'}} \times \frac{g\phi}{g_{45^\circ}},$$

in which the values of $\frac{g\phi}{g_{45^\circ}}$, taken from the following table, are to be substituted.

| ϕ | $\frac{g\phi}{g_{45^\circ}}$ | ϕ | $\frac{g\phi}{g_{45^\circ}}$ | ϕ | $\frac{g\phi}{g_{45^\circ}}$ |
|--------|------------------------------|--------|------------------------------|--------|------------------------------|
| 35° | 0.99912 | 41° | 0.99964 | 47° | 1.00018 |
| 36 | 0.99921 | 42 | 0.99972 | 48 | 1.00027 |
| 37 | 0.99929 | 43 | 0.99982 | 49 | 1.00030 |
| 38 | 0.99937 | 44 | 0.99981 | 50 | 1.00045 |
| 39 | 0.99946 | 45 | 1.00000 | 51 | 1.00054 |
| 40 | 0.99955 | 46 | 1.00009 | | |

TABLE IV.
CORRECTION TO 2403 FROM DIFFERENT CURVES.

| | I. | II. | III. | Mean = m_1 . | $B = \text{Marek.}$ | $\frac{B + m^1}{2} = m_2.$ | $m_2 - I = v_1.$ | $m_2 - II = v_2.$ | $m_2 - III = v_3.$ | v_1^2 | v_2^2 | v_3^2 |
|------|-----|-----|------|----------------|---------------------|----------------------------|------------------|-------------------|--------------------|---------|---------|---------|
| 0 | + 0 | - 3 | +00 | - 1 | + 0 | + 0 | + 0 | + 0 | + 0 | 0 | 0 | 0 |
| 5 | + 0 | + 6 | +10 | + 5 | -10 | + 0 | - 2 | -12 | -20 | 4 | 144 | 400 |
| 10 | + 2 | +12 | +20 | +11 | + 2 | + 8 | + 2 | -12 | -12 | 4 | 144 | 144 |
| 15 | + 2 | +16 | +20 | +14 | + 2 | + 8 | + 2 | -12 | -12 | 4 | 144 | 144 |
| 20 | + 6 | +20 | +10 | +15 | - 7 | + 0 | - 4 | -18 | + 2 | 16 | 324 | 4 |
| 25 | + 9 | +24 | + 5 | +13 | +18 | +19 | + 3 | -10 | + 5 | 9 | 100 | 25 |
| 30 | + 4 | +18 | - 2 | + 7 | +38 | +39 | + 1 | - 9 | + 4 | 1 | 81 | 16 |
| 35 | + 9 | +22 | + 3 | +11 | +21 | +32 | -11 | -22 | - 2 | 121 | 484 | 4 |
| 40 | +16 | +29 | +14 | +20 | + 1 | +10 | - 3 | -24 | + 6 | 9 | 576 | 36 |
| 45 | +25 | +38 | +26 | +31 | + 9 | + 9 | + 2 | - 1 | - 2 | 4 | 1 | 4 |
| 50 | +30 | +48 | +35 | +40 | + 5 | + 5 | - 7 | - 1 | + 4 | 49 | 1 | 16 |
| 55 | +44 | +54 | +38 | +45 | +00 | + 0 | + 0 | + 0 | + 0 | 0 | 0 | 0 |
| 60 | +43 | +54 | +34 | +44 | | | | | | | | |
| 65 | +37 | +50 | +19 | +38 | | | | | | | | |
| 70 | +13 | +34 | + 4 | +17 | | | | | | | | |
| 75 | + 5 | +14 | + 4 | + 8 | | | | | | | | |
| 80 | + 7 | +10 | +11 | + 9 | | | | | | | | |
| 85 | + 6 | + 3 | +10 | + 6 | | | | | | | | |
| 90 | +12 | + 6 | + 1 | + 6 | | | | | | | | |
| 95 | + 5 | +10 | + 2 | + 6 | | | | | | | | |
| 100 | +00 | +00 | +00 | +00 | | | | | | | | |
| Sums | | | | | | | | | | 217 | 1855 | 649 |

Let P_e = exterior pressure for a vertical position.

P_i = interior pressure for a vertical position.

β_e = the coefficient of exterior pressure = a quantity whose reading increases in proportion to an increase of 1 mm. in the exterior pressure.

β_i = the coefficient of interior pressure = a quantity that diminishes in proportion as the interior increases 1 mm.

Designating the exterior pressure by γ_1 , the interior pressure by γ_2 , and their sum by γ , we have

$$\gamma = \beta_i(u + nu_1) - \beta_e(B' + \frac{h}{13.6} - 760).$$

The plate upon which is represented the apparatus for comparing thermometers, as shown facing page 237, will indicate the method of obtaining the value of β_e with a very brief explanation. The bulb of a thermometer is immersed in a bottle of water with an air-tight connection. The air space at the top of the bottle is connected with an air pump by means of a small lead pipe, a branch of this pipe being connected with a glass tube which is immersed in a reservoir of mercury, which serves as a manometer. The operation of obtaining the value of β_e is as follows: The thermometer is first read under atmospheric pressure. The air is then pumped from the bottle till the column in the manometer remains at a stationary height, when the height of the column is read in millimeters, and a second reading of the thermometer is taken. We have thus an observed difference in the height of the mercurial column corresponding to an observed difference in pressure. Or:—

$$(P_1 - P_0)\beta_e = t_1 - t_2,$$

$$\beta_e = \frac{t_1 - t_2}{P_1 - P_0}.$$

Care must be taken in these observations to allow considerable time to elapse after a change of pressure takes place, in order for the glass to take its normal condition under strain, otherwise much too large values of β_e will be obtained. As much as ten minutes should be allowed for this purpose. The mean of a sufficient number of observations gave for this thermometer

$$647 \beta_e = 0.033^\circ.$$

$$\therefore \beta_e = 0.00005.$$

The value of the coefficient β_i is found by taking the difference between successive readings of a thermometer first in a vertical and then in a horizontal position.

All the observations which have been made at Breteuil agree in giving substantially the same value for β_i as for β_e for the same thermometer.

Since the coefficients for exterior and interior pressure are identical, the expression for the corrections can be simplified, giving, for a vertical point, —

$$\begin{aligned}(760 - B')\beta &= \gamma_1, \\ (u + nu_1)\beta &= \gamma_2.\end{aligned}$$

The Fundamental Interval of a Thermometer.

If the reading of a thermometer in melting ice, when corrected for calibration error and for external and internal pressure, is zero, and if the reading in steam, at 760 mm. pressure, is exactly 100°, it is obvious that no correction for fundamental interval will be required. If under these conditions the thermometer, immersed in steam, at normal pressure, reads, e.g., 100°.20, and the thermometer reads 0°.05 in melting ice, the difference between the two corrected values will give a quantity which is designated the fundamental interval of a thermometer. This difference must be distributed over the entire length of the distance between 0° and 100°. The chief difficulty in determining the true reading of a thermometer immersed in steam is the separate determination of the pressure of the steam. This is accomplished by means of a water barometer, as seen in the Regnault apparatus shown in Plate I. The method of procedure at Breteuil is as follows : —

Suppose that the reading of the barometer is 745.90 mm., that the reading of the water barometer is 12.7 mm. = 0.93 mm. for mercury, that the corrected reading of the zero point is - 0.363, and the corrected reading for steam 99°.369.

We first find the temperature corresponding to the pressure 745.90 + 0.93 = 746.83 mm. from the table following : —

| Pressure. | <i>T</i> | Δ | Pressure. | <i>T</i> | Δ |
|-----------|----------|----------|-----------|----------|----------|
| mm. | ° | | mm. | ° | |
| 740 | 99.2577 | 0.0376 | 750 | 99.6310 | 0.0371 |
| 741 | 99.2953 | 0.0374 | 751 | 99.6681 | 0.0370 |
| 742 | 99.3321 | 0.0375 | 752 | 99.7051 | 0.0370 |
| 743 | 99.3702 | 0.0373 | 753 | 99.7421 | 0.0370 |
| 744 | 99.4075 | 0.0374 | 754 | 99.7791 | 0.0369 |
| 745 | 99.4449 | 0.0373 | 755 | 99.8160 | 0.0369 |
| 746 | 99.4822 | 0.0372 | 756 | 99.8529 | 0.0368 |
| 747 | 99.5194 | 0.0373 | 757 | 99.8897 | 0.0368 |
| 748 | 99.5567 | 0.0371 | 758 | 99.9265 | 0.0368 |
| 749 | 99.5938 | 0.0372 | 759 | 99.9633 | 0.0367 |
| 750 | 99.6310 | | 760 | 1.0000 | |

| Pressure. | <i>T</i> | Δ | Pressure. | <i>T</i> | Δ |
|-----------|----------|----------|-----------|----------|----------|
| mm. | ° | | mm. | ° | |
| 760 | 100.0000 | 0.0367 | 770 | 100.3649 | 0.0363 |
| 761 | 100.0367 | 0.0366 | 771 | 100.4012 | 0.0362 |
| 762 | 100.0733 | 0.0366 | 772 | 100.4374 | 0.0362 |
| 763 | 100.1099 | 0.0366 | 773 | 100.4736 | 0.0362 |
| 764 | 100.1465 | 0.0365 | 774 | 100.5098 | 0.0361 |
| 765 | 100.1830 | 0.0364 | 775 | 100.5559 | 0.0361 |
| 766 | 100.2194 | 0.0365 | 776 | 100.5820 | 0.0360 |
| 767 | 100.2559 | 0.0364 | 777 | 100.6180 | 0.0360 |
| 768 | 100.2923 | 0.0363 | 778 | 100.6540 | 0.0360 |
| 769 | 100.3286 | 0.0363 | 779 | 100.6900 | 0.0359 |
| 770 | 100.3649 | | 780 | 100.7259 | |

From this table we find the temperature corresponding to 746.83 mm.
= 99°.512.

$$\text{True value } 1^\circ = \frac{99.512}{99.369 - (-.363)} - 1 = -0.00221.$$

Or, we may proceed as follows : —

$$\begin{aligned} \text{Corrected } 100^\circ \text{ reading} &= 99^\circ.3690 \\ \text{Correction for pressure} &= \frac{12.7 \times 0.0371}{13.6} = -0.0345 \\ \text{Reading for pressure of 760 mm.} &= 99^\circ.3345 \\ \text{Reduction from 745 to 760 mm.} &= 14.1 \times 0.0371 = +0.5231 \\ \text{True reading at } 100^\circ &= 99^\circ.8576 \\ \text{True reading at } 0^\circ &= -0.3630 \\ \text{Fundamental interval} &= 100^\circ.2206 \\ \text{Corrected } 1^\circ = \kappa &= -0.00221 \end{aligned}$$

There is one reason which seems to have considerable weight in favor of the latter method of reduction. If the reduced readings of the water barometer are subtracted from the observed temperature readings, the differences should be constant if the water barometer indicates the real steam pressure. It is just here that the observer is liable to serious error. It will be seen from the following example that the true reading of the barometer is obtained only after there has been a continuous constant steam pressure for at least twenty minutes. In the observations which follow it has been the practice to allow the thermometer to remain under a little greater than the normal pressure for about fifteen minutes, and after that under a constant pressure for about fifteen minutes longer.

The following is an example : —

OBSERVATION JANUARY 22, 1896.

| <i>T</i> | Water manometer. | <i>t</i> | Δt | True <i>t</i> . |
|----------|---------------------|----------|------------|-----------------|
| h. m. | mm. | ° | | ° |
| 9 25 | 3 | 100.218 | -0.008 | 100.210 |
| 9 28 | 2 | 100.235 | -0.005 | 100.230 |
| 9 30 | 7 | 100.292 | -0.019 | 100.273 |
| 9 32 | 8 | 100.302 | -0.021 | 100.281 |
| 9 36 | 9 | 100.320 | -0.024 | 100.296 |
| 9 38 | 9 | 100.334 | -0.024 | 100.310 |
| 9 42 | 9 | 100.327 | -0.024 | 100.303 |
| 9 44 | 8 | 100.337 | -0.024 | 100.313 |
| 9 46 | 9 | 100.348 | -0.024 | 100.324 |
| 9 48 | 9 | 100.348 | -0.024 | 100.324 |
| 9 52 | 5 | 100.340 | -0.014 | 100.326 |

Following the practice at Breteuil, the reading for the zero point has been obtained from exposure to melting ice immediately *after* the exposure to steam. For this particular thermometer the time required for recovery from the depression of the zero point is about five hours, hence in the present instance it would make little difference whether the reading before or after exposure was selected. The latter reading has been selected on the score of uniformity of practice. These observations apparently furnish an additional reason for this choice, although additional observations at low pressures are needed to confirm the apparent fact that for low pressures all the thermometers investigated by Dr. Rogers have their readings lowered at the zero point, while no such effect is seen when the reading for the zero point is taken immediately after exposure to steam. It will be seen by an examination of the following table that the fundamental interval of this thermometer remains nearly constant.

| Date. | <i>B'</i> | 100°. | | Depression. | <i>t</i> . |
|----------------------|-----------|---------|--------|-------------|------------|
| | | Before. | After. | | |
| | inches | o | o | o | o |
| Jan. 17. | 769.7 | +0.000 | -0.027 | + 0.027 | 100.110 |
| Jan. 20, morning . | 770.0 | +0.004 | -0.026 | + 0.030 | 100.108 |
| Jan. 20, afternoon . | 770.0 | +0.006 | -0.027 | + 0.033 | 100.166 |
| Jan. 21, morning . | 764.9 | +0.003 | -0.027 | + 0.030 | 100.122 |
| Jan. 21, afternoon . | 763.4 | +0.004 | -0.026 | + 0.030 | 100.147 |
| Jan. 22, morning . | 766.1 | +0.007 | -0.023 | + 0.030 | 100.152 |
| Jan. 22, afternoon . | 766.9 | +0.000 | -0.015 | + 0.015 | 100.138 |
| Jan. 23, morning . | 771.6 | +0.011 | -0.008 | + 0.019 | 100.169 |
| Jan. 23, afternoon . | 770.6 | +0.000 | -0.018 | + 0.018 | 100.152 |
| Feb. 10, morning . | 746.2 | -0.070 | +0.004 | -[0.074] | 100.129 |
| Feb. 11, morning . | 741.9 | -0.107 | -0.018 | -[0.089] | 100.135 |
| Feb. 12, morning . | 751.7 | -0.003 | -0.025 | + 0.022 | 100.166 |
| Feb. 13, morning . | 762.7 | +0.006 | -0.018 | + 0.024 | 100.150 |
| Feb. 14, morning . | 746.4 | +0.004 | -0.003 | + 0.007 | 100.152 |
| Feb. 15, morning . | 759.1 | +0.011 | +0.000 | + 0.011 | 100.136 |
| Mean | | | | + 0.019 | 100.142 |

Determination of the Fundamental Interval by means of a Thermometer whose Fundamental Interval is known.

Let us suppose that a thermometer whose constants are fully determined is immersed to a depth h in water which has a nearly constant temperature. The temperature t will be determined by the equation

$$t = n + x_n + \gamma_1 + \gamma_2 - \zeta + \eta\kappa,$$

in which ζ = the reading for the corrected zero point = $n + x_n + \gamma_1 + \gamma_2$ for 0° , and $\eta = n_1 + x_n + \gamma_1 + \gamma_2$.

Very soon afterwards place in the liquid another thermometer whose constants are all known except ζ and κ . We shall then have, since the temperature of the liquid is supposed to remain constant,

$$t = [n_1 + x_2 + \gamma_1 + \gamma_2] = \zeta_2 - n_1\kappa_2.$$

Immediately afterwards put this thermometer in melting ice and obtain a corrected reading for the zero point by the equation

$$t_0 = n_0 + x_n + \gamma_1 + \gamma_2 - \zeta_2.$$

Since for this point $\kappa_2 = 0$, for the second thermometer we shall have

$$\zeta = (n_0 + x_n + \gamma_1 + \gamma_2)$$

$$\kappa_2 = \frac{t}{(n_1 + x_n + \gamma_1 + \gamma_2) - \zeta}.$$

We take as an example the data for thermometer A given on p. D 11, Vol. I. It is required to find ζ and κ for thermometer G. The constants for these two thermometers are given on pp. D III. and D IV. For thermometer A we have

$$t = n + x_n + \gamma_1 + \gamma_2 - \zeta + \eta\kappa$$

$$13.025 + 0.1666 + 0.001 + 0.014 - 0.027 - 0.029 = 13.150.$$

For thermometer G we have

$$13.150 = 13.544 + 0.070 + 0.000 + 0.011 - \zeta = 13.626.$$

For t_0 we have

$$\zeta = n + x_n + \gamma_1 + \gamma_2$$

$$\zeta = +0.365 + 0.002 + 0.000 + 0.005 = +0.373,$$

then

$$\kappa = \frac{13.150}{13.626 - (+0.372)} - 1 = -0.00740,$$

$$\eta\kappa \text{ for } 13^\circ.6 = -0.101.$$

For a check we have

$$t = n_1 + x_n + \gamma_1 + \gamma_2 - \zeta + \eta\kappa$$

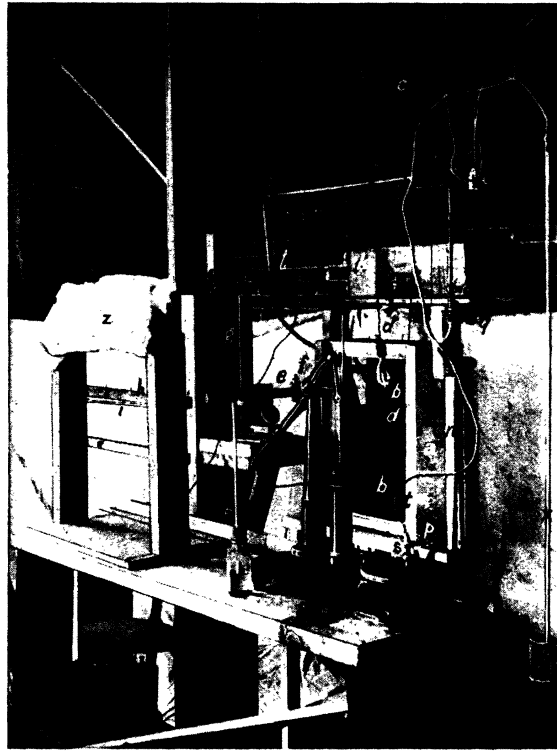
$$13^\circ.150 = 13.544 + 0.071 + 0.000 + 0.011 - 0.372 - 0.101 = 13^\circ.153.$$

We take as a second example the comparison of 2403, G. 2405, and Tounelot 62. The fundamental intervals for these thermometers are given below. Those for G. 2405 and T. 62 have been furnished by Dr. Rogers.

THE FUNDAMENTAL ELEMENTS.

| | 2403 | 2405 | T. 62 |
|--|------------------|------------------|------------------|
| | mm. | mm. | mm. |
| u = distance from center of bulb to zero of scale = | 69 | 60 | 63 |
| n' = the number of degrees of any reading = | ... | ... | ... |
| u_1 = the value of one degree in millimeters = | 4.47 | 3.29 | 3.69 |
| $n'u_1$ = the distance from zero of scale to an observed reading = | $n' \times 4.47$ | $n' \times 3.29$ | $n' \times 3.69$ |
| $u + n'u_1$ = distance of any reading from center of bulb = | ... | ... | ... |
| β = coefficient of external pressure = | 0.00005 | 0.00016 | 0.00014 |
| β_i = coefficient of internal pressure = | 0.00005 | 0.00016 | 0.00014 |
| ζ = corrected reading of zero point = | - 0.0016 | - 0.001 | +0.400 |
| κ = constant derived from the fundamental interval. = | -0.00142 | -0.00142 | -0.0038 |

PLATE I.



Apparatus for the Comparison of Thermometers, and for Determining the Coefficient of External Pressure.

(a)(a') = water tank with windows of heavy plate glass in front and rear.

(b)(b') = paddle wheels driven by the pulley (c).

(k) = ice box.

(l) = connection with water pipe.

(m) = connection with steam pipe.

(n) = connection with water tank.

(g) = upright groove, in which the sliding block *j* is moved by a weight passing over a pulley.

(e) = telescope of very short focus.

(f) = filar micrometer.

(h) = frame upon which the thermometer (i) is mounted in a horizontal position.

(z) = sheet cotton for shading (i).

(d') = wire from which the thermometers are suspended in the water tank.

(u) = Regnault's steam comparing apparatus.

(o) = water barometer for measuring the excess over normal steam pressure.

(u)(s)(r)(x)(y) = apparatus for measuring the coefficient of external pressure.

DATA DERIVED FROM OBSERVATION.

(All the thermometers were immersed in water at a depth $h = 510$ mm. Observed barometer reading, 745.6 mm. Att. ther. = $8^{\circ}.22$. Reduced barometer = $B' = 744.6$ mm. 550 mm. for water = 38 mm. for mercury.)

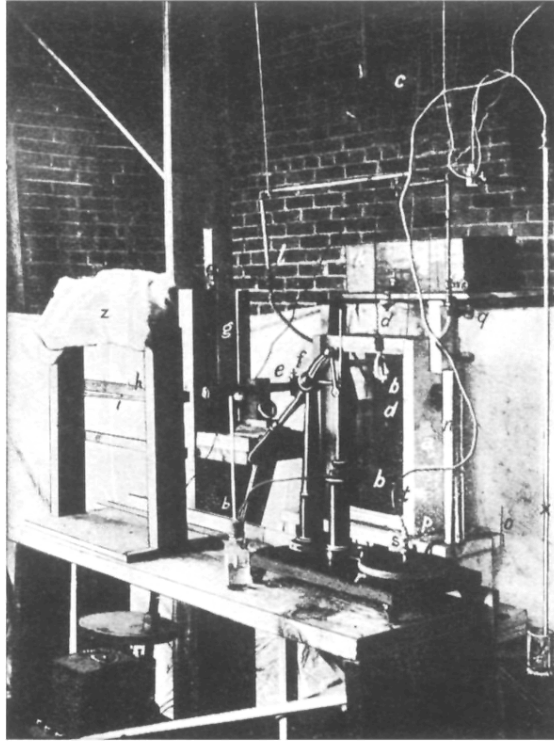
| | 2403 | 2405 | T. 62 |
|---|--------|--------|--------|
| n = | +6.727 | +6.730 | +6.259 |
| x_n = | +0.008 | -0.002 | -0.044 |
| $\beta_0 \times 15.4 = \gamma_1$ = | +0.001 | +0.002 | +0.001 |
| $-\beta_0 \times 22.6 = \gamma_1$ = | -0.001 | -0.003 | -0.002 |
| $\beta_t \times 10.19 = \gamma_2$ = | +0.005 | +0.013 | +0.012 |
| $-\zeta$ = | +0.016 | +0.000 | -0.461 |
| n = | +6.756 | +6.740 | +6.765 |
| $\eta\kappa$ = | -0.009 | -0.008 | -0.025 |
| T = | +6.747 | +6.732 | +6.740 |

In conclusion we desire to express our thanks to Dr. Rogers for assistance rendered in this investigation and for the use of the apparatus belonging to the Shannon Physical Laboratory of Colby University.

REFERENCES TO MEMOIRS OF THE INTERNATIONAL BUREAU.

| Reference Nos. | I. | II. | V. |
|----------------|-----------|-----------|-------|
| (1) | — | C 37 | 85 |
| (2) | — | C 45 | 8 |
| (3) | — | C 45-C 47 | 18-21 |
| (4) | — | C 68 | 11 |
| (5) | D 5-D 7 | C 34 | 26-37 |
| (6) | — | C 62 | — |
| (7) | A 10-A 13 | — | — |
| (8) | D 5 | — | — |
| (9) | D 7 | — | — |
| (10) | D 5 | — | — |
| (11) | A 46-A 48 | C 52-C 53 | — |
| (12) | D 9-D 11 | — | — |
| (13) | D 5-D 11 | C 54-C 63 | 49-62 |
| (14) | D 11 | — | — |
| (15) | D 10-D 11 | — | — |
| (16) | D 5 | — | — |

PLATE I.



Apparatus for the Comparison of Thermometers, and for Determining the Coefficient of External Pressure.

(a)(a') = water tank with windows of heavy plate glass in front and rear.

(b)(b') = paddle wheels driven by the pulley (c).

(k) = ice box.

(l) = connection with water pipe.

(m) = connection with steam pipe.

(n) = connection with water tank.

(g) = upright groove, in which the sliding block *j* is moved by a weight passing over a pulley.

(e) = telescope of very short focus.

(f) = filar micrometer.

(h) = frame upon which the thermometer (i) is mounted in a horizontal position.

(z) = sheet cotton for shading (i).

(d) = wire from which the thermometers are suspended in the water tank.

(u) = Regnault's steam comparing apparatus.

(o) = water barometer for measuring the excess over normal steam pressure.

(u)(s)(r)(x)(y) = apparatus for measuring the coefficient of external pressure.