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The Arithmetical Solution of Plato's Number

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stages in the process as explained to me by Mr. Madan, (i.) τὰ, (ii.) τ, (iii.) πρῶτον τὰ.

The omission of τὰ τοιάδε in A was supplied in the margin by a second hand, acknowledged to be the same as b.

(4) τὸ θερμὸν καὶ τὸ ψμχρὸν] A, τὸ θερμὸν
τὸ
καὶ ψυχρὸν B, τὸ being supplied above the line by b.

(5) ξυντρέφεται] A, συντρέφεται B, ξυντρέφεται (ξ in rasura) b.

τε
(6) οὐρανόν τε καὶ] A, om τε B, οὐρανόν καὶ b, τε being added above the line, and the accent of οὐρανὸν altered.

(7) ἀπέμαθον καὶ ταῦτα ἃ πρὸ τοῦ ᾤμην εἰδέναι] A, ἄποτ' ἔμαθον ἃ καὶ πρὸ τοῦ κ.τ.λ. B, with the following marginal note:

οὕτω δεῖ ἐν ἄλλῳ
ὥστε ἀπέμαθον καὶ ταῦτα ἃ πρὸ τοῦ ᾤμην εἴδεναι.

This note is in the hand of b, except οὕτω δεῖ, which is much later.

Mr. T. W. Allen of Queen's College, Oxford, a well-known expert in Greek Palaeography and the author of the very valuable Preface to the photographic edition of the Codex Clarkianus, has kindly re-examined this passage both in the photograph and in the MS. itself, and assures me that the corrections are all made by the same hand (b). He clearly identifies the writing with that of notes which have been generally attributed by other scholars, such as Harnack, Von Gebhardt, Heikel, and Schwartz (*Athenagorae Libellus*, Praef. p. iv) to Arethas himself. If Mr. Allen still hesitates to accept their conclusion as absolutely certain, it is, I believe, chiefly on the ground that the rich purchaser of the MS. would not have been likely to number the parchments and add the titles of the Platonic Dialogues with his own hand, instead of requiring such mechanical work to be completed by the professional scribe.

Against this objection it is urged that in other MSS. belonging to Arethas both his name and the price paid by him are added in the same writing; and especially that the D'Orville MS. of Euclid (Bodl. n. 301) written in the year 888 A.D. has the subscription by this same hand: 'Εκτησάμην
0 0 =
'Αρέθας Πατρεὺς τὴν παροῦσαν βίβλον ΝΝΔ.

Unless this use of the name Arethas with the 1st Person ἐκτησάμην is a forgery, which is not likely, the identity of the writing is a conclusive proof that the aforesaid corrections in the celebrated MS. of Plato are by the hand of Arethas himself.

Of these seven corrections occurring within twenty lines six are made to correspond with the text of Eusebius as reproduced by the scribe Baanes from some older MS. now lost.

The one remaining, n. 3, is especially remarkable. It is evident that the words τὰ τοιάδε omitted by Baanes were supplied from the older MS. in the margin by Arethas, who then proceeded to supply the word πρῶτον in his MS. of Plato in the manner described above.

It is interesting to think of the learned Archbishop in his remote Diocese in Cappadocia bestowing so much loving care upon his noble transcripts of Plato and of the early Christian Apologists, writing in the margin of his favourite passages here an ὡραῖον (schön!) and there a σημείωσαι (beachte!) as Dr. Otto Stählin points out, and making each necessary correction in the text with his own hand.

E. H. Gifford.

THE ARITHMETICAL SOLUTION OF PLATO'S NUMBER.

As I have lately had occasion to investigate the subject of Plato's *Number* afresh, and my views have in some respects altered since this matter was discussed in the *Classical Review* by Dr. Monro and myself (Vol. vi. pp. 152–156, 240–244), I have thought that it might possibly be of interest to some readers of the *Review* if I were to set down the conclusions at which I have now arrived, together with a brief account of the evidence on which they rest.

The text of the passage is as follows: ἔστι δὲ θείῳ μὲν γεννητῷ περίοδος, ἣν ἀριθμὸς περιλαμβάνει τέλειος, ἀνθρωπείῳ δὲ ἐν ᾧ πρώτῳ αὐξήσεις δυνάμεναί τε καὶ δυναστευόμεναι, τρεῖς ἀποστάσεις, τέτταρας δὲ ὅρους λαβοῦσαι, ὁμοιούντων τε καὶ ἀνομοιούντων καὶ αὐξόντων καὶ φθινόντων, πάντα προσήγορα καὶ ῥητὰ πρὸς ἄλληλα ἀπέφηναν· ὧν ἐπίτριτος πυθμὴν πεμπάδι συζυγεὶς δύο ἁρμονίας παρέχεται τρὶς αὐξηθείς, τὴν μὲν ἴσην ἰσάκις, ἑκατὸν τοσαυτάκις, τὴν δὲ ἰσομήκη μὲν τῇ, προμήκη δέ, ἑκατὸν μὲν ἀριθμῶν *ἀπὸ διαμετρῶν ῥητῶν πεμπάδος, δεομένων ἑνὸς ἑκάστων, ἀρρήτων δὲ δυοῖν, ἑκατὸν δὲ κύβων τριάδος.*

Plato's words should be thus translated: 'Now for a divine creature there is a period which is comprehended by a number that is final: but the number of a human creature is the first number in which root and square increases, having received three distances and four limits, of elements that make both like and unlike and wax and wane, render all things conversable and rational with one another; of which the numbers 4, 3, married with 5, furnish two harmonies when thrice increased, the one equal an equal number of times, so many times 100, the other of equal length one way, but oblong—on the one side of 100 squares of rational diameters of five diminished by one each, or if of irrational diameters, by two; on the other of 100 cubes of 3.'

What the number of the 'divine creature,' or in other words, the World, is, Plato does not say: but the arithmetical meaning of the words from *ἀνθρωπείῳ δέ* down to *τριάδος* may be thus expressed in *our* notation:—

(1) $3^3+4^3+5^3=216$.

(2) $(3\times4\times5)^4=3600^2=4800\times2700$.

Let us take Plato's words in detail.

αὐξήσεις δυνάμεναί τε καὶ δυναστευόμεναι means 'root and square increases,' *i.e.* either additions of root to square (e.g. $x+x^2$, $y+y^2$, $z+z^2$), or multiplications of root by square (e.g. $x\times x^2$, $y\times y^2$, $z\times z^2$). A comparison of the *Theologumena Arithmetica*, p. 39 Ast, with Proclus *in Euclidem*, p. 8, Friedlein, Plato *Theaet.* 147E, 148B, and Euclid x. def. 11 will, in my judgment, establish the truth of this statement.

The words *τρεῖς ἀποστάσεις, τέτταρας δὲ ὅρους λαβοῦσαι* shew that *αὐξήσεις δυνάμεναί τε καὶ δυναστευόμεναι* refers to *multiplications* of root by square and not to *additions* of root to square: so that the whole phrase *αὐξήσεις δυνάμεναί τε καὶ δυναστευόμεναι, τρεῖς ἀποστάσεις, τέτταρας δὲ ὅρους λαβοῦσαι* is a fantastic expression for *κυβικαὶ αὐξήσεις* or 'cubings' and nothing more.

What is the evidence for this assertion? It is as follows.

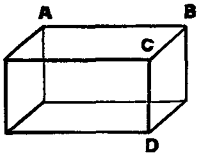
The words *ἀποστάσεις, διαστάσεις* and *διαστήματα* were used by the Greeks in the sense of 'dimensions'; and *αἱ τρεῖς διαστάσεις*, or *αἱ τρεῖς ἀποστάσεις* meant *μῆκος, πλάτος* and *βάθος*, the three dimensions of a solid body. The most precise explanation of this matter is to be found in Nicomachus *Introd. Ar.*, p. 116 Ast: *πρῶτον δὲ διάστημα γραμμὴ λέγεται· γραμμὴ γάρ ἐστι τὸ ἐφ' ἓν διαστατόν· δύο δὲ διαστήματα ἐπιφάνεια· ἐπιφάνεια γάρ ἐστι τὸ διχῇ διαστατόν· τρία δὲ διαστήματα στερεόν· στερεὸν γάρ ἐστι τὸ τριχῇ διαστατόν . . . . εἴ τι γὰρ στερεόν ἐστιν, τὰς τρεῖς διαστάσεις πάντως ἔχει, μῆκος, πλάτος καὶ βάθος· καὶ ἔμπαλιν εἴ τι ἔχει τὰς τρεῖς διαστάσεις, ἐκεῖνο πάντως στερεόν ἐστιν, ἄλλο δ' οὐδέν.* Similarly also on pp. 117, 123, 128 and *Theol. Ar.*, p. 38 the word *διαστάσεις* is used with the same connotation, as well as in Theo Smyrnaeus, pp. 24 f. Hiller, and elsewhere: and we find *τρεῖς ἀποστάσεις*, as in Plato, in *Theol. Ar.*, p. 23, where it is said that the number 4—which according to the Pythagoreans, *πρώτη ἔδειξε τὴν τοῦ στερεοῦ φύσιν—τὰς·πάσας ἀποστάσεις ἤτοι τὰς τρεῖς ἀπέστη, ὧν περαιτέρω οὐκέτι εἰσίν.* There are several other passages to the same effect in Greek writers on *ἀριθμητική*, such as Nicomachus, *Introd. Ar.*, pp. 143 f. Ast, where Nicomachus expressly refers to the Platonic Number (cf. Dr. Monro in *J. Ph.* viii. p. 276): but it is perhaps even more important to observe that the definition of body as that which has *τρεῖς διαστάσεις* is at least as old as Aristotle: see *Top.* Z 5. 142^b 24 *ὁ τοῦ σώματος ὁρισμός, τὸ ἔχον τρεῖς διαστάσεις* and *Phys.* iv. 1. 209^a 4 ff. *διαστήματα μὲν οὖν ἔχει* (*sc.* *ὁ τόπος*) *τρία, μῆκος καὶ πλάτος καὶ βάθος, οἷς ὁρίζεται σῶμα πᾶν*, together with Simplicius *in physic.* iv. 1. p. 531, 9 Diels *σῶμα—τὸ τὰς τρεῖς ἔχον διαστάσεις.* Finally, it is clear from the express testimony of Aristotle that the Pythagoreans before his day defined body in this way (*de caelo* A 1. 268^a, 7 ff. *μεγέθους δὲ τὸ μὲν ἐφ' ἓν γραμμή, τὸ δ' ἐπὶ δύο ἐπίπεδον, τὸ δ' ἐπὶ τρία σῶμα—καθάπερ γάρ φασι καὶ οἱ Πυθαγόρειοι, τὸ πᾶν καὶ τὰ πάντα τοῖς τρισὶν ὥρισται*), and I may add that the same definition is involved in a notable passage of Plato's *Laws* 894A, where the word employed is *μετάβασις* and not *ἀπόστασις*.

We may take it therefore that the three *ἀποστάσεις* of which Plato speaks are length, breadth, and thickness.

What are the four *ὅροι* or limits?

To this question a precise answer is furnished by Iamblichus *in Nicom.*, p. 93 Pistelli *στερεὸς δέ ἐστιν ἀριθμὸς ὁ τρίτον διάστημα παρὰ τὰ ἐν ἐπιπέδοις δύο προσειληφώς* (cf. *λαβοῦσαι* in Plato), *δηλονότι τετάρτου ὅρου προσγενομένου· ἐν γὰρ τέσσαρσιν ὅροις τὸ τριχῇ διαστατόν*, and by a writer in the *Theologumena Arithmetica*, p. 16 Ast, *τὸ ἐξ ὕλης καὶ εἴδους αἰσθητόν, ὅ ἐστιν ἀποτέλεσμα τριχῇ δια-*

στατόν, ἐν τέτταρσιν ὅροις ἐστίν. Thus in the accompanying figure of a solid number, which is taken from Theo Smyrnaeus, p. 42 Hiller, *AB*, *BC*, *CD* are the three ἀποστάσεις—*AB* the μῆκος, *BC* the πλάτος, and *CD* the βάθος; and the four points *A*, *B*, *C*, *D* are the four ὅροι (αἱ στιγμαὶ τῶν μεγεθῶν ὅροι Arist. *Met.* N 5. 1092^{b} 9): ἐν γὰρ τέσσαρσιν ὅροις τὸ τριχῇ διαστατόν, as Iamblichus observed.



I conclude that the whole expression αὐξήσεις—λαβοῦσαι 'root and square increases, having received three dimensions and four limits' means cubic increases. Inasmuch as solid numbers are to be produced, possessed of three dimensions and four limits, αὐξήσεις δυνάμεναί τε καὶ δυναστευόμεναι must refer to *multiplications* of root by square and not to *additions* of root to square: in other words to κυβικαὶ αὐξήσεις.

What then are the numbers to be cubed?

This information is contained or concealed in the genitives ὁμοιούντων τε καὶ ἀνομοιούντων καὶ αὐξόντων καὶ φθινόντων, which depend directly upon αὐξήσεις. The key to the interpretation of ὁμοιούντων κ.τ.λ. is supplied by Plato himself in ὧν ἐπίτριτος πυθμὴν κ.τ.λ. The antecedent of ὧν is ὁμοιούντων τε καὶ ἀνομοιούντων καὶ αὐξόντων καὶ φθινόντων, and ἐπίτριτος πυθμήν means the two numbers 4, 3: see Theo Smyrnaeus, pp. 80 f. Hiller and Proclus *in remp.* ii. p. 37 Kroll [ἔστιν οὖν οὗτος] ὁ ἐπίτριτος πυθμὴν γ′ καὶ δ′, together with Dr. Monro in *Cl. Rev.* vi. pp. 243 f. Now the most natural and obvious meaning of ὧν ἐπίτριτος πυθμήν 'of which 4, 3,' is 'of which numbers, the numbers 4, 3.' I infer, therefore, that ὁμοιούντων τε καὶ ἀνομοιούντων καὶ αὐξόντων καὶ φθινόντων, which is the antecedent to ὧν, denotes *some numbers, two of which are the numbers* 4 *and* 3.

We have thus obtained two of the numbers to be cubed, viz. 4 and 3. What is the missing number or numbers? It is clear from the partitive genitive ὧν that there is at least one other number besides 4 and 3. Plato does not tell us what the missing number is, but if we note that the numbers 4 and 3 are presently 'coupled with 5' (πεμπάδι συζυγείς), and remember that 3, 4 and 5 are the three sides of the Pythagorean triangle, which, according to Aristotle, Plutarch, Proclus and other ancient authorities, was employed by Plato in his Number, we cannot be wrong in holding that there is but one missing number, and that it is the number 5.

Why are the numbers 3, 4 and 5 said 'to make both like and unlike and wax and wane'? The full explanation of these words involves an investigation into the properties of the Pythagorean triangle, as they were conceived by some of the ancients, and lies beyond the scope of the present article. As to αὐξόντων καὶ φθινόντων, I will at present only say that these epithets are (in my belief) applied to the sides of the Pythagorean triangle regarded as cosmic agencies (κοσμικὸν τρίγωνον Proclus *in remp.* ii. p. 45 Kroll): but the epithets ὁμοιούντων τε καὶ ἀνομοιούντων have a special arithmetical meaning in the Platonic Number, and it is right to explain that meaning here. The words ὧν ἐπίτριτος πυθμὴν—τριάδος describe how the numbers 3, 4 and 5 produce, first a square (τὴν μὲν ἴσην ἰσάκις) viz. (as I believe) 3600^2, and secondly an oblong, viz. 4800 × 2700. Now, according to the Pythagoreans, square numbers are ὅμοιοι, and oblong numbers ἀνόμοιοι. The evidence is Iamblichus *in Nic. Intr. Ar.* p. 82 Pistelli οἱ δὲ παλαιοὶ ταὐτούς τε καὶ ὁμοίους αὐτοὺς (*i.e.* τοὺς τετραγώνους) ἐκάλουν διὰ τὴν περὶ τὰς πλευράς τε καὶ γωνίας ὁμοιότητα καὶ ἰσότητα, ἀνομοίους δὲ ἐκ τοῦ ἐναντίου καὶ θατέρους τοὺς ἑτερομήκεις, and also Nicomachus himself *Intr. Ar.* pp. 132 ff. Ast. That this doctrine is old, Iamblichus expressly tells us: cf. also Arist. *Met.* A 5. 986^{a} 22 ff. The numbers 3, 4 and 5 are therefore ὁμοιοῦντα because (among other reasons) they produce the square ἁρμονία, ἀνομοιοῦντα because (among other reasons) they produce the oblong ἁρμονία.

The words πάντα προσήγορα καὶ ῥητὰ πρὸς ἄλληλα ἀπέφηναν can be plentifully illustrated from Pythagorean writings. I do not now discuss them, because they do not affect the arithmetical solution of the Number in any way. For the same reason I shall not at present touch upon the question why the square and the oblong are ἁρμονίαι, merely remarking that the explanation which I gave of this matter in my *Number of Plato* was wrong.

Thus the 'number of a human creature is the first number in which cubings of 3, 4 and 5 make all things conversable and rational with one another.' Now the first number in which 3^3, 4^3 and 5^3 occur, is $3^3 + 4^3 + 5^3 = 216$; and we have a remarkable confirmation of our results, not only in

Aristotle (as I shall presently shew), but also in Aristides Quintilianus iii. p. 151 Meibom 89 Jahn, who, in explaining the properties of the Pythagorean triangle, says, in a passage where he alludes expressly to the Platonic Number, ἀλλ' εἰ καὶ τῶν πλευρῶν ἑκάστην κατὰ βάθος αὐξήσαιμεν (βάθος γὰρ ἡ σώματος φύσις), ποιήσαιμεν ἂν τὸν διακόσια δεκαέξ, ἰσάριθμον ὄντα σύνεγγυς τῷ τῶν ἑπταμήνων $(3^3+4^3+5^3=216)$. By 'the number of a human creature' Plato means the περίοδος or period during which a human creature is in the womb, as I think I have shewn in *The Number of Plato* pp. 42 f.

So much for the first part of the Platonic Number. I proceed now to the second, contained in the words from ὧν ἐπίτριτος πυθμήν down to ἑκατὸν δὲ κύβων τριάδος.

'Of which numbers' (viz. as we have seen 3, 4 and 5) 'the numbers 3, 4 coupled with 5,' means that 3, 4 and 5 are to be married *i.e. multiplied* together. Dr. Monro has said that 'there is no parallel to lead us to take συζυγείς to mean multiplied' (*Cl. Rev.* vi. p. 154). A precise parallel may now be found in Proclus *in remp.* ii. p. 54. 2 ff. Kroll ἡ δ' οὖν ἑκατοντὰς τῷ ἐλλείποντι ἀριθμῷ πρὸς αὐτὴν κατὰ τὸν ἀπὸ τῆς πεμπάδος ἀριθμὸν συζυγεῖσα ποιεῖ τὴν ἀπὸ γενέσεως ἐπὶ γένεσιν περίοδον (cf. συζεύξεις *ibid.* ii. p. 26); and the usage is in harmony with the Pythagorean habit of describing 6 as 'marriage,' because it is produced by the 'marriage' *i.e.* the 'multiplication' of the first female with the first male number (2 × 3 = 6 : see Iambl. *in Nic. Ar.* p. 34. 20 Pistelli, Arist. Quint. i. p. 151 Meibom and other passages), as well as with Euclid's οἱ γενόμενοι ἐξ to denote numbers produced by the multiplication of other numbers (*e.g.* vii. 16 ff.).

To proceed.

3 × 4 × 5 *i.e.* 60, produces, says Plato, two harmonies, when 'thrice increased.' 'Thrice increased' means here 'three times multiplied by itself'—to this point I shall return—and 60 × 60 × 60 × 60 = 12,960,000.

This number furnishes, we are told, 'two harmonies, the one equal an equal number of times, so many times 100.'

Now 12,960,000 furnishes 3600^2, and 3600^2 is 'equal an equal number of times' viz. *thirty-six* times 100,' so that τοσαυτάκις refers to 36 times. With this use of τοσαυτάκις I formerly compared *Phaedr.* 271 D (τόσα καὶ τόσα) and *Laws* 721 D (τόσῳ καὶ τόσῳ): better parallels, I think, are to be found in *Alc.* i. 108 E βέλτιον τόδε τοῦδε καὶ νῦν καὶ τοσοῦτον and Arist. *Pol.* Γ 12. 1283^a 8 τοσόνδε γὰρ μέγεθος εἰ κρεῖττον τοσοῦδε, τοσόνδε δῆλον ὡς ἴσον. None of these parallels is perfect, but the meaning which I assign to τοσαυτάκις is as natural in Greek as in English, and what Dr. Monro calls 'the ordinary interpretation of ἑκατὸν τοσαυτάκις—a hundred taken *that* number of times viz. 100 times' is, as I hope to shew hereafter, not only open to question in itself, but involves insuperable difficulties in the special context where the words occur.

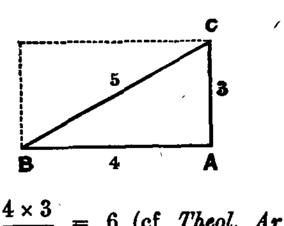
One of the two harmonies furnished by 12,960,000 is therefore, as I hold, 3600^2: what is the other? It is an oblong, one of whose sides is 100 cubes of three (ἑκατὸν δὲ κύβων τριάδος) *i.e.* $(100 \times 3^3) = 2700$, and the other '100 squares of (for ἀριθμῶν ἀπό cf. Euclid vii. 20 and Pl. *Men.* 85 B) the rational diameter of 5, diminished by 1 each, or if of irrational diameters, by 2 each.' What is the rational diameter of 5? It is the nearest rational number to the real diameter of a square whose side is five, *i.e.* to $\sqrt{50}$ by Euclid i. 47 (see Theo Smyrnaeus pp. 43 ff., Gow *Gk. Math.* p. 96 and Cantor *Gesch. d. Math.* p. 191). The nearest rational number to $\sqrt{50}$ is $7 = \sqrt{49}$: so that 'rational diameters of 5' means 'sevens.' 'A hundred squares of sevens' = 4900, and when we diminish each of the hundred squares by 1 we obtain $4900 - (1 \times 100) = 4800$, which is therefore the other side of the oblong. Now take ἀρρήτων δὲ δυοῖν. '100 squares of irrational diameters of five' $= (\sqrt{50})^2 \times 100 = 5000$. Diminish each of the hundred squares by 2, and we obtain the same result as before viz. 4800 : for $5000 - (2 \times 100) = 4800$.

The two sides of the oblong are therefore 4800 and 2700; and this harmony like the first is furnished by 60 'thrice increased,' for 60 × 60 × 60 × 60 = 4800 × 2700.

So much for Plato's words: let us now briefly discuss that part of Aristotle's criticism which has a bearing on the arithmetical solution of the Number.

The passage which concerns us is in these words: ἐν δὲ τῇ πολιτείᾳ λέγεται μὲν περὶ τῶν μεταβολῶν ὑπὸ τοῦ Σωκράτους, οὐ μέντοι λέγεται καλῶς· τῆς τε γὰρ ἀρίστης πολιτείας καὶ πρώτης οὔσης οὐ λέγει τὴν μεταβολὴν ἰδίως. φησὶ γὰρ αἴτιον εἶναι τὸ μὴ μένειν μηθὲν ἀλλ' ἔν τινι περιόδῳ μεταβάλλειν, ἀρχὴν δ' εἶναι τούτων ὧν ἐπίτριτος πυθμὴν πεμπάδι συζυγεὶς δύο ἁρμονίας παρέχεται, λέγων ὅταν ὁ τοῦ διαγράμματος ἀριθμὸς τούτου γένηται στερεός, ὡς τῆς φύσεώς ποτε φυούσης φαύλους καὶ κρείττους τῆς παιδείας, τοῦτο μὲν οὖν αὐτὸ λέγων ἴσως οὐ κακῶς· ἐνδέχεται *γὰρ εἶναί τινας οὓς παιδευθῆναι καὶ γενέσθαι σπουδαίους ἄνδρας ἀδύνατον. ἀλλ' αὕτη τί ἂν ἴδιος εἴη μεταβολὴ τῆς ὑπ' ἐκείνου λεγομένης ἀρίστης πολιτείας μᾶλλον ἢ τῶν ἄλλων πασῶν καὶ τῶν γιγνομένων πάντων*; (*Pol.* E 12. 1316ᵃ). Socrates, says Aristotle, does not assign any specific or peculiar principle of change to his best polity: 'for he says that the *cause* of change is the fact that nothing abides, but all things change in a certain cycle, and that the *beginning* of change comes from' (lit. 'is of') 'those' (elements or numbers) 'whereof 4, 3, coupled with 5, furnish two harmonies, meaning, *when the number of this diagram is made solid,* the theory being that nature sometimes produces inferior children and children who defy education. In this particular point, indeed, Socrates is no doubt right: for there may well be persons who cannot be educated and made into good men. But why should *this* be a change peculiar to the constitution which he calls the best more than to every other constitution and everything that comes into being?'

A careful examination of this passage will shew, I think, that Aristotle understood the words of Plato as we have done. 'The *beginning* of change,' he says, 'comes from those elements' (viz. the *ὁμοιούντων τε καὶ ἀνομοιούντων καὶ αὐξόντων καὶ φθινόντων* *i.e.* 3, 4, 5) 'of which the numbers 4, 3 coupled with 5, furnish two harmonies—meaning (that change begins) when the number of this diagram is made solid.' In the words of Schneider (vol. iii. p. xxix.) '*τούτων* ad *ὧν* pertinet et sensus verborum talis est: Principium mutationis positum esse in numeris—quorum sesquitertia radix etc. Deinde verba *λέγων ὅταν* declarant tempus, quo Plato initium mutationis posuerit.' Now what is 'this diagram'? It is agreed by all that the diagram is the Pythagorean triangle. What is its 'number'? 'The number of a diagram' means its area (see below), and the area of the Pythagorean triangle is $\frac{4 \times 3}{2} = 6$ (cf. *Theol. Ar.* p. 39 Ast). Make 6 solid and we obtain $6^3 = 216$, which is the number which we obtained by our investigation of the words *ἀνθρωπείῳ δὲ—ἀπέφηναν* in the passage of Plato. The only difference between Aristotle's calculation and that of Plato is that Aristotle, who was interested only in the result, and not in the processes, arrives at the number by cubing the area of the triangle, and not, like Plato, by adding together the cubes of its sides ($3^3 + 4^3 + 5^3 = 6^3 = 216$). Such a difference is to my mind exactly what we should expect: for Aristotle is in the habit of varying his predecessors' methods of expressing their results, and, in point of fact, 6^3 was itself called by the Pythagoreans the *ψυχογονικὸς κύβος* (*Theol. Ar.* p. 40).



In what sense the number 216 in Plato expresses the 'beginning of change' is a question which belongs to the interpretation of Plato's symbolism. I must content myself with saying here that the number in question is the beginning of change because it expresses the period of gestation in the human kind, and change, according to Plato, begins with the child in the womb (*ὅταν— ὑμῖν οἱ φύλακες συνοικίζωσι νύμφας νυμφίοις παρὰ καιρόν, οὐκ εὐτυχεῖς οὐδ' εὐφυεῖς παῖδες ἔσονται* 546 D). That Aristotle interpreted Plato in the same way seems to me clear from the explanatory clause *ὡς τῆς φύσεώς ποτε φυούσης φαύλους καὶ κρείττους τῆς παιδείας*, which signifies that 216 is the *ἀρχὴ μεταβολῆς* because it is connected with the production of offspring.

The above interpretation of Aristotle, which is based on that of Schneider, entirely harmonises with the results of our enquiry. What is the rival interpretation? I will give it in the words of Dr. Monro (*J. of Ph.* viii. p. 280):—

> 'Aristotle paraphrases *τρὶς αὐξηθείς* by the words *ὅταν ὁ τοῦ διαγράμματος ἀριθμὸς τούτου γένηται στερεός.* By the 'number of this figure' he cannot well mean any single number; probably he uses *ἀριθμός* in the sense of 'linear measurement,' as opposed to surfaces or solids (cf. *Rep.* p. 587 D, where *κατὰ τὸν τοῦ μήκους ἀριθμόν* is opposed to *κατὰ δύναμιν καὶ τρίτην αὔξην*). Now the most natural way of raising the Pythagorean triangle to the third dimension is by cubing each of the sides; and this process leads at once to the remarkable fact that $3^3 + 4^3 + 5^3 = 216 = 6^3$. It is difficult to resist the impression that this is what was in the mind of Plato.'

The theory which underlies this interpretation is, as the reader will observe, that *ὧν ἐπίτριτος πυθμὴν πεμπάδι συζυγεὶς—τρὶς αὐξηθείς* in Plato means $3^3 + 4^3 + 5^3 = 216$. I shall deal with the phrase *κατὰ τὸν τοῦ μήκους ἀριθμόν* (*Rep.* 587 D), on another occasion, and shew, as I think, that *ἀριθμός* has nothing to do with linear measurement in this passage of Aristotle, where there is nothing to correspond to the important words τοῦ μήκους. Meantime I will mention two obvious objections to Dr. Monro's view, each of which is in my opinion fatal to the interpretation which he advocates.

In the first place, Dr. Monro makes 'the number of this triangle' equivalent to 'the *sum* of the *numbers* of its sides.' Aristotle says simply 'number,' not 'numbers,' and gives no hint whatever that he desires us to have recourse to a process of addition. I submit that the natural and obvious meaning of ἀριθμός is 'number' and not 'numbers'; and that the ἀριθμός of a figure is proved to be the number which denotes its area by Theo Smyrnaeus p. 39 Hiller, where the number 9 is actually represented by the diagram
a a a
a a a, in which the number of letters
a a a
represents the area. Cf. also Arist. *Met.* N. 5. 1092^b 10 ff., from which it appears that this way of representing the area of figures was earlier than Aristotle, and Theophr. *Frag.* 12. 11 Wimmer.

In the second place, how does Dr. Monro cube what Aristotle calls 'the number of this diagram'? By making $3+4+5$ into $3^3+4^3+5^3$.

But, in point of fact, the cube of $3+4+5$ is 12^3, and not $3^3+4^3+5^3$. Are we to suppose that Plato and Aristotle were ignorant of this fact?

For these reasons I think that the ordinary interpretation of this passage in Aristotle is demonstrably wrong, and Schneider's, in every essential point relating to the language, unquestionably right.

On a review of the whole matter, it will, I think, be generally agreed that the corner-stone of my solution of the number is the meaning which it assigns to τρὶς αὐξηθείς. I will therefore add a few sentences by way of epilogue on this subject.

The prevalent interpretation of τρὶς αὐξηθείς seems to be 'raised to the third power,' and of τρίτη αὔξη 'the third power': see for example Dr. Monro in *Cl. Rev.* vi. p. 242. This view, in my opinion, rests on a mistranslation: for αὔξη should be translated 'increase' and not 'power' or 'dimension' or anything of the sort. The mathematical terms 'third power,' 'fourth power,' etc., were unknown to Plato. 'Power' or δύναμις alone is sometimes used with the meaning which we give to 'second power' (*Rep.* ix. 587D), but the word is so elastic that it even means 'root' in *Theaet.* 148A. See Allman *Gk. Geom.* p. 208*n.* Consequently the only safe translation of δευτέρα αὔξη and τρίτη αὔξη in Plato is 'second increase' and 'third increase.' Now 'increase' implies something to be increased, and the result will of course be different, wherever the objects which have to be increased are different. Thus in the increasing series

1, 60, 3600, 216,000, etc.,

the number 216,000 is the τρίτη αὔξη of unity; and in the increasing series with which Plato is dealing viz.

60, 3600, 216,000, 12,960,000, etc.,

the number 12,960,000 which *we* call 60^4 is the τρίτη αὔξη of 60, or in other words 60 τρὶς αὐξηθείς.

That this is 'logical,' has been admitted; but 'it is not,' says Dr. Monro, 'in accordance with the *usus loquendi*' (*Cl. Rev. l.c.* p. 154). '*We may feel sure, I think, that the "third increase" would naturally mean the third term in the increasing series rather than the fourth*' (*ib*). (The italics are mine.)

Personally, I feel quite sure that the 'third increase' did in point of fact mean to a Greek as it does to an Englishman, the *fourth* term in an increasing series, and *not* the third; and why? Because in such a series as Euclid speaks of in ix. 8 (cited by Dr. Monro *l.c.*) ἐὰν ἀπὸ μονάδος ὁποσοιοῦν ἀριθμοὶ ἑξῆς ἀνάλογον ὦσιν, ὁ μὲν τρίτος ἀπὸ τῆς μονάδος τετράγωνος ἔσται κ.τ.λ., *e.g.* 1, 60, 3600, etc., if we are to suppose, with Dr. Monro, that the third increase is the third term, we must hold that 3600 or 60^2, which is the third term, is also the τρίτη αὔξη! Against this supposition not only logic, but the *usus loquendi* itself cries out. The fact is, of course, that 60^2, which in this series is the *third term*, is the *second increase* of unity: and it is equally true that in the series 60, 3600, 216,000, 12,960,000, the number 12,960,000, or, as we call it, 60^4, is at once the *fourth term*, and the *third increase* of 60, in other words 60 τρὶς αὐξηθείς.

In conclusion, it is of course true that the idiomatic phrase τρίτη αὔξη is used once or twice in Plato with reference to what *later* mathematicians call the third dimension. The usage is, however, excessively rare: I have found no instance in Aristotle or later Greek mathematicians, and only two in Plato (*Rep.* 528 B and 587 D: cf. also [*Epin.*] 990 D τοὺς τρὶς ηὐξημένους καὶ τῇ στερεᾷ φύσει ὁμοίους sc. ἀριθμούς). But Plato employs also δευτέρα αὔξη in speaking of plane surfaces, and if we compare 528 B ὀρθῶς δὲ ἔχει ἑξῆς μετὰ δευτέραν αὔξην τρίτην λαμβάνειν with 526 C δεύτερον δὲ τὸ ἐχόμενον τούτου (ἐχόμενον is plane geometry and *τούτου* the study of Number) *σκεψώμεθα ἆρά τι προσήκει ἡμῖν* it is clear that he also regarded linear number or the line as the *πρώτη αὔξη*. What then is that something which is 'increased,' first to a line, second to a plane, and thirdly to a solid? The only possible reply is 'the unit or point': for number is a *σύστημα μονάδων* (Theo p. 18 cf. *Rep.* vii. 525A), and the Pythagoreans, as every one knows, built up the line out of points, the plane out of lines, and the solid out of planes: see Burnet, *Early Gk. Phil.* pp. 312 ff. Dr. Monro says 'there is no evidence that *αὔξη* in the idiomatic phrase *τρίτη αὔξη* originally referred to the increase of the unit or point' (*ib.* 154, 242). To me the evidence seems to be writ plain for all to read in the fundamental principles of Pythagorean physics. If it is not the point which is 'increased,' what is it?

J. ADAM.

ON ARISTOTLE, *NIC. ETH.* VII. XIV. 2 AND XII. 2.

N.E. VII. xiv. 1—2, 1154^{a} 8—21:

1154^{a} *Περὶ δὲ δὴ τῶν σωματικῶν ἡδονῶν ἐπισκεπτέον τοῖς λέγουσιν ὅτι ἔνιαί γε ἡδοναὶ αἱρεταὶ σφόδρα, οἷον αἱ καλαί, ἀλλ' οὐχ αἱ σωματικαὶ καὶ περὶ ἃς ὁ ἀκόλαστος. διὰ τί οὖν αἱ ἐναντίαι λῦπαι μοχθηραί; κακῷ γὰρ ἀγαθὸν ἐναντίον. ἢ οὕτως ἀγαθαὶ αἱ ἀναγκαῖαι, ὅτι καὶ τὸ μὴ κακὸν ἀγαθόν ἐστιν; ἢ μέχρι του ἀγαθαί; τῶν μὲν γὰρ ἕξεων καὶ κινήσεων ὅσων μὴ ἔστι τοῦ βελτίονος ὑπερβολή, οὐδὲ τῆς ἡδονῆς· ὅσων δ' ἔστι, καὶ τῆς ἡδονῆς. ἔστι δὲ τῶν σωματικῶν ἀγαθῶν ὑπερβολή, καὶ ὁ φαῦλος τῷ διώκειν τὴν ὑπερβολήν ἐστιν, ἀλλ' οὐ τὰς ἀναγκαίας· πάντες γὰρ χαίρουσί πως καὶ ὄψοις καὶ οἴνοις καὶ ἀφροδισίοις, ἀλλ' οὐχ ὡς δεῖ. ἐναντίως δ' ἐπὶ τῆς λύπης· οὐ γὰρ τὴν ὑπερβολὴν φεύγει, ἀλλ' ὅλως· οὐ γάρ ἐστι τῇ ὑπερβολῇ λύπη ἐναντία ἀλλ' ἢ τῷ διώκοντι τὴν ὑπερβολήν.*

The conclusion of this passage—*ἐναντίως δ' ἐπὶ τῆς λύπης, κ.τ.λ.*, is perhaps one of the hardest places in the Aristotelian writings.

The meaning of the last clause, *οὐ γὰρ ἐστι κ.τ.λ.* is clear enough: *ὑπερβολή* is the excess of bodily pleasure, and the man who pursues the excess of bodily pleasure is said to consider pain as opposed to excessive bodily pleasure only, because he knows no other bodily pleasure but the excessive pleasure. The difficulty lies in the relation of this clause to the preceding one—*οὐ γὰρ τὴν ὑπερβολὴν κ.τ.λ.*, whether we assume that the nominative to *φεύγει* is indefinite, or, as some think, is *ὁ φαῦλος*. If we assume the former we get, 'for it is not the excess (of bodily pain) which a man avoids, but (bodily) pain in general; for it is not to the excess only (of bodily pleasure) that (bodily) pain is opposed except for the man who pursues the excess.'

If we assume the latter we get, 'for it is not the excess (of bodily pain) which the *φαῦλος* avoids, but (bodily) pain in general; for it is not to the excess only (of bodily pleasure) that (bodily) pain is opposed, except for the man who pursues the excess: (*i.e.* except for the *φαῦλος*).'

In either case we feel there is some strange *non sequitur*.

We may endeavour to determine more precisely what the logical difficulty consists in.

The clauses, *οὐ γὰρ τὴν ὑπερβολὴν φεύγει ἀλλ' ὅλως, οὐ γάρ ἐστι τῇ ὑπερβολῇ λύπη ἐναντία ἀλλ' ἢ τῷ διώκοντι τὴν ὑπερβολήν*, are of the form

'A is true (generally), for B is not true save in an exceptional case.'

This implies that if B were true, A would not be true.

(i) Suppose the subject of *φεύγει* is indefinite.

Then A = 'it is not the excess of bodily pain which a man avoids but pain in general.'

And to deny the truth of A would naturally mean to assert:—

'it *is* the excess of bodily pain which a man avoids, not bodily pain in general.'

The truth of B means:—

'it is to excessive bodily pleasure only that bodily pain is opposed.'