XI. *The* ORBIT *and* MOTION *of the GEORGIUM SIDUS determined directly from Observations, after a very easy and simple Method.* *By* JOHN ROBISON, M. A. F. R. S. EDIN. *and Professor of Natural Philosophy in the University of* EDINBURGH.

[*Read by the Author, March* 6. 1786.]

THE accuracy of modern obſervations has diſcovered irregularities in the motions of Jupiter and Saturn, which our knowledge of the laws of planetary gravitation has not as yet enabled us to explain. I have, therefore, long thought it probable that there may be planets without the orbit of Saturn, of ſufficient magnitude to occaſion theſe irregularities. This conjecture is confirmed by the diſcovery of a new planet.

ON the 13th of March 1781, Mr HERSCHEL, an aſtronomer of great ardour and ingenuity, obſerved a Star, near the foot of Caſtor, whoſe ſteady light attracted his attention. He immediately applied to his teleſcope a higher magnifying power, and diſcovered an augmentation of its apparent diameter. Two days after, he obſerved that it had changed its place; and, taking it for a comet, he wrote an account of his obſervation to Dr MASKELYNE, Aſtronomer-royal, who got ſight of this Star on the 17th of March. An account of this diſcovery was ſoon given to the other aſtronomers of Europe, who have continued to obſerve it with unceaſing attention. I did not obtain a ſight of it till Auguſt 1782.

ALMOST at its firſt appearance, the Engliſh aſtronomers ſuppoſed it to be a Planet. They were led to this opinion by various circumſtances which rendered it very probable; ſuch as,

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its vicinity to the Ecliptic, the direction of its motion, and its being nearly ſtationary at the time of its diſcovery, in ſuch an aſpect with reſpect to the Sun, as correſponds to the ſtationary appearance of the Planets. The French aſtronomers imagined it a Comet, although it had not that train of faint light which uſually diſtinguiſhes thoſe bodies; and, in the courſe of the year 1781, endeavoured to determine the elements of its motion on this ſuppoſition, but could not find out ſuch as would correſpond with its ſucceſſive appearances. They at laſt found themſelves obliged to ſuppoſe, that it moved round the Sun in an orbit nearly circular. Mr LEXEL, Profeſſor of aſtronomy at St Peterſburg, was the firſt who attempted a computation of its motion on this principle; and ſhowed that a circular orbit, the radius of which is about nineteen times the diſtance of the earth from the ſun, would very nearly agree with all the obſervations made during the year 1781. The firſt diſtinct information which I got of it was in June 1782, from Mr MINTO, a gentleman of this place, who communicated to me a ſeries of excellent obſervations made by Profeſſor SLOP at Piſa. This ſeries contained the means of determining with accuracy the ſtationary points of the Planet in October 1781 and March 1782, and its oppoſition in December 1781. From theſe, I was enabled to aſcertain with great eaſe, the radius of its circular orbit. For, at its ſtationary appearance, we have the ſquare of the coſine of its elongation from the Sun $= \frac{r^2-1}{r^3-1}$, r being the radius, and the earth's mean diſtance being 1. The oppoſition in December 1781, gives us one place of the Planet as viewed from the Sun, independent of all hypotheſes. With theſe data, it was eaſy for me to determine the apparent place of the Star for any time, and compare it with obſervation; and the reſult of this compariſon was ſuch as to ſhow, that the opinion was very nearly true, the greateſt errors not amounting to to more than what might reaſonably be attributed to the inaccuracy of obſervation.

ASTRONOMERS were every where engaged in the ſame reſearch; and it occurred to ſome of them, that the Star might poſſibly have been obſerved before, by thoſe who were employed in making catalogues of the Zodiacal Stars. Mr BODE of Berlin had juſt publiſhed a valuable work, in which all the catalogues of the Stars were included. He had recourſe to his papers, where he had marked all the difference of theſe catalogues, in order to diſcover whether any Star, obſerved by one aſtronomer, and omitted by another, might not be this Star of Mr HERSCHEL, paying attention to thoſe differences only which he could find in the parts of the Zodiac, through which the nearly determined orbit of this newly diſcovered Planet might be ſuppoſed to paſs. Among others, he found the Star, No. 964. of MAYER's Catalogue, not obſerved by others, and but once obſerved by MAYER, who could not therefore diſcover any motion in it. Mr BODE immediately examined the heavens, and could not find this Star. He farther found, that the elements of the new Planet aſſigned to it that very apparent place, in the month of September 1756, one of the years in which MAYER was occupied with theſe obſervations. On examining the regiſter of MAYER's obſervations, it was found, that he had obſerved the Star, No. 964. on the 25th of September 1756. This was notified to Mr BODE, in September 1781. He immediately made this information public; and it has ſince been currently ſuppoſed, that the Star obſerved by MAYER was the Planet of HERSCHEL.

IT was found, even before the end of 1782, that the circular hypotheſis was not exact, and that the angular motion of the Planet round the Sun was increaſing. This ſhowed, that the Planet was not moving in a circle, but in an excentric orbit, and was approaching to the Sun. Aſtronomers, therefore, began to inveſtigate the inequality of this angular heliocentric motion, in order to diſcover the form and poſition of the ellipſe defcribed by the Planet. This was a very difficult tafk; for the very fmall inequality of the motion fhowed that the orbit was nearly circular; and the arch already defcribed was not much more than the fiftieth part of the whole circumference. The folution of the problem requires us to determine, from the variation of curvature difcoverable in this fmall arch, to what part of the circumference it belongs. This requires the utmoft accuracy in the obfervations, and great fagacity in making deductions from them *. But, taking it for granted that the 964th Star of MAYER's Catalogue was the new Planet, the problem becomes fufceptible of a very eafy folution; for that Star is fituated more than a quarter of a revolution from the place of the Planet in 1782, and fo fortunately, that almoft the whole effect of the excentricity and inequality of the motion is accumulated. Aftronomers, therefore, availed themfelves of this obfervation of Mr BODE, and quickly found, by repeated trials, elements of the motions, which correfponded perfectly with MAYER's obfervation, and all thofe made fince Mr HERSCHEL firft got fight of the Planet. But they do not all feem difpofed to confefs their obligation to Mr BODE. Some of them affect to have deduced their elements directly from obfervations, by the formulæ expreffive of the elliptical motion of the Planets, and to be agreeably furprifed with afterwards obferving the coincidence of their elements with this obfervation of MAYER. They have not given a detail of their methods of inveftigation.

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* THE firft perfon who obtained any direct information of the elliptical orbit of the Planet was the celebrated Abbé BOSCOVICH, who, in October or November 1781, deduced elements of its orbit from the obfervations of Mr MECHAIN. His method is exceedingly ingenious, and remarkable for that fimplicity and geometrical elegance which characterife all his performances. It did not come to my knowledge till the beginning of this prefent year 1787, when I found it in the Collection which he publifhed at Baffano, in 1785, in five volumes. He makes ufe of the fame phyfical principles which I employed in January 1783, to determine the orbit by the two oppofitions which had then been obferved, combined with another obfervation, made at the diftance of a fydereal year from one of the oppofitions. This method I communicated to Dr MASKELYNE in 1783.

OF all the theories of this Planet which I have ſeen, that of M. DE LA PLACE, communicated to the Royal Academy of Sciences at Paris, appears the moſt accurate, and very nearly correſponds to the obſervations which have been made ſince the time of its publication. This theory was announced to the public in the *Connoiſſance des Mouvemens Celeſtes*, as deduced directly from the recent obſervations, by a method peculiar to M. DE LA PLACE. This I hoped to find in an excellent diſſertation on the elliptical motion of the Planets, publiſhed by him in 1784. But, although I found this work full of new and valuable information, as might be expected from this eminent mathematician, I was diſappointed in my hopes of learning the proceſs by which he had deduced his theory of the new Planet. He has, indeed, inſerted in this work the elements of its orbit, and the four obſervations which he had employed for determining them, by a new method of conſidering the planetary motions, with which he was then occupied, but which he does not explain. When I compared M. DE LA PLACE's theory with thoſe obſervations, I found ſuch differences as would have allowed him to make choice of elements conſiderably different. It appears, therefore, that, before applying his method, he has corrected the obſervations on ſome juſtifiable principle, which I regret exceedingly that he has not communicated, ſince he has been ſo ſucceſsful in the uſe of it. It would, doubtleſs, have been much more deſerving of the notice of mathematicians than the empirical one which I have adopted in the ſubſequent part of this paper.

IN ſpring 1784, I framed a ſet of elements which correſponded with the obſervations made at that time with abundant accuracy. Mr MINTO, whom I have already mentioned, alſo communicated to me elements, little differing from mine, and equally accurate. Both theſe were deduced from a ſuppoſition that the Star obſerved by MAYER was the new Planet. We had, by this time, great advantages over our predeceſſors; for a much

much larger portion of the arch had been obferved; and, which was of immenfe confequence, three oppofitions had been obferved, which gave us three pofitions of the Planet, independent of all hypothefes. The arches defcribed between thefe oppofitions being thus determined, free from all uncertainty, the acceleration of the Planet's motion became known; and a method now offered of determining, by interpolation, its heliocentric place, at any intermediate moment, with very great accuracy. And now, by chufing fuch obfervations of the Star as fhould give a great difference between the heliocentric and geocentric place, the radius of the earth's orbit became a bafe, by which we could meafure, with confiderable accuracy, its diftance from the Sun. Thus, having both its pofition and diftance from the Sun, we could affign its abfolute place in the heavens, and confequently the form of its path.

IN the beginning of 1785, another oppofition was obferved, and thus a method obtained of deducing the elements directly. But this required a procefs fo extremely complicated, in order to obtain tolerable accuracy in the refult, that I had not the courage to attempt it. I waited patiently till a fifth oppofition fhould be obferved with four intercepted arches. This, I faw, would afford a method extremely fimple and eafy, and, at the fame time, fufceptible of confiderable accuracy. It is this method which I have now the honour to lay before this Society; and I hope that the Gentlemen who hear me will not think it altogether unworthy of their attention: For it is furely defirable not to reft our knowledge of the motions of this Planet on mere conjecture, whatever probability there may be of its truth from the coincidence of obfervations. I muft, at the fame time, acknowledge beforehand, that the refult of my inveftigation has not enabled me to determine the elements of its motion with perfect certainty. It has, on the contrary, convinced me, that, if we do not admit that the new Planet is the fame with

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964 of MAYER, near half a century must elapse before the elements of its motion can be determined with a precision equal to that which is attained in the case of the other Planets. But the method assigns certain limits, and these not very wide, within which all the circumstances of its motion must be comprehended. This alone must be regarded as a considerable attainment.

THE heliocentric place of the Planet in opposition to the Sun, on the 21st of December 1781, was determined by me, from observations made on the 19th and 28th of that month, by Dr MASKELYNE, combined with observations made by Professor SLOP at Pisa, on the 22d, 23d, 27th, and 28th. The heliocentric place at the opposition 1782, was determined from observations made by Dr MASKELYNE on the 14th and 28th of December, combined with those of Professor SLOP on the 22d, 25th, and 26th of that month. The place of opposition in 1783 was determined from my own observations on the 26th, 27th, 28th of December, and the 5th of January following. The place at opposition January 3d 1785, was determined from my own observations on the 28th and 29th of December, and the 1st and 6th of January. The place at opposition 1786, was determined from my own observations on the 29th, 30th, and 31st of December, and the 1st, 3d, and 8th of January. The method which I took for combining these observations, in order to get rid of the inaccuracy to which each of them was liable, was as follows: The arch described between any two successive oppositions gave me a pretty near approximation to the distance of the Planet from the Sun, by means of the Keplerian law, that the squares of the angular motions are inversely as the cubes of the distances. The heliocentric angular motion, at any opposition, must be very nearly a medium between the angular motions with which the arches, intercepted between it and the preceding and following opposition, would be uniformly described. Thus I obtained, with sufficient accuracy,

curacy, the heliocentric angular motion at the three intermediate oppofitions. The angular velocities at the two extreme oppofitions were determined with equal accuracy, by fuppofing, that the changes of angular velocity followed a regular law. Thus I was enabled to determine the geocentric motion for a few days before and after appofition, and confequently to affign, from each obfervation, the precife time and place where the Planet would be in oppofition to the Sun. Thefe determinations differed from each other in no cafe 10″. It is demonftrable, that the affumptions made for this combination of obfervations could not produce an error of 2″. I therefore, with confidence, took the means of thefe determinations for the places of the Planet, in its apparent oppofitions to the Sun.

THE times and apparent longitudes and latitudes of the Planet are expreffed in the following table:

	M. T. Ed.	Long.	Lat. N.
	h. ′ ″	s. ° ′ ″	′ ″
1781. Dec. 21.	17. 44. 33	3. 00. 52. 11	15. 07
1782. Dec. 26.	08. 56. 56	3. 05. 20. 29	18. 56
1783. Dec. 31.	00. 46. 24	3. 09. 50. 52	22. 10
1785. Jan. 3.	17. 28. 56	3. 14. 23. 02	25. 40
1786. Jan. 8.	10. 39. 31	3. 18. 57. 05	28. 52

MY manner of obferving obliged me to compare the Planet with two fixed Stars which did not differ from it, or from each other, more than one degree in declination. This obliged me to employ fome Stars which are to be found in MAYER's Catalogue alone. I have, therefore, always made ufe of this Catalogue. If, therefore, the following theory be confronted with an obfervation, where the geocentric place of the Planet is deduced from a comparifon of it with a Star *in its neighbourhood*, and if the place of this Star be deduced from BRADLEY's, or DE LA CAILLE's Catalogues, the longitude will be found about 6″ too fmall, or as much too great.

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THE manner of obfervation, and the inftrument which I make ufe of, appear to me to have feveral advantages which are not unworthy of the attention of Aftronomers. An account of them will therefore be communicated on fome future occafion.

FROM thefe places, it is eafy to determine the inclination of the Planet's orbit to the plane of the Ecliptic, and the place of its Node, which are as follow:

	s. ° ′ ″
Long. Node, Jan. 1. 1786.	2. 12. 48. 45
Inclin. Orbit,	46. 26

I WAS now enabled to reduce thefe Ecliptic places to the orbit itfelf, and thus to determine the arches of this orbit defcribed during the intervals between the oppofitions.

I THEN took the oppofition which was obferved on the 31ft of December 1783 for an epoch, to which all the obfervations fhould be reduced. The interval of time between this and the preceding oppofition was 369 *d.* 15 *h.* 49′. 28″. I counted back another equal interval, which brought me within a few minutes of the time of oppofition 1781, and I computed (by means of the heliocentric motion, already determined for that oppofition with fufficient accuracy) the place of the Planet for the beginning of the above mentioned interval. In like manner, I computed its place for two equal intervals of 369 *d.* 15 *h.* 49′ 28″, reckoned forward from the epoch. Thus I obtained four angles in the orbit, defcribed in equal intervals of time. The differences of thefe angles fhowed the inequality of the Planet's angular motion. From this inequality alone, we are to determine the chief elements of its excentric orbit.

I IMMEDIATELY found, that thefe differences, ftrictly taken, had irregularities which are inconfiftent with the moft remarkable circumftances of the Planet's motions. It appeared, therefore, that the obfervations muft be corrected, as far as is confiftent with the probability of their inaccuracy. With refpect to the obſervations of Dr MASKELYNE and Mr SLOP, made with inſtruments equal to any in Europe, this inaccuracy ſhould not be ſuppoſed greater than 5″. With reſpect to my own, I will allow it to amount to 10″.

THE queſtion is now, upon what good principle we may preſume to correct the obſervations. When the Planet appears ſtationary, we have the beſt opportunity of aſcertaining its diſtance from the Sun, by means of an imperfect knowledge of its angular motion, the earth's diſtance from the Sun affording a baſe moſt advantageouſly ſituated. Mr MINTO has communicated to me Mr SLOP's obſervations of the Planet when in this ſituation. On 1782, March, 6 *d.* 6 *h.* 14′. 56″. M. T. Greenwich, the apparent longitude of the Planet was obſerved 2 *s.* 28°. 49′. 27″. on the Ecliptic. The five obſerved oppoſitions give us the firſt and ſecond differences of the heliocentric motion at thoſe oppoſitions. By theſe means we obtain, by the uſual methods of interpolation, the heliocentric place of the Planet at the time of the above obſervation, and this without an error amounting to 2″. By comparing this with the geocentric place, we obtain the Planet's diſtance from the Sun = 18,9053. By making a ſimilar interpolation for March 7 *d.* 6 *h.* 14′. 56″, we obtain another heliocentric place of the Planet. The difference of theſe two places gives the diurnal heliocentric motion = 43″,4365. But a Planet deſcribing round the Sun a circle whoſe radius is 18, 9053, will have its diurnal motion = 43″,1647.

FROM this it is demonſtrable, that the Planet's diſtance from the Sun is greater than half the parameter of its orbit; and that its true anomaly, or diſtance from its aphelion, is more than 90° *. On the other hand, we find, from the continual acceleration of its motion, that, at the oppoſition 1785, the Planet had

* FOR the angular velocity of a body in an ellipſe, is to that of a body in a circle, at the ſame diſtance, in the ſubduplicate ratio of the half parameter to the diſtance.

had not yet arrived at its perihelion. Hence it is demonſtrable, that the differences of the arches deſcribed in equal times ſhould form a ſeries of numbers continually decreaſing, very ſlowly at firſt, but afterwards more rapidly.

UPON this principle, we may venture to correct the obſervations. In this correction there is ſtill a choice; for we may make the decreaſe of the ſeries either more or leſs rapid. The ellipſes which ariſe from the extremes of the ſeries formed upon this principle, will evidently be the limits which comprehend the principal elements of the eccentric motion; and, ſince we allow ourſelves very little liberty in the correction, it is preſumable, that theſe limits will not be very wide.

FROM the above obſervation of the Planet in its ſtationary point, we find that its angular velocity does not greatly exceed that of a Planet revolving in a circle; and a ſimilar uſe being made of Mr MASKELYNE's firſt obſervations, will ſhow, that the heliocentric motion of the Planet in April 1781 hardly exceeded the motion in a circle at the ſame diſtance. We may, therefore, preſume that its true anomaly does not much exceed 90°. Therefore, the ſeries of firſt differences, adapted to this ſituation, muſt decreaſe very ſlowly, whilſt the ſecond differences muſt increaſe alſo very ſlowly. This will appear by examining the tables of any of the Planets. I ſhall, therefore, begin by giving to the ſecond differences a very ſmall increaſe, and to the firſt differences a very ſmall diminution. This will be done by a correction not exceeding 3″ in any of the obſervations; and this muſt be allowed to be far within the limits of probability. The firſt obſervation has its longitude diminiſhed 1″; the ſecond has its longitude increaſed $2\frac{1}{2}$″; the third has its longitude increaſed by the ſame quantity, and the fourth and fifth have their longitudes increaſed 3″. The times correſponding to the above mentioned equal intervals, and the correſponding corrected longitudes, cleared from the effects of aberration and mutation, and reduced to the orbit, and to the epoch of oppofition 1783, are as follow:

	M. T. Green.	
	h. ′ ″	s. ° ′ ″
1781. Dec. 21.	17. 20. 17	3. 00. 53. 50
1782. Dec. 26.	09. 09. 45	3. 05. 21. 16,5
1783. Dec. 31.	00. 59. 13	3. 09. 50. 37,5
1785. Jan. 3.	16. 48. 41	3. 14. 21. 52
1786. Jan. 8.	08. 38. 09	3. 18. 54. 58

THESE give us the following intercepted arches, with their firft and fecond differences:

° ′ ″	′ ″	″
4. 27. 26,5		
	1. 54,5	
4. 29. 21		1
	1. 53,5	
4. 31. 14,5		2
	1. 51,5	
4. 33. 06		

FROM thefe data, the elliptical orbit of the Planet is to be conftructed. Various methods prefent themfelves of doing this, depending on the equations between the mean and true anomaly. But I found that, unlefs the quantities involving the fourth power of the excentricity were introduced into the equation, I could not determine the place of the aphelion with tolerable accuracy. The equation in this form would be almoft intractable. I therefore fearched for a method which would be more fimple, when applied to the prefent cafe, which has been rendered fo particular, by the determination already obtained of the quarter of the orbit in which the Planet has been obferved. The following method occurred to me, and is, indeed, as obvious as it is fimple, while it is alfo fufceptible of great accuracy.

LET ACP (fig. 1.) be the elliptical orbit of the Planet, P the perihelion, S the focus in which the Sun is placed, and O the centre;

centre; and let A, B, C, D, E, be the places of the Planet in its ſucceſſive oppoſitions to the Sun; draw the chords AB, BC, CD, DE, AC, CE, and the radii vectores AS, BS, CS, DS, ES. We may ſuppoſe that the points χ and γ, where the chords AC, CE, are interſected by the radii vectores, BS, DS, are in the middle of thoſe chords. For, let us ſuppoſe that thoſe chords are biſected in χ and γ by radii SB and SD, the rectilineal triangles ABS, BCS are equal, and the ſegments cut off by the chords AB, BC are very nearly equal; theſe ſegments are very ſmall in compariſon with the triangles $AB\chi$, $B\chi C$, and theſe triangles are very ſmall in compariſon with the triangles $A\chi S$, χCS. Therefore, the elliptical ſectors ABS, BCS, are very nearly equal, and B is very nearly the place of the Planet at the ſecond oppoſition.

Let the angles ASB be $= u$, BSC $= v$, CSD $= x$, DSE $= y$, ASC $= w$, CSE $= z$, $A\chi S = \chi$, and $C\gamma S = \gamma$.

Then,	$AS : A\chi$	$= \textit{ſin.}\,\chi : \textit{ſin.}\,u$,
and	$C\chi$, or $A\chi : CS$	$= \textit{ſin.}\,v : \textit{ſin.}\,\chi$.
therefore,	$AS : CS$	$= \textit{ſin.}\,v : \textit{ſin.}\,u$,
alſo,	$ES : CS$	$= \textit{ſin.}\,x : \textit{ſin.}\,y$.

Thus, we have obtained the ratio of the three diſtances AS, CS, ES, and we have the angles ASC, CSE, given by obſervation. This is all that is neceſſary for conſtructing the ellipſe, by means of the 21ſt prop. of Newton's Principia, B. I. or of a theorem to be delivered afterwards.

This ellipſe will be found to have its ſemitranſverſe axis about nineteen times the earth's diſtance from the Sun, and its excentricity about $\frac{1}{20}$ of its ſemitranſverſe axis, and the angle PSC about 73°. As it approaches very near to the form of the ellipſe really deſcribed by the Planet, we may diſcover, by its means, the errors which have ariſen from the ſuppoſition that the ſectors ASB, BSC, are equal, when $A\chi$ is equal to χB.

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For this purpofe, bifect AE in F, draw OFκ and SFc; make κc to Cc, as cS to cF; draw CφS, and draw OK parallel AE: It is evident that κc may be confidered as a ftraight line parallel to EA; the fegments EκF, FκA, are equal, and the triangles EFS, FSA, are equal; therefore the elliptical fpaces EκFS, κFSA are equal; but the triangles κcF, CcS are equal, their altitudes being reciprocally as their bafes; therefore, the elliptical fectors ACS, CSE, are equal, and C is the place of the Planet at the third oppofition. Now, cF is nearly equal to the verfed fine of cA, which is an arch of about 9°, and is therefore about $\frac{1}{80}$ of cS. κc is to cF as OK to KF; and therefore κc is nearly $\frac{1}{20}$ of cF, or $\frac{1}{1600}$ of cS. Cc is $\frac{1}{80}$ of κc, or $\frac{1}{128000}$ of cS. Therefore the angle CSc does not exceed two feconds. If a fimilar conftruction be made for the points B and D, it will be found that the angles BSb, DSd, will not exceed $\frac{1}{8}$ of a fecond. For BS, CS, DS, are nearly equal, and bH and dG are nearly $\frac{1}{4}$ of cF; therefore Bb and Dd are nearly $\frac{1}{16}$ of Cc.

Hence it is evident, that this fimple and obvious conftruction will give the elements of the orbit with all the accuracy that can be attained by any direct methods from our obfervations, becaufe the errors of obfervation are much greater than this; and if the obfervations are not equalifed according to fome probable principle, as has been attempted above, elements cannot be obtained which will be confiftent with them all. The corrections which muft be made for this equalifation are much greater than this error; and, therefore, no direct methods can give more accurate elements.

This error, fmall as it is, may be very eafily corrected, by computing its quantity in the ellipfe already conftructed. This computation

computation muſt be exceedingly near the truth, becauſe the ellipſe is very near the truth. But the trouble of this previous conſtruction may be avoided by means of the following conſiderations: The triangles $\varkappa$Fc, δGd, are nearly ſimilar; and therefore, cF : dG = $AE^2 : CE^2$ nearly; therefore the triangle $\varkappa$cF : δGd = $AE^4 : CE^4$ nearly; alſo, Sc = Sd nearly; therefore, Cc : Dd (or, φF : γG =) $AE^4 : CE^4$ nearly; but AE is nearly double of CE; therefore, φF : γG = 16 : 1 nearly; alſo, φF : χH = 16 : 1 nearly.

Now,	$CS : C\gamma = \textit{ſin.}\ \gamma : \textit{ſin.}\ x$,
and	$C\gamma : E\gamma = C\gamma : E\gamma$,
and	$E\gamma : ES = \textit{ſin.}\ y : \textit{ſin.}\ \gamma$,
therefore,	$CS : ES = C\gamma \times \textit{ſin.}\ y : E\gamma \times \textit{ſin.}\ x$.
Let	$CS : eS = \textit{ſin.}\ y : \textit{ſin.}\ x$,
then,	$ES : eS = E\gamma : C\gamma$,
and	$ES : Ee = E\gamma : C\gamma - E\gamma, = E\gamma : 2\gamma G$.

In like manner, make $CS : aS = \textit{ſin.}\ u : \textit{ſin.}\ v$, and we ſhall have $AS : Aa = A\chi : 2\chi H$ nearly, $= E\gamma : 2\gamma G$ nearly, and $Ee : Aa = ES : AS$ nearly, and therefore Ee nearly equal to Aa.

Make	$AS : So = \textit{ſin.}\ z : \textit{ſin.}\ w$,
then, (becauſe	$SE : AS = E\varphi \times \textit{ſin.}\ w : A\varphi \times \textit{ſin.}\ z$)
we have	$SE : So = E\varphi : A\varphi$,
and	$SE : Eo = E\varphi : A\varphi - E\varphi, = A\varphi : 2\varphi F$ nearly,
or	$SE : Eo = 2E\gamma : 32\gamma G, = E\gamma : 16\gamma G$ nearly.

Hence it follows that Eo is nearly equal to eight times Ee.

Laſtly, Make $aS : S\varepsilon = \textit{ſin.}\ z : \textit{ſin.}\ w$, then we ſhall have $aS : S\varepsilon = AS : So$, and $Aa : \varepsilon o = AS : SE$, and therefore εo nearly equal to Aa, or to Ee; therefore $e\varepsilon$ is nearly ſix times Ee.

Hence

HENCE may be derived the following rule for approximating to the true ratios of AS and ES to CS:

$$\begin{aligned} \text{Make}\quad CS : aS &= sin.\, u : sin.\, v, \\ CS : eS &= sin.\, y : sin.\, x, \\ eS : \alpha S &= sin.\, w : sin.\, z, \\ aS : \epsilon S &= sin.\, z : sin.\, w. \end{aligned}$$

Then make $AS = aS + \frac{a\alpha}{6}$, and $ES = eS - \frac{\epsilon e}{6}$. Then the points A, C, E, will be in the circumference of an ellipse, of which S is the focus, and O the centre, and having the sectors ASC, CSE, very nearly equal.

THE approximation will be much easier, and almost as accurate, if $\frac{1}{6}$ of the difference of the logarithms of aS and αS be added to the logarithm of aS, for the logarithm of AS, and $\frac{1}{6}$ of the difference of the logarithms of ϵS and eS be subtracted from the logarithm of eS for the logarithm of ES.

IT may even be sufficient to add $\frac{1}{6}$ of the difference of the logarithms of eS and ϵS to the logarithms of aS, and to subtract it from the logarithm of eS.

THE following Theorem may be of use for constructing the ellipse, and, I believe, is new:

LET DAP be an ellipse, (fig. 2.) of which O is the centre, S the focus, and ap the directrix; from any three points A, C, E, draw lines Aa, Cc, Ee, perpendicular to the directrix; draw the radii AS, CS, ES; draw AK, κCH, and ϵE, perpendicular to Aa, and AG, CF, perpendicular to ES, and Sp perpendicular to ap.

LET

LET AS be $= a$, CS $= c$, ES $= e$, the angle ASE $= x$, CSE $= y$, and ESP $= z$.

IT is evident that EH : EK $= \epsilon x : \epsilon A$, $=$ CS — ES : AS — ES, $= c - e : a - e$; alfo, SF $= c.cof,y$, SG $= a.cof,x$, CF $= c.fin,y$, and AG $= a.fin,x$; alfo, the angle FCH $=$ GAK, $=$ ESP, $= z$.

THEREFORE, FH $=$ CF.tan,z, $= c.fin,y.tan,z$, and GK $= a.fin,x.tan,z$; therefore, EH $= e - c.cof,y + c.fin,y.tan,z$, and EK $= e - a.cof,x + a.fin,x.tan,z$; therefore, $c - e : a - e = e - c.cof,y + c.fin,y.tan,z : e - a.cof,x + a.fin,x.tan,z$, and $(c-e).(e-a.cof,x) + (c-e).a.fin,x.tan,z = (a-e).(e-c.cof,y) + (a-e).c.fin,y.tan,z$. This gives,

$$Tan,z = \frac{(c-e).(e-a.\,cof,x) - (a-e).(e-c.\,cof,y)}{c.(a-e)\,fin,y - a.(c-e).\,fin,x}$$

Or, more conveniently for logarithms,

$$Tan,z = \frac{c.(a-e).\,cof,y - a.\,(c-e)\,cof,x - e.(a-c)}{c.(a-e).\,fin,y - a.(c-e).\,fin,x}$$

Then, by the common theorems, we have the excentricity $\epsilon =$ $= \frac{a-e}{e.\,cof,z - a.\,cof,(x+z)}$, the mean diftance being $= 1$. The aphelion and perihelion diftances are $1 + \epsilon$ and $1 - \epsilon$. By their means, we obtain the mean anomalies correfponding to the true anomalies OSA and OSE. The difference of the mean anomalies is to 360°, as the time between the appulfes of the Planet to the points A and E to the time of a fydereal revolution. The fquare of a fydereal year is to the fquare of the time of this revolution, as 1 to the cube of the Planet's mean diftance from the Sun.

This procefs gives us the following elements:

Mean Diftance, - -	19,08247
Excentricity, - -	0,9006
Periodic Time, - -	83,359 Years.

			s. ° ′ ″
Mean Anomaly at E,			4. 00. 32. 51
Longitude of the Aphelion,	for the Epoch		11. 23. 09. 51
Longitude of the Node,	1783, Dec. 31.		2. 12. 46. 14
Inclination of the Orbit,	-	-	00. 00. 46. 25

THESE elements agree with all the obſervations made ſince Mr HERSCHEL's diſcovery of the Planet, with abundant accuracy, the differences being as often, and as much in defect as in exceſs. When I compared them with MAYER's obſervation of the Star, No. 964. I found the calculated place of the Planet only 3′. 52″ to the weſtward of the Star, and 1″ to the northward. As theſe elements ſeem to be formed on good principles, I cannot help being of opinion, that that Star was the Planet now obſerved. If, in forming the elements, I had ſuppoſed that the ſecond differences of the arches were conſtant, (a ſuppoſition quite allowable,) I ſhould have obtained elements almoſt preciſely the ſame with thoſe which I formerly deduced from the ſuppoſition that the Star, No. 964. of MAYER's Catalogue, was the Planet. This aſſumption would not have occaſioned an alteration of one ſecond in any of the places above uſed.

ALTHOUGH it now appeared unneceſſary to make any farther trial, I made another correction of the obſervations, ſo as to produce a ſeries of ſecond differences, which ſhould decreaſe as rapidly as was conſiſtent with the probable inaccuracy of the obſervations. This gave me the following elements:

Mean Diſtance,	- -	19,18254
Excentricity,	- -	0,88461
		s. ° ′ ″
Mean Longitude,	1786 Jan. 1.	3. 23. 17. 03
Long. of Aphelion,	M. T. Green.	11. 17. 32. 54
		y. d. h. ′
Periodic Time,		84. 06. 04. 48

THESE elements alſo agreed very well with the obſervations ſince HERSCHEL's diſcovery; as alſo with MAYER's obſervations: But if theſe elements be compared with the obſervation of the ſtation ftation in March 1782, they produce an angular motion, which differs confiderably from what appears by interpolation, fhowing that the mean diftance is confiderably too great.

It refults from this inveftigation, that the elements of the orbit are contained between thefe extremes, and are probably much nearer to the firft fet. A confiderable time muft elapfe before they can be determined with accuracy, from obfervations made fince March 1781. But the probability that Mayer obferved the Planet is fo great, that I am decidedly of opinion that it is the fame with No. 964. of his catalogue. If this be granted, we can obtain the elements with all the accuracy that is attained in the other Planets: For the place of Mayer's Star is within fix degrees of the Aphelion, as determined by the firft fet of elements, and all the effects of its excentricity are nearly accumulated in 1781; and are therefore moft eafily deduced from the obfervations. I fhall therefore fubjoin another fet of elements accommodated to this fuppofition; they were formedby me about two years ago in the ufual way, by repeated trial, till the refult fhould agree with Mayer's obfervations, and with all the others which I had then collected. I have not found any reafon fince that time to make any change, unlefs perhaps the inclination of the orbit may be increafed about 10″.

Mean Diftance, - -	19,0858
Excentricity, - -	0,90737
Mean Longitude, 1786 Jan. 1. Noon. M. T. Green.	s. ° ′ ″ 3. 23. 41. 13
Longitude of the Aphelion, -	11. 23. 10. 38
Longitude of the Node, -	2. 12. 48. 45
Inclination of the Orbit, -	00. 00. 46. 26
Periodic Time in Days,	d. h. ′ ″ 30456. 01. 40. 48
Mean diurnal Motion,	42″,551

I may juft obferve in this place, that if I were difpofed, with fome aftronomers, to admit that the Star, No. 34. Tauri of the Britannic Catalogue, is the new Planet, the elements formed on

 the

the ſuppoſition of the moſt rapid decreaſe of the ſecond differences will agree very well with FLAMSTEAD's obſervation of that Star on December 13. 1690, being only 40″, or perhaps only 12″, to the weſtward of it. But the latitude differs more than two minutes from FLAMSTEAD's latitude, which is rightly deduced from the Zenith diſtance. This is too great an error for him to commit in the obſervation, and we ſhould therefore reject the ſuppoſition on this account alone. But there are ſtronger reaſons for rejecting it, ariſing from the diſagreement of thoſe elements with the obſervations made on the ſtations of the Planet in October 1781, and March and October 1782, which give us a very near approximation to its diſtance from the Sun. When compared with obſervations of the Planet near its ſtationary points in the Spring, they give the geocentric longitude conſiderably too great, while they give it too ſmall for the ſimilar obſervations in Autumn.

THE appearance of this Planet has ſerved to exerciſe the ingenuity of mathematicians, by a problem conſiderably different from that afforded by the motions of comets in very excentric orbits; and, by this means, has favoured the public with many improvements in analytical knowledge. My profeſſional duty has made me confine myſelf chiefly to the ſearch of ſuch methods as might be very intelligible to perſons poſſeſſed of ſmall degrees of mathematical knowledge. The method now exhibited has this advantage in an eminent degree; and therefore, although it will not engage the attention of ſkilful mathematicians, I hope it will be uſeful, becauſe it may incite beginners to a zealous proſecution of this noble ſtudy, by ſhowing them ſome of its moſt pleaſing gratifications. I may add, that the method now exhibited is one of the moſt likely to give us an accurate knowledge of the Planet's motion. Another period of four years will enable us to apply it to arches of double extent, which will diminiſh the errors ariſing from the unavoidable inaccuracy of obſervations to one fourth of their preſent quantity, and a compariſon of the new elements with thoſe now given,

given, will enable us to diminiſh them as much again. When it is conſidered, that in thoſe elements no attention has been paid to the gravitation of the Planet to the other ſix, it will ſtill more clearly appear how abundantly accurate they are for the purpoſes of aſtronomical computation.

I TOOK another method of obtaining elements, by means of the ratio of three diſtances from the Sun; namely, by interpolating heliocentric places of the Planet, for the times of its vicinity to its ſtations, and comparing theſe with its geocentric places. It is eaſy to ſee, that this method alſo is ſuſceptible of great accuracy, after having obſerved five oppoſitions, which give us ſecond and third differences of the heliocentric places, and therefore afford a proper application of the methods of interpolation. Elements deduced in this way, almoſt perfectly coincided with the above. I alſo obtained, in January 1784, a ſet of elements very nearly the ſame, by means of the three oppoſitions which had then been obſerved, and by the help of a theorem which I make uſe of in my elements of phyſical aſtronomy, *viz.* That the velocity of a body, in any point of the path which it deſcribes by the action of a centripetal force, is that which it would acquire if uniformly impelled by the centripetal force along $\frac{1}{4}$ of that chord of the oſculating circle which paſſes through the centre of forces.

I SHALL here ſubjoin tables for computing the motion of this Planet.

TABLE I. contains the Radical Mean Longitudes of the Planet, Aphelion, and Node; for the Mean Time of noon at Greenwich, at the beginning of the Aſtronomical Year, that is, for the Mean Noon of the 31ſt of December immediately preceding. It alſo contains the Mean Sydereal Motions of the Planet for months, days, and hours, and the preceſſion of the Equinoxes at the beginning of each month. The ſydereal motions are choſen in preference to the tropical, becauſe the motions of the aphelion and node are not yet known. One application of the preceſſion of the equinoctial points, is therefore ſufficient.

TABLE

TABLE II. contains the Elliptic Equation of the Planet. The argument is the Mean Anomaly, or the Mean Longitude of the Planet—the Longitude of the Aphelion.

TABLE III. contains the Logarithm of the Planet's diſtance from the Sun, the Earth's mean diſtance being 1. The argument is the Mean Anomaly of the Planet.

TABLE IV. contains the Heliocentric Latitude of the Planet, the Reduction to the Ecliptic, and the Reduction of the Logarithm of the diſtance from the Sun. The argument is the Orbital Longitude of the Planet—the Longitude of the Node.

TABLE V. contains the Geocentric Aberration of the Planet, for reducing its true to the apparent place. The argument is the Elongation of the Planet from the Sun.

E X A M P L E.

REQUIRED the heliocentric place of the Planet for 1787, January 13 *d.* 04 *h.* 56′ 00″ M. T. Greenwich.

	s. ° ′ ″		s. ° ′ ″		s. ° ′ ″
1787. M. Lon. Plan.	3. 28. 00. 12,5	Lon. Aphel.	11. 23. 11. 28	Lon. Nod.	2. 12. 49. 35
Jan. } M. Mot.	0. 00. 00. 00		3. 28. 09. 35		3. 23. 32. 35
13.	9. 13,2				
4	7,1	M. An.	4. 04. 58. 07	Arg. Lat.	1. 10. 43. 00
56′	1,7			Hel. Lat. N.	30. 15
		Log. diſt. ⊙	1.2694179		
	3. 28. 09. 34,5	Red. Log.	168		
Eq. Orbit,	4. 36. 59,3				
		Log. curt. diſt.	1. 2694011		
	3. 23. 32. 35,2				
Prec.—Red.	7,4				
Plan. for M. Eq^x.	3. 23. 32. 27,8				

IT will be remarked, that the deviations from obſervations made near the vernal ſtations are in defect, while thoſe near the autumnal ſtations are in exceſs. Hence it may be preſumed, that the mean diſtance and periodic time are ſomewhat too ſmall, and the aphelion too far advanced on the ecliptic. I did not remark this till after I had computed the tables; and it is a tedious taſk to make the computation a-new. I have publiſhed them, not in the perſuaſion that they are perfect, but becauſe none have as yet been publiſhed in Britain, and I have ſeen only thoſe of DE LA PLACE and ORIANI, both of which are leſs conſiſtent with obſervations than mine.

TABLE I.

Radical mean Places, and Motions.

	M. Lon. Plan.	Lon. Aphel.	Lon. Node.
	s ° ' "	s ° ' "	s ° ' "
1756	11. 13. 43. 43,1	11. 22. 25. 48	2. 12. 23. 35
1781	3. 02. 01. 16,5	10. 23. 06. 26	2. 12. 44. 34
1782	3. 06. 20. 59,0	11. 23. 07. 16	2. 12. 45. 24
1783	3. 10. 40. 41,0	11. 23. 08. 07	2. 12. 46. 14
1784	3. 15. 00. 23,0	11. 23. 08. 57	2. 12. 47. 05
1785	3. 19. 20. 48,0	11. 23. 09. 48	2. 12. 47. 55
1786	3. 23. 40. 30,5	11. 23. 10. 38	2. 12. 48. 45
1787	3. 28. 00. 12,5	11. 23. 11. 28	2. 12. 49. 35
1788	4. 02. 19. 54,7	11. 23. 12. 19	2. 12. 50. 26
1789	4. 06. 40. 19,5	11. 23. 13. 09	2. 12. 51. 16
1790	4. 11. 00. 01,7	11. 23. 13. 59	2. 12. 52. 06
1791	4. 15. 19. 23,9	11. 23. 14. 50	2. 12. 52. 57
1792	4. 19. 39. 06,1	11. 23. 15. 40	2. 12. 53. 47
1793	4. 23. 29. 30,9	11. 23. 16. 31	2. 12. 54. 32
1794	4. 28. 19. 13,1	11. 32. 17. 21	2. 12. 55. 28
1795	5. 02. 38. 55,3	11. 23. 18. 12	2. 12. 56. 18

D.	M. Mot.	H.	Mot.
	' "		"
1	0. 42,5	1	1,8
2	1. 25,1	2	3,6
3	2. 07,7	3	5,3
4	2. 50,2	4	7,1
5	3. 32,7	5	8,9
6	4. 15,3	6	10,6
7	4. 57,9	7	12,4
8	5. 40,5	8	14,2
9	6. 23,0	9	16,0
10	7. 05,6	10	17,7
11	7. 48,1	11	19,5
12	8. 30,7	12	21,3
13	9. 13,2	13	23,1
14	9. 55,8	14	24,8
15	10. 38,3	15	26,6
16	11. 20,9	16	28,4
17	12. 03,4	17	30,1
18	12. 46,0	18	31,9
19	13. 28,5	19	33,7
20	14. 11,1	20	35,5
21	14. 53,6	21	37,2
22	15. 36,2	22	39,0
23	16. 18,7	23	40,8
24	17. 01,3	24	42,5
25	17. 43,8	25	
26	18. 26,4	26	
27	19. 08,9	27	
28	19. 51,5	28	
29	20. 34,0	29	
30	21. 16,6	30	
31	21. 59,1	31	

Month.	M. Motion.	P. Eq.
	° ' "	"
Jan. 0	00. 00. 00,0	0,0
Feb. 0	00. 21. 59,1	4,3
Mar. 0	00. 41. 50,6	8,3
Apr. 0	1. 03. 49,8	12,5
May, 0	1. 25. 06,4	16,7
June, 0	1. 47. 05,5	20,9
July, 0	2. 08. 22,1	25,1
Aug. 0	2. 30. 21,3	29,3
Sept. 0	2. 52. 20,4	33,6
Oct. 0	3. 13. 37,0	37,8
Nov. 0	3. 35. 36,1	42,0
Dec. 0	3. 56. 52,8	46,1

N. B. In taking out the M. Mot. for any day in a leap year, after the 29th of February, reckon one day more.

TAB. II. Elliptical Equation.

Arg. M. An.

	O		I.		II.		
	—	Diff.	—	Diff.	—	Diff.	
°	° ′ ″	′ ″	° ′ ″	′ ″	° ′ ″	′ ″	°
0	0. 00. 00,0		2. 35 22,4		4. 34. 36,5		30
1	0. 05. 23,0	5. 23,0	2. 40. 07,1	4. 44,7	4. 37. 34,2	2. 57,7	29
2	0. 10. 46,0	5. 23,0	2. 44. 49,6	4. 42,5	4. 40. 27,4	2. 53,2	28
3	0. 16. 08,8	5. 22,8	2. 49. 29,5	4. 39,9	4. 43, 15,8	2. 48,4	27
4	0. 21. 31,3	5. 22,5	2. 54. 06,7	4. 37,2	4. 45. 59,5	2. 43,7	26
5	0. 26. 53,5	5. 22,2	2. 58. 41,2	4. 34,5	4. 48. 38,4	2. 38,9	25
6	0. 32. 15,4	5. 21,9	3. 03. 12,9	4. 31,7	4. 51. 12,6	2. 34,2	24
7	0. 37. 36,7	5. 21,3	3. 07. 41,7	4. 28,8	4. 53. 41,7	2. 29,1	23
8	0. 42. 57,4	5. 20,7	3. 12. 07,5	4. 25.8	4. 56. 05,7	2. 24,0	22
9	0. 48. 17,5	5. 20,1	3. 16. 30,3	4. 22,8	4. 58. 24,9	2. 19,2	21
10	0. 53. 36,8	5. 19,3	3. 20. 49,9	4. 19,6	5. 00. 39,0	2. 14,1	20
11	0. 58. 55,3	5. 18,5	3. 25. 06,3	4. 16,4	5. 02. 47,9	2. 08,9	19
12	1. 04. 12,9	5. 17,6	3. 29. 19,5	4. 13,2	5. 04. 51,7	2. 03,8	18
13	1. 09. 29,5	5. 16,6	3. 33. 29,3	4. 09,8	5. 06. 50,2	1. 58,5	17
14	1. 14. 45,0	5. 15,5	3. 37. 35,7	4. 06,4	5. 08. 43,4	1. 53,2	16
15	1. 19. 59,4	5. 14,4	3. 41. 38,7	4. 03,0	5. 10. 31,3	1. 47,9	15
16	1. 25. 12,6	5. 13.2	3. 45. 38,0	3. 59,3	5. 12. 13,9	1. 42,6	14
17	1. 30. 24,4	5. 11,8	3. 49. 33,8	3. 55,8	5. 13. 51,0	1. 37,1	13
18	1. 35. 34,9	5. 10,5	3. 53. 25,8	3. 52,0	5. 15. 22,7	1. 31,7	12
19	1. 40. 43,9	5. 09,0	3. 57. 14,1	3. 48,3	5. 16. 48,8	1. 26,1	11
20	1. 45. 51,3	5. 07,4	4. 00. 58,4	3. 44,3	5. 18. 09,5	1. 20,7	10
21	1. 50. 57,0	5. 05,7	4. 04. 39,0	3. 40,6	5. 19. 24,5	1. 15,0	9
22	1. 56. 01,1	5. 04,1	4. 08. 15,5	3. 36,5	5. 20. 33,9	1. 09,4	8
23	2. 01. 03,4	5. 02,3	4. 11. 48,0	3. 32,5	5. 21. 37,7	1. 03,8	7
24	2. 06. 03,7	5. 00,3	4. 15. 16,4	3. 28,4	5. 22. 35,7	0. 58,0	6
25	2. 11. 02,2	4. 58,5	4. 18. 40,6	3. 24,2	5. 23. 28,0	0. 52,3	5
26	2. 15. 58,5	4. 56,3	4. 22. 00,5	3. 19,9	5. 24. 14,6	0. 46,6	4
27	2. 20. 52,8	4. 54,3	4. 25. 16,0	3. 15,5	5. 24. 55,3	0. 40,7	3
28	2. 25. 44,9	4. 52,1	4. 28. 27,2	3. 11,2	5. 25. 30,2	0. 34,9	2
29	2. 30. 34,8	4. 49,9	4. 31. 34,2	3. 07,0	5. 25. 59,3	0. 29,1	1
30	2. 35. 22,4	4. 47,6	4. 34. 36,5	3. 02,3	5. 26. 22,4	0. 23,1	0
	+		+		+		
	XI.		X.		IX.		

TAB. II. ELLIPTICAL EQUATION.

Arg. M. An.

	III.		IV.		V.		
	—	Diff.	—	Diff.	—	Diff.	
°	° ′ ″	′ ″	° ′ ″	′ ″	° ′ ″	′ ″	°
0	5. 26. 22,4	0. 17,4	4. 51. 23,4	2. 42,8	2. 52. 12,4	5. 08,1	30
1	5. 26. 39,8	0. 11,4	4. 48. 40,6	2. 48,4	2. 47. 04,3	5. 12,0	29
2	5. 26. 51,2	0. 05,4	4. 45. 52,2	2. 53,8	2. 41. 52,3	5. 15,3	28
3	5. 26. 56,6	0. 00,6	4. 42. 58,4	3. 00,1	2. 36. 37,0	5. 18,7	27
4	5. 26. 56,0	0. 08,6	4. 39. 58,3	3. 05,0	2. 31. 18,3	5. 21,9	26
5	5. 26. 48,4	0. 11,5	4. 36. 53,3	3. 10,9	2. 25. 56,4	5. 25,1	25
6	5. 26. 36,9	0. 18,6	4. 33. 42,4	3. 16,4	2. 20. 31,3	5. 28,1	24
7	5. 26. 18,3	0. 24,6	4. 30. 26,0	3. 21,7	2. 15. 03,2	5. 31,0	23
8	5. 25. 53,7	0. 30,7	4. 27, 04,3	3. 27,3	2. 09. 32,2	5. 33,8	22
9	5. 25. 23,0	0. 36,8	4. 23. 37,0	3. 32,5	2. 03. 58,4	5. 36,5	21
10	5. 24. 46,2	0. 42,7	4. 20. 04,5	3. 38,0	1. 58. 21,9	5. 39,0	20
11	5. 24. 03,5	0. 48,9	4. 16. 26,5	3. 43,2	1. 52. 42,9	5. 41,5	19
12	5. 23. 14,6	0. 55,2	4. 12. 43,3	3. 48,3	1. 47. 01,4	5. 43,7	18
13	5. 22. 19,4	1. 00,7	4. 08. 55,0	3. 53,4	1. 41. 17,7	5. 46,0	17
14	5. 21. 18,7	1. 07,2	4. 05. 01,6	3. 58,6	1. 35. 31,7	5. 47,9	16
15	5. 20. 11,5	1. 13,1	4. 01. 03,0	4. 03,4	1. 29. 43,6	5. 50,0	15
16	5. 18. 58,4	1. 19,1	3, 56. 59,6	4. 08,4	1. 23. 53,6	5. 51,9	14
17	5. 17. 39,3	1. 25,3	3. 52. 51,2	4. 13,2	1. 18. 01,7	5. 53,5	13
18	5. 16. 14,0	1. 31,5	3. 48. 38,0	4. 18,0	1. 12. 08,2	5. 55,1	12
19	5. 14. 42,5	1. 37,3	3. 44. 20,0	4. 22,7	1. 06. 13,1	5. 56,6	11
20	5. 13. 05,2	1. 43,3	3. 39. 57,3	4. 27,2	1. 00. 16,5	5. 57,9	10
21	5. 11. 21,9	1. 49,6	3. 35. 30,1	4. 31,7	0. 54. 18,6	5. 59,0	9
22	5. 09. 32,3	1. 55,3	3. 30. 58,4	4. 36,2	0. 48. 19,6	6. 00,2	8
23	5. 07. 37,0	2. 01,4	3. 26. 22,2	4. 40,6	0. 42. 19,4	6. 01,0	7
24	5. 05. 35,6	2. 07,4	3. 21. 41,6	4. 44,8	0. 36. 18,4	6. 01,9	6
25	5. 03, 28,2	2. 12,9	3. 16. 56,8	4. 48,9	0. 30. 16,5	6. 02,5	5
26	5. 01. 15,3	2. 19,5	3. 12. 07,9	4. 53,0	0. 24. 14,0	6. 03,0	4
27	4. 58. 55,8	2. 24,8	3. 07. 14,9	4. 57,0	0. 18. 11,0	6. 03,5	3
28	4. 56. 31,0	2. 31,0	3. 02. 17,9	5. 00,8	0. 12. 07,5	6. 03,7	2
29	4. 54. 00,0	2. 36,6	2. 57. 17,1	5. 04,7	0. 06. 03,8	6, 03,8	1
30	4. 51. 23,4		2. 52. 12,4		0. 00. 00,0		0
	+		+		+		
	VIII.		VII.		VI.		

T A B. III. Logarithm of the PLANET's Diſtance from the SUN.

Arg. M. An.

	O		I.		II.		
°	Logar.	Diff.	Logar.	Diff.	Logar.	Diff.	°
0	1.3008817		1.2984548		1.2916063		30
1	1.3008790	27	1.2982929	1619	1.2913149	2914	29
2	1.3008708	82	1.2981260	1669	1.2910200	2949	28
3	1,3008571	137	1.2979542	1718	1.2907217	2983	27
4	1.3008379	192	1.2977775	1767	1.2904202	3015	26
5	1.3008133	246	1.2975959	1816	1.2901155	3047	25
6	1.3007833	300	1.2974094	1865	1.2898077	3078	24
7	1.3007478	355	1.2972182	1912	1.2894968	3109	23
8	1.3007068	410	1.2970224	1959	1.2891828	3140	22
9	1.3006604	464	1.2968218	2006	1.2888659	3169	21
10	1.3006085	519	1.2966165	2053	1.2885462	3197	20
11	1.3005512	573	1.2964065	2100	1.2882237	3225	19
12	1.3004886	626	1.2961920	2145	1.2878985	3252	18
13	1.3004206	680	1.2959730	2190	1.2875708	3277	17
14	1.3003472	734	1.2957495	2235	1.2872406	3302	16
15	1.3002684	788	1.2955216	2279	1.2869080	3326	15
16	1.3001842	842	1.2952892	2324	1.2865730	3350	14
17	1.3000947	895	1.2950525	2367	1.2862358	3372	13
18	1.2999999	948	2.2948116	2409	1.2858964	3394	12
19	1.2998998	1001	1.2945664	2452	1.2855550	3414	11
20	1.2997945	1053	1.2943170	2494	1.2852116	3434	10
21	1.2996839	1106	1.2940635	2535	1.2848662	3454	9
22	1.2995679	1160	1.2938060	2575	1.2845190	3472	8
23	1.2994467	1212	1.2935445	2615	1.2841702	3488	7
24	1.2993203	1264	1.2932790	2655	1.2838198	3504	6
25	1.2991888	1315	1.2930095	2695	1.2834680	3518	5
26	1.2990522	1366	1.2927362	2733	1.2831148	3532	4
27	1.2989105	1417	1.2924592	2770	1.2827603	3545	3
28	1.2987636	1469	1.2921786	2806	1.2824045	3558	2
29	1.2986117	1519	1.2918943	2843	1.2820476	3569	1
30	1.2984548	1569	1.2916063	2879	1.2816896	3580	0
	Logar.	Diff.	Logar.	Diff.	Logar.	Diff.	
	XI.		X.		IX.		

TAB. III. Logarithm of the PLANET's Diſtance from the SUN.

Arg. M. An.

	III.		IV.		V.		
°	Logar.	Diff.	Logar.	Diff.	Logar.	Diff.	°
0	1.2816896		1.2710423		1.2627235		30
1	1.2813308	3588	1.2707098	3325	1.2625225	2010	29
2	1.2809712	3596	1.2703799	3299	1.2623274	1951	28
3	1.2806110	3602	1.2700528	3271	1.2621382	1892	27
4	1.2802503	3607	1.2697288	3240	1.2619552	1830	26
5	1.2798891	3612	1.2694078	3210	1.2617785	1767	25
6	1.2795275	3616	1.2690899	3179	1.2616080	1705	24
7	1.2791656	3619	1.2687754	3145	1.2614439	1641	23
8	1.2788036	3620	1.2684644	3110	1.2612863	1576	22
9	1.2784416	3620	1.2681570	3074	1.2611352	1511	21
10	1.2780797	3619	1.2678533	3037	1.2609906	1446	20
11	1.2777181	3616	1.2675534	2999	1.2608526	1380	19
12	1.2773567	3614	1.2672574	2960	1.2607214	1312	18
13	1.2769958	3609	1.2669653	2921	1.2605969	1245	17
14	1.2766354	3604	1.2666773	2880	1.2604793	1176	16
15	1.2762757	3597	1.2663936	2837	1.2603686	1107	15
16	1.2759168	3589	1.2661143	2793	1.2602647	1039	14
17	1.2755589	3579	1.2658396	2747	1.2601677	970	13
18	1.2752020	3569	1.2655695	2701	1.2600777	900	12
19	1.2748461	3559	1.2653041	2654	1.2599948	829	11
20	1.2744915	3546	1.2650434	2607	1.2599190	758	10
21	1.2741384	3531	1.2647875	2559	1.2598504	688	9
22	1.2737868	3516	1.2645367	2508	1.2597888	616	8
23	1.2734367	3501	1.2642911	2456	1.2597345	543	7
24	1.2730884	3483	1.2640507	2404	1.2596874	471	6
25	1.2727420	3464	1.2638157	2350	1.2596475	399	5
26	1.2723976	3444	1.2635861	2296	1.2596148	327	4
27	1.2720553	3423	1.2633620	2241	1.2595893	255	3
28	1.2717152	3401	1.2631434	2186	1.2595711	182	2
29	1.2713775	3377	1.2629305	2129	1.2595601	110	1
30	1.2710423	3352	1.2627235	2070	1.2595564	37	0
	Logar.	Diff.	Logar.	Diff.	Logar.	Diff.	
	VIII.		VII.		VI.		

TABLE IV.

°	0. *N.* VI. *S.* Lat.	— Red.	— R. log	I. *N.* VII. *S.* Lat.	— Red.	— R. log	II. *N.* VIII. *S.* Lat.	— Red.	— R. log	°
°	′ ″.	″		′ ″	″		′ ″	″		°
0	00. 00	0	0	23. 12	8	99	40. 12	8	297	30
1	00. 49	0	0	23. 54	8	105	40. 36	8	303	29
2	1. 37	1	1	24. 36	8	111	40. 59	8	309	28
3	2. 26	1	1	25. 17	9	117	41. 21	8	314	27
4	3. 14	1	2	25. 57	9	124	41. 43	7	320	26
5	4. 03	2	3	26. 37	9	130	42. 04	7	325	25
6	4. 51	2	4	27. 17	9	137	42. 24	7	330	24
7	5. 39	2	6	27. 56	9	143	42. 43	7	335	23
8	6. 28	3	8	28. 35	9	150	43. 02	7	340	22
9	7. 16	3	10	29. 13	9	157	43. 20	6	345	21
10	8. 04	3	12	29. 50	9	164	43. 37	6	349	20
11	8. 51	4	14	30. 27	9	170	43. 53	6	354	19
12	9. 39	4	17	31. 03	9	177	44. 08	6	358	18
13	10. 26	4	20	31. 29	9	184	44. 23	5	362	17
14	11. 14	4	23	32. 14	9	191	44. 37	5	366	16
15	12. 01	5	26	32. 49	9	198	44. 50	5	369	15
16	12. 48	5	30	33. 23	9	205	45. 02	4	372	14
17	13. 34	5	33	33. 57	9	211	45. 13	4	376	13
18	14. 21	6	38	34. 30	9	219	45. 24	4	379	12
19	15. 07	6	42	35. 02	9	226	45. 34	4	381	11
20	15. 53	6	46	35. 33	9	233	45. 43	3	384	10
21	16. 38	6	51	36. 04	9	239	45. 51	3	386	9
22	17. 23	7	55	36. 34	9	246	45. 58	3	389	8
23	18. 08	7	61	37. 04	9	253	46. 04	2	390	7
24	18. 53	7	65	37. 33	9	259	46. 10	2	392	6
25	19. 37	7	71	38. 01	9	265	46. 15	2	393	5
26	20. 21	7	76	38. 29	9	272	46. 18	1	394	4
27	21. 04	8	82	38. 56	9	278	46. 21	1	394	3
28	21. 47	8	88	39. 22	8	285	46. 23	1	394	2
29	22. 30	8	93	39. 47	8	291	46. 25	0	395	1
30	23. 12	8	99	40. 12	8	297	46. 26	0	396	0
	XI. *S.* V. *N.*	+	—	X. *S.* IV. *N.*	+	—	IX. *S.* III. *N.*	+	—	

TAB. V.

Elong.		Ab.
		″
0.	00	—24
	10	—23
	20	—21
I.	00	—19
	10	—17
	10	—15
II.	20	—13
	10	—10
	20	— 7
III.	00	— 3
	10	— 0
	20	+ 3
IV.	00	+ 6
	01	+ 9
	02	+ 11
V.	00	+ 12
	10	+ 14
	20	+ 15
VI.	00	+ 16